5

Automobile Ownership

The car has become an article of dress without which we feel uncertain, unclad, and incomplete.

Marshall McLuhan, Canadian Educator and Philosopher

What do you think Marshall McLuhan meant in his quote?

What do you think?

Answers might include that the car is much more than a means of transportation. It has become a mode of self-expression as well as a mode of transportation. People pride themselves in automobile ownership; many even see it as a status symbol.

TEACHING RESOURCES

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The automobile is part of the American way of life. Many people commute to jobs that require them to own a car. Some students drive several miles to school. Stores and businesses are clustered in central locations often not near residential neighborhoods. When there is no mass transit system readily available to you, an automobile can provide convenient and necessary transportation.

Owning an automobile is a tremendous responsibility. The costs of gas, repairs, and insurance are high. Driving an automobile can also be dangerous. As a driver, you have a responsibility to yourself, your passengers, pedestrians, and other motorists. So, before embarking upon that first automobile purchase, you need to be aware of the physics and finances of operating a car. Being equipped with this knowledge will make your years on the road safer, less expensive, and more enjoyable.

Really?

How much does it cost to fill your car's gas tank today? Did your parents ever tell you stories about gas prices when they were young? Can you imagine people in gas lines in 1973, furious that gas prices had risen to over 50 cents per gallon?

The table shows the average price per gallon of gasoline from 1950–2005. Gas prices vary from region to region. They even differ from gas station to gas station, depending on the services the station provides and the neighborhood in which it is. Therefore, use the table as a general guide to gas prices.

Imagine what it would cost to fill a tank in any of the years listed in the table. Imagine what new cars cost! The first Corvette, the 1953 model, had a base price of $3,498. There were only 300 of these cars manufactured. It cost about $5 to fill its 18-gallon gas tank! The 1953 Corvette buyer had an easy time picking a color. The car came in one color only—white.

Year	Price per Gallon ($)
1950	0.27
1955	0.30
1960	0.31
1965	0.31
1970	0.35
1975	0.53
1980	1.13
1985	1.19
1990	1.13
1995	1.14
2000	1.66
2005	2.33

Source: NBC

CHAPTER OVERVIEW

This chapter offers 9 lessons pertaining to the automobile. Students explore formulas of varying degrees of mathematical sophistication as they work on pricing structures, insurance issues, automobile depreciation, and data that can assist them in making wise and safe driving decisions.

REALLY? REALLY!

The variability of gasoline prices has been of interest over the past few years. Looking at the table, students will notice a slow and small increase in prices in the early years. The seventies marked an era of increased automobile consumption and worldwide awareness of the power that oil ownership held in international relations. Prices have continued to rise. The inclusion of the data on the first Corvette, a highly desirable car, underscores how much things have changed in a relatively short period of time.

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Really!

Classified Ads

Objectives
- Compute the cost of classified ads for used cars.
- Compute the cost of sales tax on automobiles.

Key Terms
- sales tax
- domain
- piecewise function
- split function
- cusp

EXAMINE THE QUESTION

Classified ads provide a central place where buyers and sellers can be matched up according to what they want. They can then negotiate, agree on a price, and exchange their wares.

CLASS DISCUSSION

If you were looking for a 2009 Mustang, imagine trying to find someone who was selling one without using a classified ad.

What are some other options you might be looking for? Do you know their abbreviations?

HOW DO BUYERS AND SELLERS USE CLASSIFIED ADS FOR AUTOMOBILES?

Most teenagers cannot wait to get their own set of "wheels." New cars are expensive, so many people buy used cars when they purchase their first car. They can buy used cars from a dealer or by looking at the classified ads in the newspaper or on the Internet.

Classified ads in newspapers use abbreviations to save space and lower the cost of the ad. Take a look at your local newspaper's classified ad section and see how many of the abbreviations you understand.

Words such as *mint* and *immaculate* are often used to describe cars in excellent condition. A car with many options is often listed as *loaded*. The number of thousands of miles the car has been driven is abbreviated as K. An ad that says "34K" tells you that the car has been driven a total of 34,000 miles. Take a look at some other abbreviations used in classified ads for used cars.

ac	air conditioning
auto	automatic transmission
cruise	cruise control
CD	compact disc player
cyl	number of cylinders
dr	number of doors
GPS	navigation system
lthr	leather interior
p/ant	power antenna
p/locks	power door locks
p/mirrors	power mirrors
p/seats	power seats
ps	power steering
pw	power windows

The asking price is usually given in the advertisement. *Negotiable* means that the seller is willing to bargain with you. *Firm* means that the owner is unwilling to change the price. *Sacrifice* means that the seller needs to sell the car quickly and believes that the price is lower than the car's worth.

By knowing what these expressions mean, you will be able to skim the classified ads and focus on the ones that describe the used car that would be best for you.

Skills and Strategies

Here you will learn some of the steps that may be involved when buying or selling a used car. You can contact your state's Department of Motor Vehicles to find specific information about cars in your state. In some states the buyer of a used car must pay a **sales tax** on the car.

EXAMPLE 1

Kerry purchased a used car for $7,400 and had to pay $8\frac{1}{2}$% sales tax. How much tax did she pay?

SOLUTION   To find the sales tax, multiply the price of the item by the sales tax rate, expressed as a decimal.

$$\text{Sales tax} = \text{Price of item} \times \text{Sales tax rate}$$
$$= 7,400 \times 0.085 = 629.00$$

Kerry must pay $629.00 in sales tax. This money goes to the state, not the seller of the car. Be sure you consider the sales tax expense on a car you are planning to purchase. It can be thousands of dollars on a new car.

■ CHECK YOUR UNDERSTANDING

The sales tax rate in Mary Ann's state is 4%. If she purchases a car for x dollars, express the total cost of the car with sales tax algebraically.

EXAMPLE 2

The cost of a classified ad is determined by its length. John plans to sell his car and places a 5-line ad. The newspaper charges $31 for the first two lines and $6 per extra line to run the ad for one week. What will John's ad cost to run for two weeks?

SOLUTION   Subtract to find the number of lines over 2 lines.

$$5 - 2 = 3$$

Multiply 3 by $6 to find the cost of the extra 3 lines.

$$3(6) = 18$$

Add to find the cost of running the ad for one week.

$$31 + 18 = 49$$

Multiply by 2 to get the cost for the two-week ad.

$$49(2) = 98$$

The ad will cost John $98.

■ CHECK YOUR UNDERSTANDING

Ramon plans to sell his car and places an ad with x lines. The newspaper charges y dollars for the first g lines and p dollars per extra line to run the ad for a week. If $x > g$, express the cost of running the ad for a week.

Recall that the **domain** is the set of values that can be input into a function.

EXAMPLE 3

Jason works for the *Glen Oaks News* and is writing a program to compute ad costs. He needs to enter an algebraic representation of the costs of an ad. His company charges $42.50 for up to five lines for a classified ad. Each additional line costs $7. Express the cost of an ad with x lines as a function of x algebraically.

SOLUTION The algebraic representation of the classified ad cost function requires two rules. One rule is for ads with five or fewer lines and the other rule is for ads with more than five lines. You can view these two conditions as two different domains.

You will find the equation for the cost when $x \leq 5$, and then find the equation for the cost when $x > 5$. These are the two different domains.

Let $c(x)$ represent the cost of the classified ad. In this situation, x must be an integer.

If the ad has five or fewer lines, the cost is $42.50.

$$c(x) = 42.50 \text{ when } x \leq 5$$

If the ad has more than five lines, the cost is $42.50 plus the cost of the lines over 5 lines. Note that the domain is given by the inequality that follows *when* in the statement of the function.

If x is the number of lines, then the number of lines over five can be expressed as $x - 5$. These extra lines cost $7 each.

$$c(x) = 42.50 + 7(x - 5) \text{ when } x > 5$$

These two equations can be written in mathematical shorthand using a **piecewise function**. Piecewise functions are sometimes called **split functions**.

A piecewise function gives a set of rules for each domain of the function. Notice that $c(x)$ is computed differently depending on the value of x. Here $c(x)$ is expressed as a piecewise function.

$$c(x) = \begin{cases} 42.50 & \text{when } x \leq 5 \\ 42.50 + 7(x - 5) & \text{when } x > 5 \end{cases}$$

The domain is defined by the inequalities that follow *when* in the above statement.

■ CHECK YOUR UNDERSTANDING

The *Smithtown News* charges $38 for a classified ad that is 4 or fewer lines long. Each line above four lines costs an additional $6.25. Express the cost of an ad as a piecewise function.

EXAMPLE 4

Roxanne set up the following piecewise function which represents the cost of an auto classified from her hometown newspaper.

$$c(x) = \begin{cases} 41.55 & \text{when } x \leq 6 \\ 41.55 + 5.50(x - 6) & \text{when } x > 6 \end{cases}$$

If x is the number of lines in the ad, use words to express the price $c(x)$ of a classified ad from this paper.

SOLUTION Look at the two domains. Look at the function rule in the first line. The inequality $x \leq 6$ tells you that the cost is $41.55 if the number of lines is less than or equal to 6.

Next, look at the second line. The expression $x - 6$ gives the number of lines over six. That expression is multiplied by 5.50, so the cost of each extra line must be $5.50. The inequality $x > 6$ tells you that the cost is $41.55 for the first six lines, and $5.50 for each line over six lines.

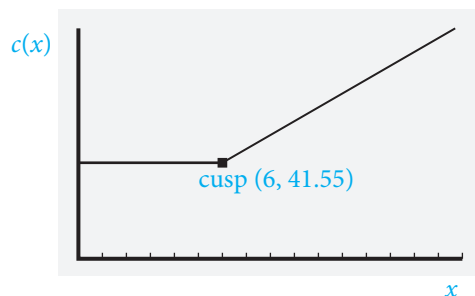■ **CHECK YOUR UNDERSTANDING**

The following piecewise function gives the price $p(w)$ of a classified ad in a classic car magazine. If w is the number of lines in the ad, use words to express the price $p(w)$ of a classified ad from this paper.

$$p(w) = \begin{cases} 60 & \text{when } w \leq 5 \\ 60 + 8(w - 5) & \text{when } w > 5 \end{cases}$$

EXAMPLE 5

Graph the piecewise function Roxanne created in Example 4.

SOLUTION Use your graphing calculator to display functions with more than one domain.

cusp (6, 41.55)

Notice that the graph is composed of two straight lines that meet at the point (6, 41.55). The point where the two lines meet is called a **cusp** because it resembles the sharp cusp on a tooth.

■ **CHECK YOUR UNDERSTANDING**

Find the cusp of the graph of the following piecewise function.

$$c(x) = \begin{cases} 42.50 & \text{when } x \leq 5 \\ 42.50 + 7(x - 5) & \text{when } x > 5 \end{cases}$$

Applications

In auto sales, appearance is everything, or almost everything. It is certainly the most important single factor in a consumer's decision to buy this or that make.

Harley Earl, Designer/Inventor of the Corvette
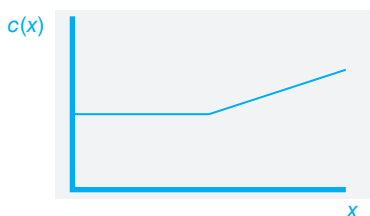
1. Interpret the quote in the context of what you learned. See margin.

2. The *North Shore News* charges $19.50 for a two-line classified ad. Each additional line costs $7. How much does a six-line ad cost? $47.50

3. The *Antique Auto News* charges $45 for a three-line classified ad. Each additional line costs $8.50. For an extra $40, a seller can include a photo in the ad. How much would a four-line ad with a photo cost? $93.50

4. A local newspaper charges g dollars for a four-line classified ad. Each additional line costs d dollars. Write an expression for the cost of a seven-line ad. $g + 3d$

5. The *Auto Times* charges g dollars for a classified ad with m or less lines. Each additional line is d dollars. If $x > m$, express the cost of an x-line ad algebraically. $g + d(x - m)$

6. Samantha purchased a used car for $4,200. Her state charges 4% tax for the car, $47 for license plates, and $35 for a state safety and emissions inspection. How much does Samantha need to pay for these extra charges, not including the price of the car? $250

7. Ralph placed a classified ad to sell his used Honda Odyssey minivan for $18,500. After two weeks, he didn't sell the minivan, and the newspaper suggested lowering the price 5%. What would the new price be if Ralph reduced it according to the suggestion? $17,575

8. The *Bayside Bugle* charges by the word to run classified ads. The newspaper charges $18 for the first 20 words and $0.35 for each additional word. How much would a 27-word classified ad cost? $20.45

9. A local newspaper charges by the character for its classified ads. Letters, numbers, spaces, and punctuation each count as one character. They charge $46 for the first 200 characters and $0.15 for each additional character.
   a. If x represents the number of characters in the ad, express the cost $c(x)$ of an ad as a piecewise function. See margin.
   b. Graph the function from part a. See margin.
   c. Find the coordinates of the cusp in the graph in part b. (200, 46)

10. The *Kings Park Register* gives senior citizens a 10% discount on classified ads. Mr. Quadrino, a senior citizen, is selling his car and wants to take out a four-line ad. The paper charges $6.50 per line. What is the price of the ad for Mr. Quadrino? $23.40

11. The *Good Ole Times* magazine charges for classified ads by the "column inch." A column inch is as wide as one column, and it is one inch high. The cost is $67 per column inch. How much would the magazine charge to print a $2\frac{1}{2}$-inch ad? $167.50

12. Leslie placed this ad in the *Collector Car Monthly*.

> 1957 Chevrolet Nomad station wagon. Tropical Turquoise, 6 cyl. auto, PS, PW, AM/FM, repainted, rebuilt transmission, restored two-tone interior. Mint! Moving, sacrifice, $52,900. 555-4231

  a. If the newspaper charges $48 for the first three lines and $5 for each extra line, how much will this ad cost Leslie? $58

  b. Ruth buys the car for 8% less than the advertised price. How much does she pay? $48,668

  c. Ruth must pay her state 6% sales tax on the sale. How much must she pay in sales tax? $2,920.08

13. The *Online Car Auctioneer* charges a commission for classified ads. If the car sells, the seller is charged 4% of the *advertised* price, not of the price for which the car actually sells. If the car doesn't sell, the seller pays nothing. If Barbara advertises her Cadillac for $12,000 and sells it for $11,200, how much must she pay for the ad? $480

14. The cost of an ad in a local paper is given by the piecewise function

$$c(x) = \begin{cases} 38 & \text{when } x \le 4 \\ 38 + 6.25(x - 4) & \text{when } x > 4 \end{cases}$$

  a. Find the cost of a three-line ad. $38

  b. Find the difference in cost between a one-line ad and a four-line ad. $0

  c. Find the cost of a seven-line ad. $56.75

  d. Graph this function on your graphing calculator. See margin.

  e. Find the coordinates of the cusp from the graph in part d. (4, 38)

15. Express the following classified ad rate as a piecewise function. Use a let statement to identify what *x* and *y* represent.

  $29 for the first five lines, and $6.75 for each additional line. See margin.

16. The piecewise function describes a newspaper's classified ad rates.

$$y = \begin{cases} 21.50 & \text{when } x \le 3 \\ 21.50 + 5(x - 3) & \text{when } x > 3 \end{cases}$$

  a. If *x* represents the number of lines, and *y* represents the cost, translate the function into words. See margin.

  b. If the function is graphed, what are the coordinates of the cusp? (3, 21.50)

17. A local *Pennysaver* charges $11 for each of the first three lines of a classified ad, and $5 for each additional line.

  a. What is the price of a two-line ad? $22

  b. What is the price of a five-line ad? $43

  c. If *x* is the number of lines in the ad, express the cost *c(x)* of the ad as a piecewise function. See margin.

18. The *Position Posted* online job website charges $15 to place a classified ad plus $2.50 for each of the first five lines, and $8 for each additional line after the fifth line. If *x* is the number of lines in the ad, write a piecewise function for the cost of the ad, *c(x)*.

$$c(x) = \begin{cases} 15 + 2.5(5) & \text{when } x \le 5 \\ 27.5 + 8(x - 5) & \text{when } x > 5 \end{cases}$$

5-2 Buy or Sell a Car

Objectives

• Compute mean, median, mode, range, quartiles, and interquartile range.

Key Terms

- statistics
- data
- measures of central tendency
- mean
- arithmetic average
- outlier
- median
- ascending order
- descending order
- skew
- resistant
- range
- quartiles
- lower quartile
- upper quartile
- subscripts
- interquartile range (IQR)
- mode
- bimodal

HOW CAN STATISTICS HELP YOU NEGOTIATE THE SALE OR PURCHASE OF A CAR?

You are planning to buy a used car. How can you tell what a reasonable price is for the car you want to buy? You can find a lot of information about used car prices on the Internet. You can also visit a used car dealer. The price of any car depends heavily on its condition and how desirable it is in the marketplace.

You will probably spend a few weeks shopping for your car. You can determine a reasonable price for a particular car by examining the prices of those and similar cars listed in classified ads.

The *Kelley Blue Book* (www.kbb.com) and Edmunds (edmunds.com) are two of many excellent sources on the Internet you can use to find the value of a used car. Ask questions as you do your research. You can contact sellers to find out about their cars. Be smart in your search, and if possible, bring a knowledgeable person with you when you go to test drive a used car.

As you search, compile a list of advertised prices for the cars you want. Then, you can use **statistics** to help analyze the numbers, or **data**, that you compile. **Measures of central tendency** are single numbers designed to represent a "typical" value for the data.

You will find less variability in the prices of new cars, because all new cars are in the same condition. The price you will pay is based on the sticker price of the car. Different dealers can give different prices, and it is best to compare deals when buying a new car.

NIKNIKON/ISTOCKPHOTO.COM

Skills and Strategies

Used car prices vary greatly, and a skilled negotiator will have an advantage when buying or selling a used car.

EXAMPLE 1

Jason wants to sell his Ford SUV. He compiles these prices from the Internet for cars similar to his: $11,000, $9,900, $12,100, $10,500, and $9,000. What is a reasonable price for Jason to consider for his SUV?

SOLUTION Jason should start by finding the **mean** or **arithmetic average** of the five prices. The mean is often called the average.

Add and then divide by 5.

$$\frac{11{,}000 + 9{,}900 + 12{,}000 + 10{,}500 + 9{,}000}{5} = 10{,}500$$

The mean is $10,500. Jason can adjust this mean price based on the condition of his car, the mileage it has on it, and the options it has.

■ CHECK YOUR UNDERSTANDING

Maxine compiled a list of these car prices: $7,500, $6,500, $5,750, $4,900, $6,250, and $4,200. Find the mean of the prices.

EXAMPLE 2

Dory is looking for a classic 1967 Firebird. She finds these prices on the Internet: $18,000, $77,000, $22,000, $21,200, $19,000, $17,500, and $22,500. She computes the mean as $28,171.43. This number doesn't seem to be a good representative of the data. How can she find a better representation?

SOLUTION There is an **outlier**—a piece of data that is extremely different than the rest of the data. When there are outliers, the mean is often not a good representation. In these cases you can use the **median**—the middle score—to best represent the data.

To find the median, arrange the values in **ascending order** (from least to greatest), or **descending order** (from greatest to least).

Pair the numbers starting from the ends of the list as shown, and circle the middle number that remains after the numbers are paired.

The median is the circled number. Notice there is the same number of scores below the median as there are above the median.

The median is $21,200. This price is a better representation of the data. When the mean of a data set is not equal to the median, the data is **skewed**.

The median is unaffected by the outlier. If the $77,000 price was $977,000, the median would remain the same. The median is **resistant** to extreme numbers.

17,500   18,000   12,000   (21,200)   22,000   22,500   77,000

■ **CHECK YOUR UNDERSTANDING**

Find the mean and median of the following prices for a used car extended warranty: $1,200, $1,650, $1,500, $2,000, $1,400, $1,850, and $1,600. Is the data skewed?

EXAMPLE 3

Find the median of the following used car prices: $6,700, $5,800, $9,100, $8,650, $7,700, and $7,800.

SOLUTION Put the numbers in ascending order. Then, pair the numbers. Since there is an even number of scores, there is no number left alone in the middle. Circle the last two numbers that were paired.
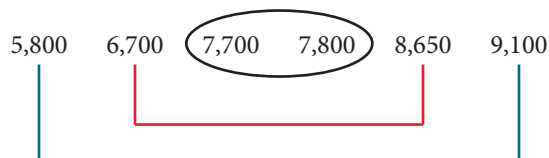
To find the median, find the mean of the two innermost circled numbers.

Add and then divide by 2. $\dfrac{7,700 + 7,800}{2} = 7,750$

The median is $7,750. Again, notice that there is the same number of scores below the median as there are above the median, and the median is resistant to extreme scores.

■ **CHECK YOUR UNDERSTANDING**

Find the median of these prices: $10,200, $9,300, $11,900, $2,999, $17,200, and $9,600.

EXAMPLE 4

Prices found online for the same GPS navigation system are $295, $345, $199, $225, and $200. Find the range of the GPS prices.

SOLUTION The **range** of a data set is a measure that shows dispersion (how spread out the data are). The range is the difference between the greatest and least numbers in the data.

The greatest price is $345 and the least is $199. The range is the difference between these two prices. Therefore, the range is $146, because 345 – 199 = 146.

■ **CHECK YOUR UNDERSTANDING**

Find the range of the used car prices in Example 3.

Quartiles

If you want to find out more about how the numbers are dispersed, you can use **quartiles**. Quartiles are three values represented by Q_1, Q_2, and Q_3 that divide the distribution into four subsets that each contain 25% of the data.

EXAMPLE 5

Find the quartiles for the tire pressures of cars at an auto clinic.

15, 17, 21, 25, 31, 32, 32, 32, 34

Tire pressure is measured in psi—pounds per square inch.

SOLUTION   The numbers are in ascending order.

- Q_1 is the first quartile or **lower quartile**, and 25% of the numbers in the data set are at or below Q_1.
- Q_2 is the second quartile. Half the numbers are below Q_2, and half are above, so Q_2 is equal to the *median.*
- Q_3 is the third quartile, or **upper quartile**, and 75% of the numbers are at or below Q_3.
- Q_4 is the *maximum value* in the data set because 100% of the numbers are at or below that number.

The **subscripts** are used to name each quartile.

To find the quartiles, first find Q_2. Because Q_2 equals the median, $Q_2 = 31$.

For Q_1, find the median of the numbers below the median, which are 15, 17, 21, and 25. The median of these numbers is $Q_1 = 19$.

Add and then divide by 2. $\dfrac{17 + 21}{2} = 19$

For Q_3, find the median of the numbers in the data set that are above the median, which are 32, 32, 32, 34. The two middle numbers are 32, so $Q_3 = 32$.

The maximum value in the data set is 34. So, $Q_4 = 34$. The quartile values are $Q_1 = 19$, $Q_2 = 31$, $Q_3 = 32$, and $Q_4 = 34$.

You can use your graphing calculator to find quartiles.

■ CHECK YOUR UNDERSTANDING

What percent of the numbers in a data set are above Q_3?

EXAMPLE 6

What is the difference between Q_1 and Q_3 from the data set in Example 5?

SOLUTION   The difference $Q_3 - Q_1$ is the **interquartile range (IQR)**. The interquartile range gives the range of the middle 50% of the numbers. A small interquartile range means that the middle 50% of the numbers are clustered together. A large interquartile range means that the middle 50% of the numbers are more spread out. To find the interquartile range, subtract. The interquartile range is $Q_3 - Q_1 = 32 - 19 = 13$.

■ **CHECK YOUR UNDERSTANDING**

Find the interquartile range for the data in Example 3.

EXAMPLE 7

Find the outliers for these tire prices:

$45, $88, $109, $129, $146, $189, $202, $218, and $545

SOLUTION The interquartile range is used to identify outliers. Outliers may occur on the lower or upper end of the data set. The numbers are in ascending order. The median, Q_2, is $146.

$$Q_1 = \frac{88 + 109}{2} = 98.5 \qquad\qquad Q_3 = \frac{202 + 218}{2} = 210$$

$$IQR = 210 - 98.5 = 111.5$$

Use $Q_1 - 1.5(IQR)$ to compute the boundary for lower outliers.

$$98.5 - 1.5(111.5) = -68.75$$

Any number below -68.75 is an outlier. There are no lower outliers.

Use $Q_3 + 1.5(IQR)$ to compute the boundary for upper outliers.

$$210 + 1.5(111.5) = 377.25$$

Any number above 377.25 is an upper outlier, so $545 is an upper outlier.

■ **CHECK YOUR UNDERSTANDING**

The store that charged $545 for a tire in Example 7 had a sale and lowered its price to $399. Is the new price an upper outlier?

EXAMPLE 8

Each year, the 880 seniors in North Shore High School vote for one of the 110 teachers to receive the annual yearbook dedication. The teacher who receives the most votes wins. Can a teacher who receives 9 votes win, if every senior votes?

SOLUTION The **mode** is the most-occurring item and is often used with non-numerical variables, such as the winning teacher.

If each of the 880 votes were split among the 110 teachers, each teacher would get 8 votes. If one teacher received 7 votes, another received 9 votes, and everyone else received 8 votes, the teacher with 9 votes would win. A set can have no mode.

If there are two modes, the set is **bimodal**.

■ **CHECK YOUR UNDERSTANDING**

Find the mode of the tire pressures from Example 5.

Applications

> *Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write.*
>
> **H.G. Wells,** English Science Fiction Author

1. Interpret the quote in the context of what you learned. See margin.

2. Find the mean, median, mode, and range for each data set given.
  a. 7, 12, 1, 7, 6, 5, 11 mean = 7; median = 7; mode = 7; range = 11
  b. 85, 105, 95, 90, 115 mean = 98; median = 95; no mode; range = 30
  c. 10, 14, 16, 16, 8, 9, 11, 12, 3 mean = 11; median = 11; mode = 16; range = 13
  d. 10, 8, 7, 5, 9, 10, 7 mean = 8; median = 8; mode = 7 and 10; range = 5
  e. 45, 50, 40, 35, 75 mean = 49; median = 45; no mode; range = 40
  f. 15, 11, 11, 16, 16, 9 mean = 13; median = 13; mode = 11 and 16; range = 7

3. Which of the data sets from Exercise 2 are skewed? b and e

4. Courtney wants to sell her grandfather's antique 1932 Ford. She begins to set her price by looking at ads and finds these prices: $24,600, $19,000, $33,000, $15,000, and $20,000. What is the mean price? $22,320

5. Five Smithtown High School students are saving up to buy their first cars. They all have after-school jobs, and their weekly salaries are listed in the table.

Emily	$110
Sam	$145
Danielle	$130
Katie	$160
Stephanie	$400

  a. What is the mean weekly salary for these students? $189
  b. What is the median salary? $145
  c. Whose salary would you consider to be an outlier? Stephanie's
  d. Which number do you think is a better representation of the data, the mean or the median? median
  e. Explain your answer to part d. Because there is an outlier, the median is a better representation than the mean.

6. Rosanne is selling her Corvette. She wants to include a photo of her car in the ad. Three publications give her prices for her ad with the photograph:

Lake Success Shopsaver	$59.00
Glen Head Buyer	$71.00
Floral Park Moneysaver	$50.00

  a. What is the mean price of these ads? Round to the nearest cent. $60.00
  b. What would it cost her to run all three ads? $180
  c. If each of the three newspapers used the mean price as their ad price, what would it cost Rosanne to run ads in all three papers? $180
  d. Find the range of these ad prices. $21.00

7. Dan's parents are going to pay for half of his car if he gets a 90 average in math for all four marking periods and the final exam. All grades are weighted equally. Here are his grades for the first four quarters: 91, 82, 90, and 89. What grade does he need on his final exam to have a 90 average? 98

Exercise 8
Remind students about the credit chapter they recently completed. If someone has to save years for a car, what would they drive while they are saving?

Exercises 14–18
Students can create their lists and trade with other students. Then have them perform the required computations to see if their numbers satisfy the problems posed.

8. Elliot is saving to buy a used car next year on his 18th birthday. He plans on spending $6,000. How much must he save each week, if he plans to work the entire year with only two weeks off? $120

9. The mean of five numbers is 16. If four of the numbers are 13, 20, 11 and 21, what is the fifth number? 15

10. The quartiles of a data set are $Q_1 = 50$, $Q_2 = 72$, $Q_3 = 110$, and $Q_4 = 140$. Find the interquartile range. 60

11. The following list of prices is for a used original radio for a 1955 Thunderbird. The prices vary depending on the condition of the radio.

    $210, $210, $320, $200, $300, $10, $340,
    $300, $245, $325, $700, $250, $240, $200

    a. Find the mean of the radio prices. $275
    b. Find the median of the radio prices. $247.50
    c. Find the mode of the radio prices. $200, $210, and $300
    d. Find the four quartiles. $Q_1 = 210$, $Q_2 = 247.50$, $Q_3 = 320$, $Q_4 = 700$
    e. Find the interquartile range for this data set. 110
    f. Find the boundary for the lower outliers. Are there any lower outliers? $45; yes, there is one lower outlier, $10.
    g. Find the boundary for the upper outliers. Are there any upper outliers? $485; yes, there is one upper outlier, $700.

12. Bill is looking for original taillights for his 1932 Ford. The prices vary depending on the condition. He finds these prices:
    $450, $100, $180, $600, $300, $350, $300, and $400.
    a. Find the four quartiles. $Q_1 = 240$; $Q_2 = 325$; $Q_3 = 425$; $Q_4 = 600$
    b. Find the interquartile range. 185
    c. Find the boundary for the lower outliers. Are there any lower outliers? –$37.50; there are no lower outliers.
    d. Find the boundary for the upper outliers. Are there any upper outliers? $702.50; there are no upper outliers.

13. Eliza wants to sell a used car stereo online. From her research on the website she will post to, she found 8 similar stereos listed. She decides to list her stereo for 20% less than the mean price of the stereos already for sale on the site. Let x represent the sum of the prices of the stereos she found in her research. Write an expression to calculate the price she will list as the cost of her stereo. $\frac{x}{8} - 0.2\left(\frac{x}{8}\right)$ or $0.8\left(\frac{x}{8}\right)$

14. Create a list of five different numbers whose mean is 50. Answers vary.

15. Create a list of six different numbers whose median is 10. Answers vary.

16. Create a list of five numbers whose mean and median are both 12. Answers vary.

17. Create a list of numbers whose mean, median, and mode are all 10. Answers vary.

18. Create a list of numbers with two upper outliers and one lower outlier. Answers vary.

19. Explain why you cannot find the range of a data set if you are given the four quartiles. You need the least number, which is not one of the quartiles.

Graph Frequency Distributions

5-3

Key Terms

- frequency distribution
- frequency
- stem-and-leaf plot
- box-and-whisker plot
- boxplot
- modified boxplot

WHY ARE GRAPHS USED SO FREQUENTLY IN MATHEMATICS, AND IN DAILY LIFE?

Think of all the graphs you have seen in your mathematics textbooks over the years. Think of all of the graphs you have seen in newspapers, magazines, online, and on television. Why are graphs so prevalent? The answer is simple: "a picture is worth a thousand words." Graphs gather and present information in an easy-to-see format that can be interpreted quicker than information from a long list.

In your mathematical career, you have worked with bar graphs, histograms, circle graphs, and line graphs. Earlier in this book you learned about scatterplots. Trends in data that a long list can hide can be seen on a graph.

In the previous lesson you learned about measures of central tendency and measures of dispersion. In this lesson, you will learn about two graphs that present information about central tendency and dispersion pictorially. You can use these graphs to help negotiate car purchases and sales. If the graph supports your position, it can give the other party involved a quick look at the point you are trying to make.

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Price, p ($)	Frequency, f
540	1
550	4
600	3
675	1
700	7
750	1
775	2
800	1
870	1
900	2
990	6
1,000	1
1,200	3
Total	33

Skills and Strategies

Here you will learn how to organize data using a table and draw two types of graphs to display how the data is distributed.

EXAMPLE 1

Jerry wants to purchase a car stereo. He found 33 ads for the stereo he wants and arranged the prices in ascending order:

$540 $550 $550 $550 $550 $600 $600 $600 $675 $700 $700 $700

$700 $700 $700 $700 $750 $775 $775 $800 $870 $900 $900 $990

$990 $990 $990 $990 $990 $1,000 $1,200 $1,200 $1,200

He is analyzing the prices, but having trouble because there are so many numbers. How can he organize his prices in a helpful format?

SOLUTION   Jerry can set up a **frequency distribution**. A frequency distribution is a table that gives each price and the **frequency**—the number of stereos that are advertised at each price.

Jerry adds the numbers in the frequency column to find the total frequency—the total number of pieces of data in his data set. He wants to make sure he did not accidentally leave out a price.

Because there are 33 prices in the set, and the sum of the frequencies is 33, Jerry concludes his frequency distribution is correct.

■ CHECK YOUR UNDERSTANDING

Use the frequency distribution from Example 1 to find the number of car stereos selling for less than $800.

EXAMPLE 2

Find the mean of the car stereos prices from Example 1.

SOLUTION   Jerry creates another column in his table for the product of the first two column entries.

The sum of the entries in the third column, 26,425, is used to find the mean. This is the same sum you would find if you added the original 33 prices. Divide by 33 to find the mean, and round to the nearest cent.

$$26,425 \div 33 \approx 800.76$$

The mean of the prices is $800.76.

You can use your graphing calculator to find the mean, median, and quartiles.

Price, p ($)	Frequency, f	Product, pf
540	1	540
550	4	2,200
600	3	1,800
675	1	675
700	7	4,900
750	1	750
775	2	1,550
800	1	800
870	1	870
900	2	1,800
990	6	5,940
1,000	1	1,000
1,200	3	3,600
Total	33	26,425

Jerry, from Example 1, decides he is not interested in any of the car stereos priced below $650 because they are in poor condition and need too much work. Find the mean of the data set that remains after those prices are removed.

EXAMPLE 3

Rod was doing Internet research on the number of gasoline price changes per year in gas stations in his county. He found the following graph, called a **stem-and-leaf plot**. What are the mean and the median of this distribution?

1	1 1 2 3 7 9
2	0 3 6 6
3	8 8 9 9 9 9 9
4	0
5	2 2 4 5 5 5 6 7
6	3 4 4
7	2

$$5|2 = 52$$

SOLUTION   A stem-and-leaf plot displays data differently than a frequency table. To read the stem-and-leaf plot, look at the first row. In this plot, the numbers to the left of the vertical line represent the tens place digit, and are the stems. The numbers to the right of the vertical line represent the digits in the ones place, in ascending order, and are the leaves. The first row represents these numbers.

> 11, 11, 12, 13, 17, 19

The second row represents these numbers.

> 20, 23, 26, 26

The last row represents the number 72.

With one quick look at a stem-and-leaf plot, you can tell if there are many low numbers, many high numbers, or many numbers clustered in the center. Upon further investigation, you can find the total frequency and every piece of data in the data set. This allows you to find the mean, median, mode, range, and quartile values.

By counting the leaves, the entries on the right side of the vertical line, you find the frequency is 30. Add the data represented in the plot and divide to find the mean. The sum is 1,188.

Divide by 30 to find the mean. $1,188 \div 30 = 39.6$

The stem-and-leaf plot presents the numbers in ascending order. To find the median, locate the middle number. The frequency, 30, is even, so find the mean of the numbers in the 15th and 16th positions. The two middle numbers are both 39, so the median is 39.

Stem-and-leaf plots may have a slightly different look depending on what information is displayed. A stem-and-leaf plot should include a legend or key that describes how to read it.

■ **CHECK YOUR UNDERSTANDING**

Find the range and the upper and lower quartiles for the stem-and-leaf plot shown in Example 3.

EXAMPLE 4

Box-and-whisker plots should be drawn to scale whenever possible. Explain that the mean cannot be computed using the information from a box-and-whisker plot. Also the median is not necessarily midway between the first and third quartiles. This is a common misconception.

You should provide several examples of boxplots where $Q_3 - Q_2$ does not equal $Q_2 - Q_1$ to show students all boxplots are not symmetrical.

CHECK YOUR UNDERSTANDING

Answer 75%

EXAMPLE 4

Rod, from Example 3, found another graph called a **box-and-whisker plot**, or **boxplot**. It is shown below.
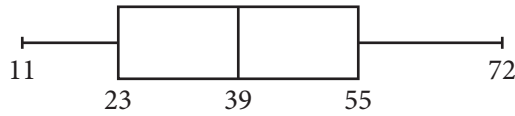
Find the interquartile range of the distribution.

SOLUTION   Look at the information presented on the box-and-whisker plot.
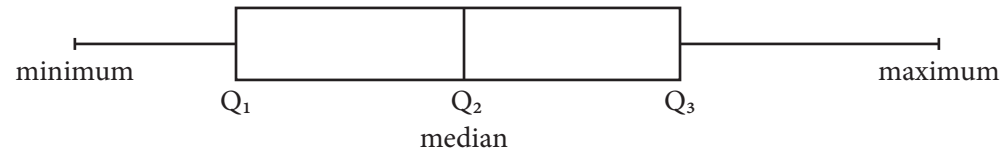
The box-and-whisker plot shows all four quartiles and the *least* number. It should be drawn to scale, so it changes shape depending on the distribution. Recall that the interquartile range can be computed by subtracting Q_1 from Q_3. The box part of the diagram helps you find the interquartile range, because it displays Q_1 and Q_3.

$$Q_3 - Q_1 = 55 - 23 = 32$$

The interquartile range is 32. That means 50% of all the gas prices are within this range. Notice that you can also find the range using a boxplot, but you cannot find the mean from a boxplot.

You can use the statistics menu on your graphing calculator to draw a box-and-whisker plot.

■ **CHECK YOUR UNDERSTANDING**

Based on the box-and-whisker plot from Example 4, what percent of the gas stations had 55 or fewer price changes?

EXAMPLE 5

The following box-and-whisker plot gives the purchase prices of the cars of 114 seniors at West High School. Are any of the car prices outliers?

$3,000 $43,000
$5,200 $7,000 $9,100
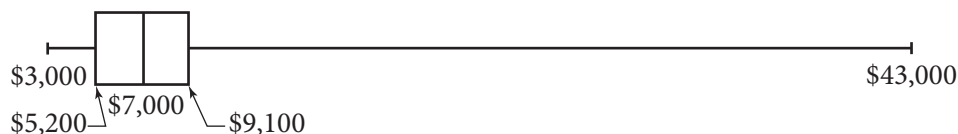
SOLUTION Quartiles are shown on the boxplot, so you can find the interquartile range. The interquartile range is

$$\text{IQR} = Q_3 - Q_1 = 9{,}100 - 5{,}200 = 3{,}900$$

The boundary for lower outliers is

$$Q_1 - 1.5(\text{IQR}) = 5{,}200 - 1.5(3{,}900) = -650$$

There are no lower outliers.

The boundary for upper outliers is

$$Q_3 + 1.5(\text{IQR}) = 9{,}100 + 1.5(3{,}900) = 14{,}950$$

There is at least one upper outlier, the high price of $43,000. From this boxplot, you cannot tell if there are any others, because the boxplot does not give all the original data. Boxplots are drawn to scale, so the long whisker on the right means that there could be more than one outlier.

If you want to show outliers on a boxplot, you can create a **modified boxplot**. A modified boxplot shows all the numbers that are outliers as single points past the whiskers. In the following modified boxplot, $43,000 is the only outlier. The greatest price less than $43,000 is $12,500.

$3,000 $12,500 $43,000
$5,200 $7,000 $9,100

If there were three upper outliers, the modified boxplot would have three dots to the right of the whisker.

$3,000 $12,500 $21,000 $43,000
$5,200 $7,000 $9,100 $16,000
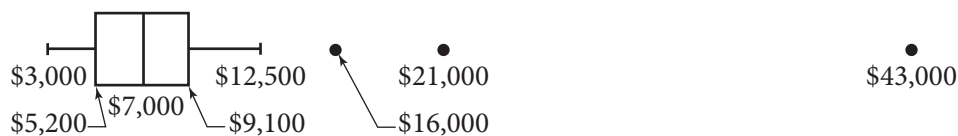
Modified boxplots give more information than standard box-and-whisker plots. Your calculator can draw modified boxplots.

■ **CHECK YOUR UNDERSTANDING**

Examine the modified boxplot. Is 400 an outlier?
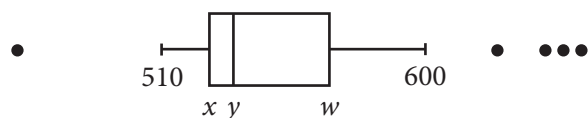
510   600
x y   w

Applications

It is a capital mistake to theorize before one has data.
Sir Arthur Conan Doyle, Scottish Author (Sherlock Holmes novels)

1. Interpret the quote in the context of what you learned. See margin.

2. Look at the frequency table in Example 2. Imagine the 33 prices listed in ascending order. If the prices were numbered using subscripts from p_1 to p_{33}, the middle price would be price number p_{17}. Use the frequency table to find the median. $750

3. Find the mode of the distribution from Example 1. $700

4. Find the range of the distribution from Example 1. $660

5. Martina found the mean of the data from Example 1 by adding the prices in the first column and dividing by the number of prices she added. Her answer was incorrect. Explain what error she made. Martina did not take into account the frequency of each price.

6. Brian looked up prices of thirteen used Chevrolet HHR "retro" trucks in the classified ads and found these prices: $8,500, $8,500, $8,500, $9,900, $10,800, $10,800, $11,000, $12,500, $12,500, $13,000, $13,000, $14,500, and $23,000.
   a. Make a frequency table for this data set. See margin.
   b. Find the mean. Round to the nearest dollar. $12,038
   c. Find the median. $11,000
   d. Find the mode. $8,500
   e. Find the range. $14,500
   f. Find the four quartiles. Q_1 = $9,200; Q_2 = $11,000; Q_3 = $13,000; Q_4 = $23,000
   g. Find the interquartile range. $3,800
   h. Find the boundary for the upper outliers. $18,700
   i. Find the boundary for the lower outliers. $3,500
   j. How many outliers are there? 1
   k. Draw a modified box-and-whisker plot. Label it. See additional answers.

7. Enter the data from Example 1 in your calculator.
   a. Create a box-and-whisker plot using the data from Example 1. See additional answers.
   b. How would you determine if it would be appropriate to create a modified boxplot for this data? See margin.
   c. How many outliers are there in this distribution? 0

8. Megan has a friend at work who is selling a used Honda. The car has 60,000 miles on it. Megan comparison shops and finds these prices for the same car.
   a. Find the mean price of the 5 prices listed. $18,320
   b. How many of these cars are priced below the mean? 3
   c. Find the median price. $18,000
   d. How many of these cars are priced below the median? 2

Price
$22,000
$19,000
$18,000
$16,700
$15,900

9. Megan, from Exercise 8, decides to get more information about the cars she researched. The table has prices and mileages for the same used car. In addition to the statistics she has learned in this chapter, Megan decides to use her linear regression skills from Chapter 2 to see if there is a relationship between the prices and the mileage. She hopes to use this knowledge to negotiate with sellers.

Mileage, x	Price, y
21,000	$22,000
30,000	$19,000
40,000	$18,000
51,000	$16,700
55,000	$15,900

   a. Enter the data into your calculator. Find the regression equation. Round to the nearest hundredth. $y = -0.16x + 24,722.26$
   b. Find the correlation coefficient r. Round to three decimal places. $r = -0.970$
   c. Is the regression equation a good predictor of price, given the mileage? Explain. Yes; r is close to -1.
   d. The car Megan is considering has 60,000 miles on it and the price is $19,000. Discuss her negotiating strategy. Explain on what grounds she should try to get a lower price. See margin.

10. The Cold Spring High School student government polled randomly selected seniors and asked them how much money they spent on gas in the last week. The following stem-and-leaf plot shows the data they collected.

```
1 | 7
4 | 1 1 2 9
5 | 3 3 3 3 3
6 | 1 3 4 6 7 8 9
7 | 1 3 5 5 7
8 | 2 2 3 4

   8| 2 = 82
```

   a. How many students were polled? 26
   b. Find the mean to the nearest cent. $62.12
   c. Find the median. $65
   d. Find the mode. $53
   e. Find the range. $67
   f. Find the four quartiles. $Q_1 = \$53; Q_2 = \$65; Q_3 = \$75; Q_4 = \$84$
   g. What percent of the students spent $53 or more on gas? 75%
   h. Find the interquartile range. $22
   i. What percent of the students spent from $53 to $75 on gas? 50%
   j. Find the boundary for the lower outliers. $20
   k. Find the boundary for the upper outliers. $108
   l. How many outliers are there? 1
   m. Draw a modified boxplot. See additional answers.

11. A group of randomly-selected recent college graduates were asked how much the monthly payment is on their student loan. The responses are shown in the stem-and-leaf plot.
   a. What is the total frequency? 18
   b. How many people had monthly payments between $210 and $219? 0
   c. What is the mode monthly payment? $226
   d. What is the median monthly payment? $189.50

```
16 | 2 3 5 5
17 | 5 7 8
18 | 7 7
19 | 2 4 6
20 | 1 1
21 |
22 | 5 6 6 6

  16| 2 = $162
```

12. Express the mean of the data set shown in the frequency table algebraically. $\dfrac{xy + 5w + 64 + 18v}{y + v + 9}$

Price	Frequency
x	y
w	5
16	4
18	v

5-4 Automobile Insurance

Objectives

- Learn about different types of auto insurance coverage.

- Compute insurance costs.

- Compute payments on insurance claims.

Key Terms

- liable
- negligent
- automobile insurance
- premium
- claim
- liability insurance
- bodily injury liability (BI)
- property damage liability (PD)
- uninsured/underinsured motorist protection (UMP)
- personal injury protection (PIP)
- no-fault insurance
- comprehensive insurance
- collision insurance
- car-rental insurance
- emergency road service insurance
- actuary
- surcharge
- deductible

EXAMINE THE QUESTION

What types of damages can you cause while driving? Initiate a discussion on "what could go wrong" when you are operating an automobile.

CLASS DISCUSSION

Do you know how much auto body work costs? How much a fire hydrant or lamp post costs?

What do you know about the cost of doctors and hospitals?

Discuss what specific items are covered by comprehensive insurance, and ask students if they personally know of any car damages caused by the listed disasters. You could refer to the intense damage caused to automobiles in New York City the day of the September 11 attacks, and how comprehensive insurance covered them.

WHY IS HAVING AUTO INSURANCE SO IMPORTANT?

Even responsible drivers run the risk of injuring themselves, hurting other people, and damaging property. By law, drivers are **liable** (responsible) to pay for the damages they cause with their automobiles. You could also be sued for being **negligent** (at fault) if you cause an accident.

Drivers purchase **automobile insurance** because most drivers cannot afford the costs that could result from an auto accident. An automobile insurance policy is a contract between a driver and an insurance company. The driver agrees to pay a fee (called the **premium**) and the company agrees to cover certain accident-related costs when the driver makes a **claim** (a request for money). **Liability insurance** is the most important coverage. States set minimum liability requirements. Insurance regulations vary by state. Liability insurance is required unless you can prove financial responsibility otherwise. Several types of coverages are available.

- **Bodily Injury Liability (BI)** BI liability covers bodily injury. If you are at fault in an automobile accident, you are responsible for paying the medical expenses of anyone injured in the accident. You can purchase as much BI liability as you want.

- **Property Damage Liability (PD)** This coverage pays for damage you cause to other people's property. You are financially responsible if you damage a telephone pole, fire hydrant, another car, or any other property. You can purchase as much PD liability insurance as you want.

- **Uninsured/Underinsured Motorist Protection (UMP)** This coverage pays for injuries to you or your passengers caused by a driver who has no insurance or does not have enough insurance to cover your medical losses.

- **Personal Injury Protection (PIP)** This is coverage, mandatory in some states, that pays for any physical injuries you or your passengers sustain while in the vehicle, even if you are not involved in a traffic accident. It compensates you regardless of who is at fault, so it is sometimes called **no-fault insurance**. Your PIP insurance will cover you and people injured in, on, around, or under your car for medical treatment.
- **Comprehensive Insurance** This covers the repair or replacement of parts of your car damaged by vandalism, fire, flood, wind, earthquakes, falling objects, riots, hail, damage from trees, and other disasters. It also covers your car if it is stolen. If your car is older, comprehensive coverage may not be cost-effective.
- **Collision Insurance** This pays you for the repair or replacement of your car if it's damaged in a collision with another vehicle or object, or if it overturns, no matter who is at fault. If you took out a loan to purchase your car, the lender will probably require you to have collision coverage. If your car is older, collision coverage may not be a worthwhile expense.
- **Car-Rental Insurance** This pays you for part of the cost of a rented car if your car is disabled because of a collision or comprehensive-covered repair.
- **Emergency Road Service Insurance** This coverage pays for towing or road service when your car is disabled. Only the road service fee is covered. Gas, oil, part, and labor are not covered.

Auto insurance companies are in business to make a profit. The company loses money if a high percentage of insured drivers get into accidents. Insurance companies classify drivers according to their age, sex, marital status, driving record, and locality. Statisticians called **actuaries** predict how often customers, based on these criteria, will submit claims.

Skills and Strategies

Once you learn more about auto insurance, you'll understand how you can save money and comparison shop for different insurance policies.

EXAMPLE 1

Kwan's annual premium is $1,284. If he pays quarterly, there is a $1 per payment **surcharge** (extra fee). What is the quarterly payment?

SOLUTION

| Divide the annual premium by 4 | $1,284 \div 4 = 321$ |
| Add on the $1 surcharge. | $321 + 1 = 322$ |

Each of the four quarterly payments is $322.

■ CHECK YOUR UNDERSTANDING

Leon's annual premium is x dollars. If he pays his premium semiannually, there is a y-dollar surcharge on each semiannual payment. Express the amount of his semiannual payment algebraically.

CHECK YOUR UNDERSTANDING

Answer $x + y$

This amount will be paid in full if the coverage limit on the property damage is greater than $x + y$. The w-dollar damage is not covered under PD.

EXAMPLE 3

The deductible is per accident. Collision and comprehensive are sold on a deductible basis.

CHECK YOUR UNDERSTANDING

Answer $y - x$

EXAMPLE 2

Stan DeMille has $25,000 worth of property damage liability insurance. He caused an accident that damaged a $2,000 fire hydrant and did $5,600 worth of damage to another car. How much of the damage must Stan pay?

SOLUTION Find the sum of the damages.

$$2,000 + 5,600 = 7,600$$

$7,600 < $25,000, so the company will pay for all of the damage and Stan will pay nothing. Notice that this $25,000 coverage is *per accident*.

■ **CHECK YOUR UNDERSTANDING**

Keith ran his car into a telephone pole that had a bicycle leaning against it which was also damaged. The pole will cost x dollars to fix, the bicycle will cost y dollars to replace, and there was w dollars damage to the car. Express algebraically the amount that can be claimed under Keith's property damage liability insurance.

Deductibles

When you purchase an automobile insurance policy, you must choose a *deductible* amount that will be part of the policy. The **deductible** is the amount that the policy owner must pay before the insurance policy pays any money. Once an owner has paid the deductible amount, the insurance company pays the rest of the cost to get the repairs done. Collision insurance only covers damage to the policy owner's car, not property damaged, or another driver's vehicle. If a driver has $500 deductible and the repairs to this car cost $2,200, the driver pays the first $500 and the insurance company pays the balance, $2,200 − 500$, or $1,700.

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EXAMPLE 3

Peter has $1,000 deductible collision insurance. Peter backs his car into his garage and causes $4,300 worth of damage to the car. How much will his insurance company have to pay?

SOLUTION Subtract the deductible, which is $1,000, because Peter must pay that amount.

$$4,300 - 1,000 = 3,300$$

The company must pay $3,300.

■ **CHECK YOUR UNDERSTANDING**

Manuel has an x-dollar deductible on his comprehensive insurance. His car is stolen and never recovered. The value of his car is y dollars where $y > x$. How much must the insurance company pay him for his stolen car?

Bodily Injury and Property Damage

Bodily injury insurance coverage uses two numbers with a slash between them. The first number is the maximum amount per accident the insurance company will pay, in thousands of dollars, to any one person who is hurt and sues you due to your driving negligence. The second number represents the maximum amount per accident your insurance company will pay in total to all people who sue as a result of the accident. Sometimes, bodily injury and property damage are combined into a three number system with two slashes. The numbers 100/300/25 represent 100/300 BI insurance and $25,000 PD insurance.

EXAMPLE 4

Bob was in an auto accident caused by his negligence. He has 100/300 bodily injury insurance. The three people injured in the accident sued. One person was awarded $140,000, and each of the other two was awarded $75,000. How much does the insurance company pay?

SOLUTION Bob has 100/300 BI, so the company only pays $100,000 to the person who was awarded $140,000. The other two injured persons were awarded a total of $150,000. Each was under $100,000. The most Bob's company would pay out for any BI claim is $300,000.

Add the awarded amounts. $100,000 + 75,000 + 75,000 = 250,000$

$250,000 < $300,000, so the insurance company pays $250,000. The remaining $40,000 owed to one of the injured is Bob's responsibility.

■ CHECK YOUR UNDERSTANDING

Joan has 50/100 BI liability insurance. She hurts 28 children riding a school bus, and each child is awarded $10,000 as a result of a lawsuit. How much will the insurance company pay in total for this lawsuit?

EXAMPLE 5

Desmond has a policy with 50/150 BI, $50,000 PD, and $50,000 PIP. He causes an accident in which he hurts 7 people in a minivan and 4 people in his own car, including himself. The eleven people who are hurt have minor injuries and do not sue Desmond. The total medical bill for all involved is $53,233. How much does the insurance company pay?

SOLUTION Desmond is covered by his PIP, which has a limit of $50,000 per person, per accident. PIP takes care of medical payments without regard to who is at fault. The company pays the entire $53,233, as long as no individual person requests more than $50,000.

Notice that the bodily injury numbers were not relevant in this scenario.

■ CHECK YOUR UNDERSTANDING

Pat has 50/100 BI liability insurance and $100,000 PIP insurance. She hurts 28 children in a school bus and is not sued. However, if each child needs $10,000 for medical care, how much will the insurance company pay in total for these medical claims?

Applications

Never lend your car to anyone to whom you have given birth.
Erma Bombeck, Humor Writer

1. Interpret the quote in the context of what you learned. See margin.

2. Rachel has $25,000 worth of property damage insurance. She causes $32,000 worth of damage to a sports car in an accident.
   a. How much of the damages will the insurance company have to pay? $25,000
   b. How much will Rachel have to pay? $7,000

3. Ronald Kivetsky bought a new car and received these price quotes from his insurance company.
   a. What is the annual premium? $1,467
   b. What is the semiannual premium? $733.50
   c. How much less would Ronald's semiannual payments be if he dropped the optional collision insurance? $205

4. Gloria pays her insurance three times each year. The first payment is 40% of the annual premium, and each of the next two payments is 30% of the annual premium. If the annual premium is $924, find the amounts of the three payments. $369.60; $277.20; $277.20

5. Ruth Fanelli has decided to drop her collision insurance because her car is getting old. Her total annual premium is $916, of which $170.60 covers collision insurance.
   a. What will her annual premium be after she drops the collision insurance? $745.40
   b. What will her quarterly payments be after she drops the collision coverage? $186.35

6. Gary Lieberman has $10,000 worth of property damage insurance. He collides with two parked cars and causes $12,000 worth of damage. How much money must Gary pay after the insurance company pays its share? $2,000

7. Craig Rosenberg has a personal injury protection policy that covers each person in, on, around, or under his car for medical expenses as a result of an accident. Each person can collect up to $50,000. Craig is involved in an accident and three people are hurt. One person has $23,000 of medical expenses, one person has $500 worth of medical expenses, and Craig himself has medical expenses totaling $70,000. How much money must the insurance company pay out for these three people? $73,500

8. Leslie has comprehensive insurance with a $500 deductible on her van. On Halloween her van is vandalized, and the damages total $1,766. Leslie submits a claim to her insurance company.
   a. How much must Leslie pay for the repair? $500
   b. How much must the insurance company pay? $1,266

9. Felix Madison has $10,000 worth of property damage insurance and a $1,000 deductible collision insurance policy. He had a tire blow-out while driving and crashed into a $1,400 fire hydrant. The crash caused $1,600 in damages to his car.

    a. Which insurance covers the damage to the fire hydrant? property damage

    b. How much will the insurance company pay for the fire hydrant? $1,400

    c. Which insurance covers the damage to the car? collision

    d. How much will the insurance company pay for the damage to the car? $600

10. Jared's car slides into a stop sign during an ice storm. There is x dollars damage to his car, where $x > 1,000$, and the stop sign will cost y dollars to replace. Jared has $25,000 worth of PD insurance, a $1,000 deductible on his collision and comprehensive insurance, and $50,000 no-fault insurance.

    a. Which insurance covers the damage to the sign? PD

    b. How much will his company pay for the stop sign? See margin.

    c. Which insurance covers the damage to his car? collision

    d. How much will his company pay for the damage to the car? $x - 1,000$

11. Eric must pay his p dollar annual insurance premium by himself. He works at a job after school.

    a. Express how much he must save each month to pay this premium algebraically. $\frac{p}{12}$

    b. If he gets into a few accidents and his company raises his insurance 15%, express how much he must save each month to meet this new premium algebraically. $0.15\left(\frac{p}{12}\right)$

12. Mollie has 100/300/50 liability insurance and $50,000 PIP insurance. She drives through a stop sign and hits a telephone pole and bounces into a minivan with 8 people inside. Some are seriously hurt and sue her. Others have minor injuries. Three passengers in Mollie's car are also hurt.

    a. The pole will cost $7,000 to replace. Mollie also did $6,700 worth of damage to the minivan. What insurance will cover this, and how much will the company pay? property damage; $13,700

    b. The minivan's driver was a concert violinist. The injury to his hand means he can never work again. He sues for $4,000,000 and is awarded that money in court. What type of insurance covers this, and how much will the insurance company pay? BI; $100,000

    c. The minivan's driver (from part b) had medical bills totaling $60,000 from his hospital trip and physical therapy after the accident. What type of insurance covers this, and how much will the insurance company pay? PIP; $50,000

    d. The three passengers in Mollie's car are hurt and each requires $12,000 worth of medical attention. What insurance covers this, and how much will the company pay? PIP; $36,000

13. Julianne currently pays x dollars for her annual premium. She will be away at college for the upcoming year and will only use the car when she is home on vacations. Her insurance company offers her a 35% discount for her annual premium. Express algebraically the amount she must save each month to pay the new, lower premium. $\frac{x - 0.35x}{12}$ or $\frac{0.65x}{12}$

14. The Schuster family just bought a third car. The annual premium would have been x dollars to insure the car, but they are entitled to a 10% discount since they have other cars with the company.

   a. Express their annual premium after the discount algebraically. $x - 0.1x$, or $0.9x$

   b. If they pay their premium quarterly and have to pay a y-dollar surcharge for this arrangement, express their quarterly payment algebraically. $\frac{0.9x}{4} + y$

15. Marc currently pays x dollars per year for auto insurance. Next year, his rates are going to increase 15%. If he completes a defensive driver course, the insurance company will lower his rate by d dollars.

   a. Express his annual premium for next year algebraically if he completes the course. $x + 0.15x - d$ or $1.15x - d$

   b. Express his semiannual premium for next year algebraically if he does not complete the course. $\frac{1.15x}{2}$

16. The stem-and-leaf plot gives the semiannual premiums for the girls and boys in Van Buren High School who currently drive. It is called a *back-to-back stem-and-leaf plot*, and combines two stem-and-leaf plots. The numbers between the two vertical lines represent the hundreds and tens digits. The numbers on the extreme left show the units digits for the girls. Notice they are written in ascending order as you move out from the middle. The numbers on the extreme right show the units digits for the boys.

```
9 8 1 1 1 1 | 87 | 1 2 2
        3 2 | 88 | 2 4 6 7
      7 5 4 | 89 | 1 3
7 6 6 6 6 5 | 90 | 2 7 7 7
```
```
        1| 87 = $871
        87| 1 = $871
```

   a. How many girls at Van Buren HS drive? 17

   b. How many boys at Van Buren HS drive? 13

   c. Find the range of the annual premiums for all of the students. $36

17. The following stem-and-leaf plot gives the number of juniors who took a driver education course at Guy Patterson High School over the last two decades. Construct a box-and-whisker plot based on the data.
   See additional answers.

```
4 | 1 2 3 5 5 6
5 | 1 1 8 8 8 9
6 | 0 0 0 1 2 5
7 | 1 1
```
```
    1| 5 = 15
```

18. Express the boundary for the upper outliers algebraically, using the modified box-and-whisker plot given below. $d + 1.5(d - c)$

Linear Automobile Depreciation

5-5

Key Terms
- depreciate
- appreciate
- straight line depreciation
- slope
- straight line depreciation equation

Objectives
- Write, interpret, and graph a straight line depreciation equation.

What is the value of your car?

Most cars will not be worth their purchase prices as they get older. Most cars **depreciate**; that is, they lose value over time. Some collectible cars increase in value over time, or **appreciate**. The simplest form of depreciation is **straight line depreciation**. When a car loses the same amount of value each year, the scatterplot that models this depreciation appears linear. By determining the equation of this linear model, you can find the value of the car at any time in its lifespan. There are many factors contributing to the depreciation of an automobile. The condition of the car, mileage, and make of the car are only a few of those factors. The straight line depreciation equation is a mathematical model that can be used as a starting point in examining auto depreciation.

In Chapter 2, you used the intercepts of linear equations when graphing expense and demand functions. Recall that the horizontal intercept always has the form $(a, 0)$ and the vertical intercept always has the form $(0, b)$. In addition to intercepts, straight lines also have slope. The **slope** of the line is the numerical value for the inclination or declination of that line. It is expressed as a ratio of the change in the vertical variable over the change in the horizontal variable from one point on the line to the next. Traditionally, the horizontal axis is called the x-axis and the vertical axis is called the y-axis. Using those variable names, the slope of a line would be represented by the following ratio.

$$\text{Slope} = \frac{\text{Change in } y\text{-value}}{\text{Change in } x\text{-value}}$$

If the coordinates of the two points are (x_1, y_1) and (x_2, y_2), then the slope can be modeled mathematically by the following ratio.

Slope ratio $\dfrac{y_2 - y_1}{x_2 - x_1}$

The independent variable in a car's depreciation equation is *time in years* and the dependent variable is *car value*. By identifying the intercepts and slope of a straight line depreciation model, you will be able to determine the equation that represents the depreciation.

EXAMINE THE QUESTION

How do the automobile industry, car dealers, and individual owners define "car value"? The value of a car can be both personal and monetary. Ask the students what makes a car personally valuable? Ask students what factors might contribute to the monetary value of a car?

CLASS DISCUSSION

Ask students to list items that would appreciate/depreciate over time. Discuss the fact that straight line depreciation is merely a depreciation model and may not be the exact way that cars lose value over time. We introduce it here as a means for beginning the discussion on depreciation.

Use this opportunity to review the general equation for a straight line, $y = mx + b$. Discuss the slope, the intercepts, and the fact that all points on the line must satisfy the equation.

Skills and Strategies

Here you will learn how to determine and use a **straight line depreciation equation**.

EXAMPLE 1

Suppose that you purchase a car for $27,000. According to your online research, this make and model of car loses all of its marketable value after 12 years. That is, it depreciates to a value of zero dollars 12 years after the purchase date. If this car depreciates in a straight line form, what are the intercepts of the depreciation equation?

SOLUTION   Let *x* represent the time in years. The minimum *x*-value is 0 years, the purchase year of the car. Because the car totally depreciates after 12 years, the maximum *x*-value will be 12.

In a straight line depreciation equation, the intercepts are

(0, maximum car value) and (maximum lifespan, 0)

Let *y* represent the value of the car at any time during its lifetime. The minimum *y*-value is zero dollars and the maximum *y*-value is the purchase price of $27,000. Knowing this information, you can identify the intercepts as (0, 27,000) and (12, 0).

■ CHECK YOUR UNDERSTANDING

A car sells for *D* dollars and totally depreciates after *T* years. If this car straight line depreciates, what are the intercepts of the straight line depreciation equation?

EXAMPLE 2

Determine the slope of the straight line depreciation equation for the situation in Example 1.

SOLUTION   Two points determine a line, so you only need two points to determine the slope of a line. Let the coordinates of the *y*-intercept be the first point. That is, $(x_1, y_1) = (0, 27{,}000)$. Let the coordinates of the *x*-intercept be the second point. That is, $(x_2, y_2) = (12, 0)$.

Use the slope ratio. $\dfrac{y_2 - y_1}{x_2 - x_1}$

Substitute and simplify. $\dfrac{0 - 27{,}000}{12 - 0} = \dfrac{-27{,}000}{12} = -2{,}250$

The slope of the depreciation line is $\dfrac{-2{,}250}{1}$.

■ CHECK YOUR UNDERSTANDING

Write the slope of the straight line depreciation equation that models the situation in which a car is purchased for *D* dollars and totally depreciates after *T* years.

EXAMPLE 3

Write the straight line depreciation equation for the situation discussed in Examples 1 and 2. Then draw the graph of the equation.

SOLUTION   The general form for the equation of a straight line is

$$y = mx + b$$

where m represents the slope of the line and b represents the y-intercept.

The slope is $-2{,}250$, and the y-intercept is 27,000. Therefore, the straight line depreciation equation is

$$y = -2{,}250x + 27{,}000$$

To graph the equation on a graphing calculator, first determine an appropriate graphing window. Use your maximum and minimum x- and y-values as a starting point. Choose x- and y-values that are larger than the maximum values you have determined so that you get a complete picture of the graph. One such pair could be a maximum of \$30,000 on the y-axis and 15 on the x-axis as shown in the graph. Because time and car value are both positive numbers, the minimum x- and y-values will be zero.

■ CHECK YOUR UNDERSTANDING

Write and graph the straight line depreciation equation for a car that was purchased for \$22,000 and totally depreciates after 11 years.

EXAMPLE 4

Suppose that Jack purchased a car five years ago at a price of \$27,600. According to research on this make and model, similar cars have straight line depreciated to zero value after 12 years. How much will this car be worth after 66 months?

SOLUTION   Determine the straight line depreciation equation. The intercepts are (0, 27,600) and (12, 0). Determine the slope.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 27{,}600}{12 - 0} = \frac{-27{,}600}{12} = -2{,}300$$

Therefore, the straight line depreciation equation is $y = -2{,}300x + 27{,}600$. Because x represents years, it is necessary to convert 66 months into years by dividing by 12.

$$\frac{66}{12} = 5.5$$

Therefore, 66 months is equivalent to 5.5 years.

Use the depreciation equation.	$y = -2{,}300x + 27{,}600$
Substitute 5.5 for x.	$y = -2{,}300(5.5) + 27{,}600$
Simplify.	$y = 14{,}950$

The car will be worth \$14,950 after 66 months.
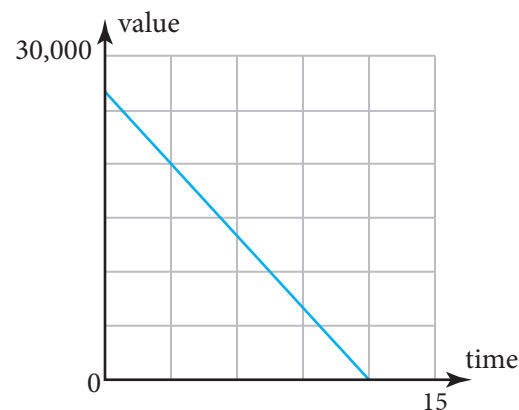
slope as a fraction with 1 in the denominator. Indicate that it is not necessary to do so and eliminating the 1 makes the equation easier to interpret. Once the equation is established, ask students for the real-world significance of the numbers −2,250 and 27,000.

CHECK YOUR UNDERSTANDING

Answer   The equation is $y = -2{,}000x + 22{,}000$. The graph is
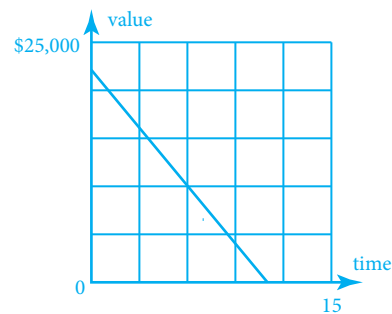
EXAMPLE 4

Alert students that the time is in months but the variable, x, represents time in years. Elicit from them how to change 66 months into years.

■ **CHECK YOUR UNDERSTANDING**

A car sells for $18,495 dollars and straight line depreciates to zero after 9 years. Write the straight line depreciation equation for this car and an expression for the value of the car after *W* months.

EXAMPLE 5

The straight line depreciation equation for a car is $y = -4,000x + 32,000$. In approximately how many years will the car's value decrease by 25%?

SOLUTION The original value of the car is the *y*-intercept, 32,000. You must determine the actual value of the car after it has dropped by 25%. This can be done in two ways.

You can find 25% of the original value of the car and then subtract that amount from the original value.

or

You can notice that once the car drops in value by 25%, it will be worth 75% of its original value.

$0.25 \times 32,000 = 8,000$

$32,000 - 8,000 = 24,000$

$0.75 \times 32,000 = 24,000$

The value is $24,000.

The value is $24,000.

You are trying to determine a length of time. Solve the depreciation equation for *x*.

Use the depreciation equation. $\qquad y = -4,000x + 32,000$

Substitute 24,000 for *y*. $\qquad 24,000 = -4,000x + 32,000$

Subtract 32,000 from each side. $\qquad -8,000 = -4,000x$

Divide each side by –4,000. $\qquad \dfrac{-8,000}{-4,000} = \dfrac{-4,000x}{-4,000}$

Simplify. $\qquad 2 = x$

The car will depreciate by 25% after 2 years.

■ **CHECK YOUR UNDERSTANDING**

Write an algebraic expression that represents the length of time it will take the car in Example 5 to have a value of *D* dollars.

Automobile Expense Function

In Chapter 2 you learned about expense functions. You can create an expense function for an automobile. While there are many expenses that contribute to the running and upkeep of a car, for the purposes here, the expense function is composed of the fixed expense down payment that you make when you purchase a car and the variable expense monthly payment that you make to the lending institution. Looking at the linear expense and depreciation functions simultaneously will give you insight into the value of your automotive investment.

Celine bought a new car for $33,600. She made a $4,000 down payment and pays $560 each month for 5 years to pay off her loan. She knows from her research that the make and model of the car she purchased straight line depreciates to zero over 10 years.

a. Create an expense and depreciation function.

b. Graph these functions on the same axes.

c. Interpret the region before, at, and after the intersection point.

SOLUTION

a. Let x represent time in months and y represent dollars. Celine's expense function is the sum of her monthly payments over this time period and her initial down payment.

Expense function $y = 560x + 4{,}000$

The time, x, is in months rather than years. Express Celine's depreciation function in terms of months as well. Celine's car totally depreciates after 10 years, or 120 months. To determine her monthly depreciation amount, divide the original car value by 120.

$$\frac{33{,}600}{120} = 280$$

Celine's car depreciates $280 per month. To calculate the slope of the depreciation equation, use the intercepts (0, 33,600) and (120, 0).

Slope $\frac{0 - 33{,}600}{120 - 0} = \frac{-33{,}600}{120} = -280$

Notice that the slope is the negative of the monthly depreciation amount. The straight line depreciation function for Celine's car is as follows.

Depreciation function $y = -280x + 33{,}600$

b. Determine an appropriate graphing window by using the largest coordinates of the intercepts for both functions to set up the horizontal and vertical axes. Graph both functions as shown.

c. Using a graphing calculator, the coordinates of the intersection point, rounded to the nearest hundredth, are (35.24, 23,733.33). This means that after a little more than 35 months, both your expenses and the car's value are the same. In the region before the intersection point, the expenses are lower than the value of the car. The region after the intersection point indicates a period of time that the value of the car is less than what you have invested in it.

expense function
depreciation function

EXAMPLE 6

Students create both an expense and a depreciation function. They then plot them on the same coordinate grid and interpret the interactions of the graphs. You may want to review the component parts of an expense function (variable and fixed expenses) and how they contribute to the creation of an automobile expense function. Expense functions were introduced and used in Chapter 2.

CHECK YOUR UNDERSTANDING

Answer You can include your monthly maintenance and insurance costs with the monthly payment amount. This will yield a slope that is greater than the one depicted in the graph. Since the expense graph will be steeper, the intersection point will have an x-value that is less than 35.24. This will indicate that the value of the car will be less than the expenses-to-date earlier in the car's lifetime.
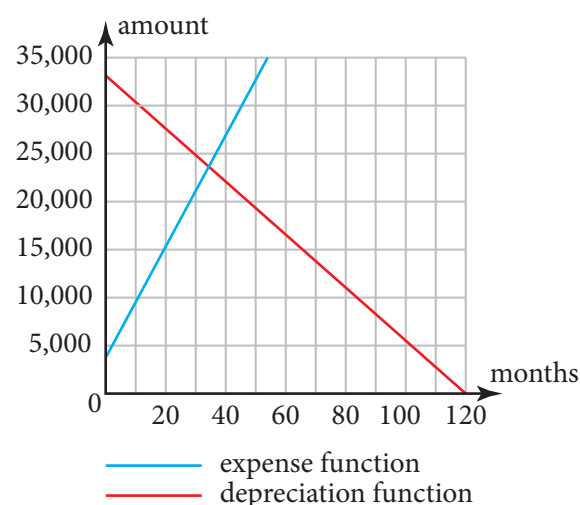
■ **CHECK YOUR UNDERSTANDING**

How might the expense function be altered so that it reflects a more accurate amount spent over time? What effect might that have on the graphs?

Applications

If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost $100 [and] get a million miles per gallon.

Michael Moncur, Internet Consultant

1. How might those words apply to what you have learned? See margin.

2. Delia purchased a new car for $25,350. This make and model straight line depreciates to zero after 13 years.
   a. Identify the coordinates of the *x*- and *y*-intercepts for the depreciation equation. (0, 25,350) and (13, 0)
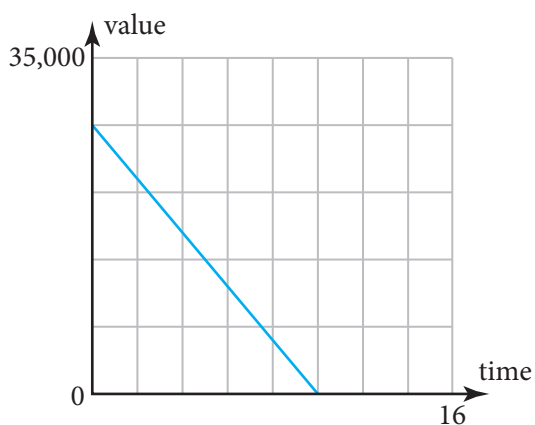   b. Determine the slope of the depreciation equation. −1,950
   c. Write the straight line depreciation equation that models this situation. $y = -1{,}950x + 25{,}350$
   d. Draw the graph of the straight line depreciation equation. See additional answers.

3. Vince purchased a used car for $11,200. This make and model used car straight line depreciates to zero after 7 years.
   a. Identify the coordinates of the *x*- and *y*-intercepts for the depreciation equation. (0, 11,200) and (7, 0)
   b. Determine the slope of the depreciation equation. −1,600
   c. Write the straight line depreciation equation that models this situation. $y = -1{,}600x + 11{,}200$
   d. Draw the graph of the straight line depreciation equation.
      See additional answers.

4. Examine the straight line depreciation graph for a car.
   a. At what price was the car purchased? $28,000
   b. After how many years does the car totally depreciate? 10 years
   c. Write the equation of the straight line depreciation graph shown. $y = -2{,}800x + 28{,}000$

5. The straight line depreciation equation for a luxury car is $y = -3{,}400x + 85{,}000$.
   a. What is the original price of the car? $85,000
   b. How much value does the car lose per year? $3,400
   c. How many years will it take for the car to totally depreciate? 25 years

6. The straight line depreciation equation for a motorcycle is $y = -2{,}150x + 17{,}200$.
   a. What is the original price of the motorcycle? $17,200
   b. How much value does the motorcycle lose per year? $2,150
   c. How many years will it take for the motorcycle to totally depreciate? 8 years

7. The straight line depreciation equation for a car is $y = -2{,}750x + 22{,}000$.
   a. What is the car worth after 5 years? $8,250
   b. What is the car worth after 8 years? $0
   c. Suppose that *A* represents a length of time in years when the car still has value. Write an algebraic expression to represent the value of the car after *A* years. −2,750*A* + 22,000

8. The straight line depreciation equation for a car is
$y = -2,680x + 26,800$.

   a. How much is the car worth after 48 months? $16,080

   b. How much is the car worth after 75 months? $10,050

   c. Suppose that M represents the length of time in months when the car still has value. Write an algebraic expression to represent the value of this car after M months. $-2,680\left(\dfrac{M}{12}\right) + 26,800$

9. The graph of a straight line depreciation equation is shown.

   a. Use the graph to approximate the value of the car after 4 years. $12,800

   b. Use the graph to approximate the value of the car after 5 years. $9,600

   c. Use the graph to approximate when the car will be worth half its original value. 4 years

10. A car is originally worth $34,450. It takes 13 years for this car to totally depreciate.

   a. Write the straight line depreciation equation for this situation. $y = -2,650x + 34,450$

   b. How long will it take for the car to be worth half its value? 6.5 years

   c. How long will it take for the car to be worth $10,000? Round your answer to the nearest tenth of a year. 9.2 years

11. The original price of a car is entered into spreadsheet cell A1 and the length of time it takes to totally depreciate is entered into cell B1.

   a. Write the spreadsheet formula that calculates the amount that the car depreciates each year. =A1/B1

   b. The spreadsheet user is instructed to enter a length of time in years that is within the car's lifetime in cell C1. Write the spreadsheet formula that will calculate the car's value after that period of time. =-(A1/B1)*C1+A1

12. The original price of a car is entered into spreadsheet cell A1 and the annual depreciation amount in cell B1.

   a. Write the spreadsheet formula to determine the number of years it will take for the car to totally depreciate. =A1/B1

   b. The spreadsheet user is instructed to enter a car value in cell D1. Write the spreadsheet formula to compute how long it will take for the car to depreciate to that value. =(D1–A1)/(–B1)

   c. The spreadsheet user is instructed to enter a percent into cell E1. Write the spreadsheet formula to compute the length of time it will take for the car to decrease by that percent. =(((100–E1)/100)*A1–A1)/(–B1)

13. Winnie purchased a new car for $54,000. She has determined that it straight line depreciates to zero over 10 years. When she purchased the car, she made an $8,000 down payment and financed the rest with a 4-year loan at 4.875%. You can use the monthly payment formula from the last chapter to determine the monthly payment to the nearest cent. depreciation: $y = -450x + 54,000$; expense: $y = 1,056.74x + 8,000$

   a. Create an expense and depreciation function.

   b. Graph these functions on the same axes. See additional answers.

   c. Interpret the region before, at, and after the intersection point in light of the context of this situation. See margin.

5-6 Historical and Exponential Depreciation

Objectives

- Write, interpret, and graph an exponential depreciation equation.

- Manipulate the exponential depreciation equation in order to determine time, original price, and depreciated value.

Key Terms

- dollar value
- historical data
- historical depreciation
- exponential decay
- exponential depreciation

Age	Value ($)	Age	Value ($)
1	24,230	6	15,245
2	22,355	7	14,075
3	20,645	8	13,100
4	18,070	9	12,325
5	16,265	10	11,525

HOW DOES YOUR CAR LOSE ITS VALUE?

In the previous lesson, you examined the depreciation of cars where the car lost the same amount of **dollar value** each year. That may not always be the case. You can often get a good idea of how a car loses its value by looking at prices from the past. This information is known as **historical data**, and the devaluation of a car when using this type of data is called **historical depreciation**.

There are many websites that list the prices of used cars. One well-known site is *Kelley Blue Book.* Before the Internet, the *Kelley Blue Book* was an actual book of historical car prices that could be used to determine the current value of a used car. Today, the website gives the same information in a much easier to access format.

Examine the data of used car prices for a Chevrolet Corvette 2-door Coupe in good condition. The table shows the age of the car in years and the value of the car at that time. The prices quoted are for cars with similar usage for their age and offered for sale in the same geographic location.

The scatterplot of this data is shown. Notice that it is not linear, but rather appears to be curved. The car values seem to have a greater drop at the beginning of the car's lifetime and less as each year passes. Notice that the depreciation is not constant from year to year. This scatterplot models an **exponential decay** function. Rather than the value decreasing by the same dollar amount each year, it decreases by the same percentage each year. In the context of auto devaluation, such a model is known as **exponential depreciation**. The general form of the exponential depreciation equation is

$$y = A(1 - r)^x$$

where A is the starting value of the car, r is the percent of depreciation expressed as a decimal, x is the elapsed time in years, and y is the car value after x years.

Skills and Strategies

The extent to which the exponential depreciation model fits the historical data varies from situation to situation. Here you will learn how to determine and use an exponential depreciation model.

EXAMPLE 1

Determine an exponential depreciation equation that models the data in the table from the previous page.

SOLUTION   The exponential depreciation function can be determined using exponential regression calculated by hand, by computer software, or by a graphing calculator. When you use the statistics feature on a graphing calculator, the data is entered into two lists as shown. (Note that only 7 of the 10 data are shown on the calculator screen.) The independent variable is the age of the car and the dependent variable is the car value.

L1	L2	L3
1	24230	------
2	22355	
3	20645	
4	18070	
5	16265	
6	15245	
7	14075	

L2(1)=24230

```
ExpReg
 y=a*b^x
 a=25921.87218
 b=.9189620427
```

The exponential regression equation is displayed in the graphing calculator screen at the right. Notice that the general form of the exponential regression equation used by the calculator is slightly different than the one introduced on the previous page. For ease of use, the numbers are rounded to the nearest hundredth. Using the format $y = a \times b^x$, where $a = 25{,}921.87$ and $b = 0.92$, the exponential depreciation function is $y = 25{,}921.87 \times (0.92)^x$. The graph of this function, superimposed over the scatterplot, appears to be a good fit.

■ **CHECK YOUR UNDERSTANDING**

How might a better-fitting exponential depreciation equation look when superimposed over the same scatterplot?

EXAMPLE 2

What is the depreciation percentage for the 10 years of car prices as modeled by the exponential depreciation equation found in Example 1?

SOLUTION   The exponential decay function was introduced as $y = A(1 - r)^x$. The graphing calculator uses the format $y = a*b^x$. Both formats are identical if you recognize that $b = 1 - r$.

Use the equation and solve for *r*. $b = 1 - r$

Subtract 1 from each side. $b - 1 = 1 - r - 1$

Simplify. $b - 1 = -r$

Divide each side by −1. $\dfrac{b - 1}{-1} = \dfrac{-r}{-1}$

Simplify. $1 - b = r$

Since b is approximately 0.92, then $1 - 0.92 = 0.08$. The Corvette depreciated by about 8% per year.

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■ **CHECK YOUR UNDERSTANDING**

After entering a set of automobile value data into a graphing calculator, the following exponential regression equation information is given: $y = a*b\char`\^x$, $a = 32{,}567.98722$, $b = 0.875378566$. Round the values to the nearest hundredth. Determine the depreciation percentage.

EXAMPLE 3

Eamon purchased a four-year-old car for $16,400. When the car was new, it sold for $23,000. Find the depreciation rate to the nearest tenth of a percent.

SOLUTION   Let r equal the depreciation rate expressed as a decimal. The exponential depreciation formula for this situation is $16{,}400 = 23{,}000\,(1 - r)^4$. Notice that the variable r is in the base of an exponential expression. To solve for r, you must first isolate that expression.

Use the exponential depreciation formula.

$$16{,}400 = 23{,}000(1 - r)^4$$

Divide each side by 23,000.

$$\frac{16{,}400}{23{,}000} = \frac{23{,}000(1 - r)^4}{23{,}000}$$

Simplify.

$$\frac{16{,}400}{23{,}000} = (1 - r)^4$$

To solve for r, you need to undo the exponent of 4 to which the expression $1 - r$ has been raised by raising each side of the equation to the reciprocal of 4, or $\frac{1}{4}$.

$$\left(\frac{16{,}400}{23{,}000}\right)^{\frac{1}{4}} = \left((1 - r)^4\right)^{\frac{1}{4}}$$

To simplify a power raised to an exponent, multiply the exponents. The exponent on the right side of the equation is 1.

$$\left(\frac{16{,}400}{23{,}000}\right)^{\frac{1}{4}} = (1 - r)^1$$

Simplify.

$$\left(\frac{16{,}400}{23{,}000}\right)^{\frac{1}{4}} = 1 - r$$

Subtract 1 from each side.

$$\left(\frac{16{,}400}{23{,}000}\right)^{\frac{1}{4}} - 1 = -r$$

Divide both sides by −1.

$$\frac{\left(\frac{16{,}400}{23{,}000}\right)^{\frac{1}{4}} - 1}{-1} = \frac{-r}{-1}$$

Simplify.

$$1 - \left(\frac{16{,}400}{23{,}000}\right)^{\frac{1}{4}} = r$$

Calculate.

$$0.0810772512 = r$$

Because r represents a percent expressed as a decimal, the depreciation rate rounded to the nearest tenth of a percent is 8.1%.

EXAMPLE 4

A car originally sold for $26,600. It depreciates exponentially at a rate of 5.5% per year. When purchasing the car, Richard put $6,000 down and pays $400 per month to pay off the balance. After how many years will his car value equal the amount he paid to date for the car?

SOLUTION   This problem is similar to Example 6 in Lesson 5-5. To find the solution, you need to set up both an expense equation and a depreciation equation.

The exponential depreciation equation is

$$y = 26{,}600(1 - 0.055)^x$$

where x represents time in years.

The expense equation is

$$y = 400x + 6{,}000$$

where x represents the number of months that have passed.

To graph these two equations on the same axes, the independent variable in each equation must represent the same unit of time.

If you let x represent time in years, then to make the expense equation work, you need to determine the yearly payment rather than the monthly payment.

Over the course of the year, Richard will have paid $400(12)$, or $4,800, in car payments.

The new yearly expense equation is

$$y = 4{,}800x + 6{,}000$$

where x is time in years.

Use the graph shown to determine an appropriate viewing window to use on your graphing calculator. Use the calculation feature to find the coordinates of the point of intersection.

After approximately 3.3 years (about 40 months), Richard will have paid about $22,022.74 toward his loan payments and the car will have a value of that same amount.

CHECK YOUR UNDERSTANDING

Describe the situation pictured above after 4 years.

EXAMPLE 4

This example lends itself to a graphical solution. The algebraic solution is beyond the scope of this course. The intersection of the exponential depreciation equation and the expense equation yields the point at which the values are the same.

Make sure that you emphasize that the depreciation equation uses time in years and the original statement of the expense equation uses time in months. Therefore, the monthly payment of $400 must be converted to a yearly payment of $4,800 in order that both equations can be graphed on the same axes.
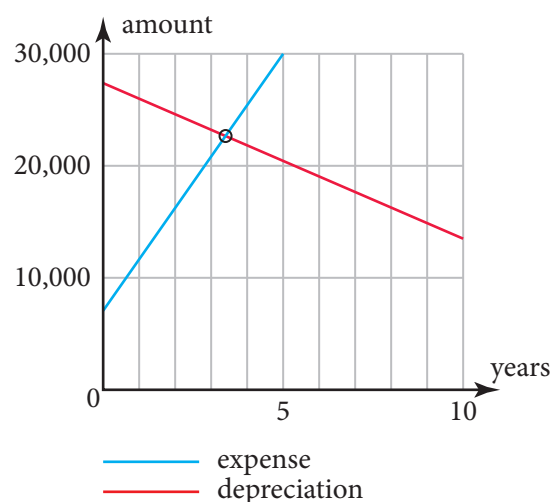
CHECK YOUR UNDERSTANDING

Answer The amount he has paid toward the loan of the car is more than what the car is worth.

ANSWERS (for page 257)

1. To some people, the value of a car is more than monetary. It is emotional as well. While a car may depreciate monetarily, some people find that it has increased in personal value.

EXAMPLE 5

A car exponentially depreciates at a rate of 6% per year. Beth purchased a 5-year-old car for $18,000. What was the original price of the car when it was new?

SOLUTION

Use the exponential depreciation equation. $\quad\quad y = A(1 - r)^x$

Substitute 18,000 for y, 0.06 for r, and 5 for x. $\quad 18{,}000 = A(1 - 0.06)^5$

Simplify. $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 18{,}000 = A(0.94)^5$

Divide each side by $(0.94)^5$. $\quad\quad\quad \dfrac{18{,}000}{0.94^5} = \dfrac{A(0.94)^5}{0.94^5}$

Simplify and calculate to the nearest cent. $\quad 24{,}526.37 = A$

The original price of this car was approximately $24,526.37.

■ **CHECK YOUR UNDERSTANDING**

A car depreciates exponentially at a rate of 5% per year. If the car is worth $30,000 after 9 months, what was the original price of the car?

EXAMPLE 6

Leah and Josh bought a used car valued at $20,000. When this car was new, it sold for $24,000. If the car depreciates exponentially at a rate of 8% per year, approximately how old is the car?

SOLUTION You need to solve for the variable x in the exponential depreciation equation

$$y = A(1 - r)^x$$

In the last chapter, you learned that solving for an exponent requires the use of natural logarithms. The length of time, x, can be determined using the following formula.

$$x = \frac{\ln\left(\dfrac{y}{A}\right)}{\ln(1 - r)}$$

Because y equals the value of the car after x years, $y = 20{,}000$. The new car price, A, is $24,000. The variable r represents the depreciation rate expressed as a decimal. Therefore, $r = 0.08$.

Substitute and calculate. $\quad x = \dfrac{\ln\left(\dfrac{20{,}000}{24{,}000}\right)}{\ln(1 - 0.08)} \approx 2.19$

At the time of the purchase, the car was about 2.19 years old.

■ **CHECK YOUR UNDERSTANDING**

How old would the car in Example 4 be had it been purchased at half its value?

Applications

I once bought an old car back after I sold it because I missed it so much and I had forgotten that it never ran . . .I just wanted it back. I could only remember what was good about it.
Connie Chung, Television News Commentator

1. How might the quote apply to what you have learned? *See margin.*

2. Seamus bought a car that originally sold for $40,000. It exponentially depreciates at a rate of 7.75% per year. Write the exponential depreciation equation for this car. $y = 40{,}000(1 - 0.0775)^x$

3. Shannon's new car sold for $28,000. Her online research indicates that the car will depreciate exponentially at a rate of $5\frac{1}{4}$% per year. Write the exponential depreciation formula for Shannon's car.
$y = 28{,}000(1 - 0.0525)^x$

4. Chris purchased a used car for $19,700. The car depreciates exponentially by 10% per year. How much will the car be worth after 6 years? Round your answer to the nearest penny. $10,469.39

5. Laura's new car cost her $21,000. She was told that this make and model depreciates exponentially at a rate of $8\frac{5}{8}$% per year. How much will her car be worth after 100 months? $9,903.32

6. Lisa purchased a used car for D dollars. The car depreciates exponentially at a rate of E% per year. Write an expression for the value of the car in 5 years, in A years, and in M months. $D(1 - E/100)^5$; $D(1 - E/100)^A$; $D(1 - E/100)^{(M/12)}$

7. A graphing calculator has determined this exponential regression equation based upon car value data: $y = a{*}b^x$, $a = 20{,}952.11$, and $b = 0.785$. What is the rate of depreciation for this car? How much is this car worth after 6 years; 78 months; w years? 21.5%; $4,902.82; $4,343.91; $20{,}952.11 \times 0.785^w$

8. A graphing calculator has determined this exponential regression equation based upon car value data: $y = a{*}b^x$, $a = 18{,}547.23$, and $b = 0.8625$. What is the rate of depreciation for this car? How much is this car worth after 6 years, 78 months, and w months? 13.75%; $7,635.43; $7,091.09; $18{,}547.23 \times 0.8625^{(w/12)}$

9. The historical prices of a car are recorded for 11 years as shown.
   a. Construct a scatterplot for the data. *See margin.*
   b. Determine the exponential depreciation equation that models this data. Round to the nearest hundredth. *See margin.*
   c. Determine the depreciation rate. approximately 11%
   d. Predict the value of this car after $3\frac{1}{2}$ years. $11,902.01

10. The historical prices of a car are recorded for 17 years as shown.
   a. Construct a scatterplot for the data. *See margin.*
   b. Determine the exponential depreciation formula that models this data. Round to the nearest hundredth. *See margin.*
   c. Determine the depreciation rate. approximately 11%
   d. Predict the value of this car after 140 months. $10,843.12

ANSWERS

9a.
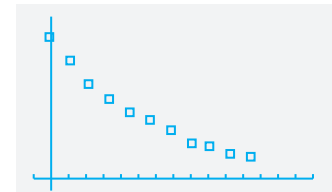
9b. $y = 17{,}895.97{*}(0.89)^x$

10a.
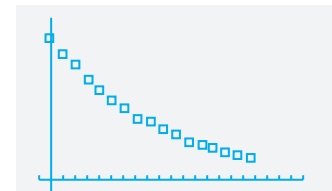
10b. $y = 42{,}228.36{*}0.89^x$

Age	Value ($)	Age	Value ($)
0	19,000	6	8,600
1	16,325	7	7,200
2	13,700	8	6,900
3	12,000	9	6,000
4	10,500	10	5,600
5	9,700		

Age	Value	Age	Value
0	42,000	9	14,800
1	37,420	10	13,000
2	34,000	11	11,245
3	29,400	12	10,211
4	26,200	13	9,400
5	23,700	14	8,100
6	20,990	15	7,500
7	18,200	16	6,290
8	16,876		

TEACH

Exercises 11–13
These problems require students to determine the original value of the car. Encourage them to use Example 5 as a model.

ANSWERS

20a. After 1967, the car appreciates exponentially in value.
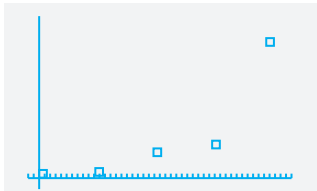
11. Raphael purchased a 3-year-old car for $16,000. He was told that this make and model depreciates exponentially at a rate of 5.45% per year. What was the original price of the car when it was new? $18,929.34

12. The car that Diana bought is 8 years old. She paid $6,700. This make and model depreciates exponentially at a rate of 14.15% per year. What was the original price of the car when it was new? $22,706.62

13. Chaz bought a two-year-old car. He paid D dollars. This make and model depreciates at a rate of E percent per year. Write an expression for the original selling price of the car when it was new. $D/(1 - E/100)^2$

14. What is the exponential depreciation rate, expressed as a percent to the nearest tenth of a percent, for a car that originally sells for $30,000 when new but exponentially depreciates after 5 years to $18,700? 9%

15. What is the exponential depreciation rate, expressed as a percent to the nearest tenth of a percent, for a car that originally sells for $52,000 when new but exponentially depreciates to $45,000 after 32 months? 5.3%

16. A new car sells for $27,300. It exponentially depreciates at a rate of 6.1% to $22,100. How long did it take for the car to depreciate to this amount? Round your answer to the nearest tenth of a year. 3.4 years

17. Amber bought a used car valued at $16,000. When this car was new, it was sold for $28,000. If the car depreciates exponentially at a rate of 9% per year, approximately how old is the car? 5.9 years

18. A car originally sold for $25,900. It depreciates exponentially at a rate of 8.2% per year. Nina put $10,000 down and pays $550 per month to pay off the balance. After how many years will her car value equal the amount she paid for the car to that point? What will that value be? 1.8 years; $22,131.10

19. Jazmine's car originally sold for $46,600. It depreciates exponentially at a rate of 10.3% per year. Jazmine put $12,000 down and pays $800 per month to pay off the balance. After how many years will her car value equal the amount she paid to date for the car? What will that value be? 2.5 years; $35,651.67

20. The July 2008 issue of *Hemmings Motor News* included a feature story on the 1957 Cadillac Eldorado Brougham. When sold as a new car in 1957, the price was $13,074. It depreciated in value over the next few years. Then, in 1967, something interesting began to happen as seen in this table of values.

Year	Value
1967	$2,500
1977	$5,500
1987	$18,500
1997	$25,000
2007	$95,000

   a. Construct a scatterplot for the data. Let 1967 be year 1, 1977 be year 11, 1987 be year 21, and so on. What do you notice about the trend? See margin.

   b. Find an exponential regression equation that models this situation. Round the numbers to the nearest hundredth. $y = 2{,}262.70 \times 1.09^x$

   c. What kind of a rate has been used? What is the value of that rate to the nearest tenth of a percent? exponential appreciation rate; 9%

Driving Data

5-7

Key Terms

- odometer
- electronic odometer
- mechanical odometer
- trip odometer
- speedometer
- fuel economy measurement
- miles per gallon (mpg)
- kilometers per liter (km/L)
- English Standard System
- Metric System
- distance formula
- currency exchange rate

Objectives

- Write, interpret, and use the distance formula.

- Use the formula for the relationship between distance, fuel economy, and gas usage.

WHAT DATA IS IMPORTANT TO A DRIVER?

The dashboard of an automobile is an information center. It supplies data on fuel, speed, time, and engine-operating conditions. It can also give information on the inside and outside temperature. Some cars even have a global positioning system mounted into the dashboard. This can help the driver find destinations or map out alternate routes. Your cellular phone can be wirelessly connected to your car so that you can send and receive hands-free calls. There have been many advances in the information that the driver has available to make trips safer, smarter, and more energy efficient.

The **odometer** indicates the distance a car has traveled since it left the factory. All automobiles have either an electronic or mechanical odometer. Some dashboard odometers can give readings in both miles and kilometers. An **electronic odometer** gives the readings digitally. A **mechanical odometer** consists of a set of cylinders that turn to indicate the distance traveled. Many cars also have a **trip odometer** which can be reset at the beginning of each trip. The trip odometer gives you the accumulated distance traveled on a particular trip. The **speedometer** tells you the rate at which the car is traveling. The rate, or speed, is reported in miles per hour (mi/h or mph) or kilometers per hour (km/h or kph).

Drivers are concerned not only with distance traveled and speed, but also with the amount of gasoline used. Gasoline is sold by the gallon or the liter. Over the past 20 years, the price of gasoline has changed dramatically. Economizing on fuel is a financial necessity. Car buyers are usually interested **fuel economy measurements**. These are calculated in **miles per gallon (mi/g** or **mpg)** or **kilometers per liter (km/L)**. In order to understand these fuel economy measurements, it is necessary to have a good sense of distances in both the **English Standard System** of measurement used in the United States, and the **Metric System** of measurement used in most countries throughout the world.

EXAMINE THE QUESTION

This is a very important question that will help you understand what students see as significant driving information. Newer cars give a variety of dashboard data that is helpful in knowing about average speed, time traveled, fuel efficiency, temperature, and so on. This question goes beyond the dashboard numbers and asks students to come up with other data that might at some time save their lives.

CLASS DISCUSSION

Based upon the equivalencies stated here, which is a greater distance—a mile or a kilometer? If a sign read "100 miles to the Canadian Border", would the numeral used to represent the number of kilometers be greater than 100 or less than 100?

A mile equals 5,280 feet. A meter is a little more than 3 feet. Driving distances are not reported in feet or meters, but in miles and kilometers. A kilometer is equal to 1,000 meters. Miles and kilometers can be compared as follows.

1 kilometer ≈ 0.621371 mile
1 mile ≈ 1.60934 kilometers

The distance from Seattle, Washington, to Vancouver, British Columbia, is about 176 kilometers or 110 miles. When traveling, it is important to use the correct measurement system. Miles per gallon is a unit of measurement that gives the number of miles a car can be driven on one gallon of gas. A car that gets 28 mpg can travel about 28 miles on one gallon. A car that gets 11.9 km/L can travel about 11.9 kilometers on one liter. There are about 3.8 liters in a gallon and 0.26 gallons in a liter. When shopping for a new car, always ask for the fuel estimate.

Skills and Strategies

A smart automobile owner is aware that a working knowledge of driving data can help reduce the costs of automobile ownership. Here you will learn how to use and interpret driving data.

EXAMPLE 1

A car travels at an average rate of speed of 50 miles per hour for 6 hours. How far does this car travel?

SOLUTION The distance that a car travels is a function of its speed and the time traveled. This relationship is shown in the **distance formula**

$$D = R \times T$$

where D represents the distance traveled, R represents the rate at which the car is traveling, and T is the time in hours.

Substitute 50 for D and 6 for T. $D = 50 \times 6$

Calculate. $D = 300$

The car travels 300 miles.

■ CHECK YOUR UNDERSTANDING

A car is traveling at R miles per hour for M minutes. Write an algebraic expression for the distance traveled.

EXAMPLE 2

Jack lives in New York and will be attending college in Atlanta, Georgia. The driving distance between the two cities is 883 miles. Jack knows that the speed limit varies on the roads he will travel from 50 mi/h to 65 mi/h. He figures that he will average about 60 mi/h on his trip. At this average rate, for how long will he be driving? Express your answer rounded to the nearest tenth of an hour and to the nearest minute.

SOLUTION

Use the distance formula.	$D = R \times T$
Divide each side by R.	$\dfrac{D}{R} = \dfrac{R \times T}{R}$
Simplify.	$\dfrac{D}{R} = T$
Substitute 883 for D and 60 for R.	$\dfrac{883}{60} = T$
Calculate.	$14.71\overline{6} = T$

The answer is a non-terminating, repeating decimal as indicated by the bar over the digit 6. The time rounded to the nearest tenth of an hour is 14.7 hours.

If you are using a calculator and the display reads 14.71666667, the calculator has rounded the last digit, but it stores the repeating decimal in its memory. Because you know that the exact time is between 14 and 15 hours, use only the decimal portion of the answer. Once the answer is on the calculator screen, subtract the whole number portion.

$$14.7166666667 - 14 = 0.7166666667$$

The number of sixes displayed will depend upon the accuracy of your calculator. There are 60 minutes in an hour, so multiply by 60.

$$0.7166666667 \times 60 = 43$$

The decimal portion of the hour is 43 minutes. Jack will be driving for 14 hours and 43 minutes.

■ CHECK YOUR UNDERSTANDING

Danielle drove from Atlanta, Georgia, to Denver, Colorado, which is a distance of 1,401 miles. If she averaged 58 miles per hour on her trip, how long is her driving time to the nearest minute?

EXAMPLE 3

Kate left Albany, New York, and traveled to Montreal, Quebec. The distance from Albany to the Canadian border is approximately 176 miles. The distance from the Canadian border to Montreal, Quebec, is approximately 65 kilometers. If the entire trip took her about $3\frac{3}{4}$ hours, what was her average speed for the trip?

SOLUTION Kate's average speed can be reported in miles per hour or kilometers per hour. To report her speed in miles per hour, convert the entire distance to miles. To change 65 kilometers to miles, multiply by the conversion factor 0.621371.

$$65 \times 0.621371 = 40.389115$$

The distance from the Canadian border to Montreal is approximately 40.4 miles. Kate's total driving distance is the sum of the distances from Albany to the Canadian border and from the Canadian border to Montreal.

$$176 + 40.4 = 216.4 \text{ miles}$$

EXAMPLE 4

This example provides an opportunity to show how to utilize the distance formula in another way. Students need to understand that the rate they are using in this problem is fuel economy measurement per gallons of fuel used.

Now, solve for the rate. Let $D = 216.4$ and $T = 3.75$.

Use the distance formula.	$D = R \times T$
Divide each side by T.	$\dfrac{D}{T} = \dfrac{R \times T}{T}$
Simplify.	$\dfrac{D}{T} = R$
Substitute 216.4 for D and 3.75 for T.	$\dfrac{216.4}{3.75} = R$
Calculate.	$57.7 \approx R$

Kate traveled at approximately 58 miles per hour.

Follow the same reasoning to determine her speed in kilometers per hour. To change the portion of the trip reported in miles to kilometers, multiply 176 by the conversion factor 1.60934.

$$176 \times 1.60934 \approx 283.2$$

There are approximately 283.2 kilometers in 176 miles.

The distance from Albany to Montreal is 283.2 + 65, or 348.2 kilometers.

Let $D = 348.2$ and $T = 3.75$ in the distance formula.

$$\frac{348.2}{3.75} = R$$
$$92.85\overline{3} = R$$

Kate traveled approximately 93 kilometers per hour.

■ CHECK YOUR UNDERSTANDING

In Example 3 above, could Kate's km/h have been calculated by multiplying her miles per hour by the conversion factor? Explain your answer.

EXAMPLE 4

Juan has a hybrid car that averages 40 miles per gallon. His car has a 12-gallon tank. How far can he travel on one full tank of gas?

SOLUTION The distance traveled can also be expressed as a function of the fuel economy measurement and the number of gallons used.

$$\text{Distance} = \text{miles per gallon} \times \text{gallons}$$
$$\text{Distance} = \text{kilometers per liter} \times \text{liters}$$

Therefore, the distance that Juan can travel on one tank of gas is the product of his miles per gallon and the tank size in gallons.

$$\text{Distance} = 40 \times 12 = 480 \text{ miles}$$

When traveling at an average rate of 40 mpg, one full tank of gas in Juan's hybrid car can take him 480 miles.

■ CHECK YOUR UNDERSTANDING

Lily drove a total of 500 miles on g gallons of gas. Express her fuel economy measurement in miles per gallon as an algebraic expression.

EXAMPLE 5

When Barbara uses her car for business, she must keep accurate records so that she will be reimbursed for her car expenses. When she started her trip, the odometer read 23,787.8. When she ended the trip it read 24,108.6. Barbara's car gets 32 miles per gallon. Her tank was full at the beginning of the trip. When she filled the tank, it cost her $40.10. What price did she pay per gallon of gas on this fill-up?

SOLUTION   Begin by computing the distance Barbara traveled. Find the difference between her ending and beginning odometer readings.

$$24,108.6 - 23,787.8 = 320.8$$

Barbara traveled 320.8 miles.

Since Barbara's car gets 32 mpg, you can determine the number of gallons of gas used on the trip with the formula

$$D = M \times G$$

where D is the distance traveled, M is the miles per gallon, and G is the number of gallons used.

Use the formula.	$D = M \times G$
Substitute 320.8 for D and 32 for M.	$320.8 = 32G$
Divide each side by 32.	$\dfrac{320.8}{32} = \dfrac{32G}{32}$
Simplify.	$\dfrac{320.8}{32} = G$
Calculate.	$10.025 = G$

Barbara used 10.025 gallons of gas on this trip.

If her total gas bill was $40.10, divide this total amount by the number of gallons used to get the price per gallon paid.

$$\frac{40.10}{10.025} = \text{Price per gallon}$$

Barbara paid $4 per gallon for this fill-up.

■ CHECK YOUR UNDERSTANDING

Suppose a person begins a trip with an odometer reading of A miles and ends the trip with an odometer reading of B miles. If the car gets C miles per gallon and the fill-up of gas for this trip cost D dollars, write an algebraic expression that represents the price per gallon.

Students may have difficulty understanding when to multiply and when to divide to convert properly. Walking through this example using dimensional analysis will help them visualize the conversion process because they can see how the units cancel.

CHECK YOUR UNDERSTANDING

Answer $4.55 per gallon

EXAMPLE 7

Example 6 converts Mexican pesos per liter to U.S. dollars per gallon. Example 7 converts U.S. dollars per gallon to Mexican pesos per liter. Explaining to students that these are inverse operations may help them to more easily understand the conversions.

CHECK YOUR UNDERSTANDING

Answer approximately 0.70 per liter

EXAMPLE 6

David is driving in Mexico on his vacation. He notices that gas costs 8.50 Mexican pesos per liter. What is this equivalent to in U.S. dollars?

SOLUTION   David must find the current currency exchange rate. The **currency exchange rate** is a number that expresses the price of one country's currency calculated in another country's currency. Up-to-date exchange rates are available on the Internet.

David needs to know what 1 U.S. dollar (USD) is worth in Mexican pesos. For the time of his travel, 1 USD = 13.3 Mexican pesos. Divide the foreign currency amount paid for gas by the exchange rate.

$$8.50 \div 13.3 \approx 0.64$$

Each liter would cost him about 64 cents of U.S. currency. He knows there are approximately 3.8 liters in a gallon, so he can multiply 0.64×3.8 to determine the equivalent gas price if it was purchased with U.S. dollars per gallon.

The price of 8.50 Mexican pesos per liter is approximately $2.43 per gallon.

■ CHECK YOUR UNDERSTANDING

On a trip through Canada, Angie noticed that the average price of gas per liter was 1.28 Canadian dollars. If 1 USD is equivalent to approximately 1.07 Canadian dollars, what is the equivalent gas price per gallon in U.S. currency?

EXAMPLE 7

David knows that the price of gas in his home town is about $2.90 per gallon. How can he compare this price to the price paid in Example 6 for a liter?

SOLUTION   David needs to express the U.S. gas price as a price in USD per liter. There are approximately 3.8 liters in a gallon. Divide the price per gallon by 3.8 to determine the price per liter in USD.

$$2.90 \div 3.8 \approx 0.76$$

His home town gas price is equivalent to about 0.76 USD per liter. So gas is less expensive in Mexico, $0.64 < $0.76.

To compare the prices in pesos, multiply the USD amount by the exchange rate.

Exchange rate was 13.3. $0.76 \times 13.3 \approx 10.11$

The gas in his home town would sell for about 10.11 Mexican pesos. Just as the comparison in USD showed, the comparison in pesos shows that gas is less expensive in Mexico, 8.50 < 10.11.

■ CHECK YOUR UNDERSTANDING

In the Example 6 Check Your Understanding, Angie knew that the price of gas in her home town was $2.50 per gallon. What is the equivalent price in Canadian dollars per liter?

Applications

> *Is it sufficient that you have learned to drive the car, or shall we look and see what is under the hood? Most people go through life without ever knowing.*
>
> **June Singer,** Analyst and Writer

1. How might the quote apply to what you have learned? See margin.

2. Arthur travels for 3 hours on the freeway. His average speed is 55 mi/h. How far does he travel? 165 miles

3. Yolanda is planning a 778-mile trip to visit her daughter in Maryland. She plans to average 50 miles per hour. At that speed, approximately how long will the trip take? Express your answer to the nearest tenth of an hour. Then express your answer to the nearest minute. 15.6 h and 15 h 34 min

4. Steve's SUV has a 17-gallon gas tank. The SUV gets an estimated 24 miles per gallon. Approximately how far can the SUV run on half a tank of gas? 204 mi

5. Becky is planning a 2,100-mile trip to St. Louis to visit a college. Her car averages 30 miles per gallon. About how many gallons will her car use on the trip? 70

6. Robbie's car gets M miles per gallon. Write an algebraic expression that represents the number of gallons he would use when traveling 270 miles. $\frac{270}{M}$

7. Michael used his car for business last weekend. When he reports the exact number of miles he traveled, the company will pay him 52 cents for each mile. At the beginning of the weekend, the odometer in Michael's car read 74,902.6 miles. At the end of the weekend, it read 75,421.1 miles.
   a. How many miles did Michael drive during the weekend? 518.5 miles
   b. How much money should his company pay him for the driving? $269.62

8. Lenny's car gets approximately 20 miles per gallon. He is planning a 750-mile trip.
   a. About how many gallons of gas should Lenny plan to buy? 37.5 gallons
   b. At an average price of $4.10 per gallon, how much should Lenny expect to spend for gas? $153.75

9. Francois' car gets about 11 kilometers per liter. She is planning a 1,200-kilometer trip.
   a. About how many liters of gas should Francois plan to buy? Round your answer to the nearest liter. 109 liters
   b. At an average price of $1.45 per liter, how much should Francois expect to spend for gas? $158.05

10. Nola's car gets approximately 42 miles per gallon. She is planning to drive x miles to visit her friends.
    a. What expression represents the number of gallons of gas she should expect to buy? $\frac{x}{42}$
    b. At an average price of $2.38 per gallon, write an expression for the amount that Nola will spend for gas. $\frac{2.38x}{42}$

TEACH

Exercise 13
Students will need to use the problem-solving strategy of working backwards in order to solve this problem.

Exercise 15
Before assigning this problem, ask students how they would determine an appropriate graphing window.

11. Jason uses his car for business. He must keep accurate records so his company will reimburse him for his car expenses. When he started his trip, the odometer read 42,876.1. When he ended the trip it read 43,156.1. Jason's car gets 35 miles per gallon. His tank was full at the beginning of the trip. When he filled the tank, it cost $34.24. What price did he pay per gallon of gas on this fill-up? **$4.28**

12. Complete the chart for entries a–l. **See margin.**

Number of gallons purchased	Price per gallon	Total gas cost	Number of people in car pool	Gas cost per person
10	$3.99	**a.**	4	**g.**
12	$4.08	**b.**	5	**h.**
17	$4.15	**c.**	3	**i.**
26	$4.30	**d.**	6	**j.**
15	D	**e.**	4	**k.**
G	P	**f.**	C	**l.**

13. Alexandra uses her car for business. She knows that her tank was full when she started her business trip, but she forgot to write down the odometer reading at the beginning of the trip. When the trip was over, the odometer read 13,020.5. Alexandra's car gets 25 miles per gallon. When she filled up the tank with gas that cost $4.15 per gallon, her total bill for the trip was $59.76. Determine Alexandra's beginning odometer reading. **12,660.5**

14. Bill left Burlington, Vermont, and traveled to Ottawa, Ontario, the capital of Canada. The distance from Burlington to the Canadian border is approximately 42 miles. The distance from the Canadian border to Ottawa is approximately 280 kilometers. If it took him 4.3 hours to complete the trip, what was his average speed in miles per hour? **about 50 mi/h**

15. A car averages 56 mi/h on a trip.
  a. Write an equation that shows the relationship between distance, rate, and time for this situation. $D = 56T$
  b. Let time be the independent variable and distance be the dependent variable. Draw and label the graph of this equation. **See additional answers.**
  c. Use the graph to determine approximately how far this car would travel after 14 hours. **about 800 miles**
  d. Use the graph to determine the approximate length of time a 500-mile trip would take. **approximately 9 hours**

16. A spreadsheet has been created so that the user enters information in the stated cells.
  a. Write a formula to calculate the speed of the car for the trip in cell C1. **=(A2−A1)/A4**
  b. Write a formula to calculate the number of gallons of gas used in cell C2. **=(A2−A1)/A3**
  c. Write a formula to calculate the total cost of gas for the trip in cell C3. **=C2*A5**

	A	B	C
1		Starting odometer reading in A1	
2		Ending odometer reading in A2	
3		Fuel efficiency measure in mpg in A3	
4		Duration of trip in A4	
5		Gas price per gallon in A5	

Use the following information to complete Exercises 17–22. Round all answers to two decimal places.

1 USD ≈ 1.07 Canadian dollars (CAD)   1 USD ≈ 89.85 Japanese yen (JPY)

1 USD ≈ 0.69 Euros (EUR)   1 USD ≈ 7.34 South African rand (ZAR)

1 USD ≈ 1.16 Australian dollars (AUD)   1 USD ≈ 1.00 Swiss franc (CHF)

17. Complete the chart. See margin.

USD	CAD	EUR	AUD
3.80	a.	b.	c.
15.75	d.	e.	f.
20.00	g.	h.	i.
178.50	j.	k.	l.
250.00	m.	n.	p.
5500.00	q.	r.	s.

18. Complete the chart. See margin.

Foreign Currency	USD Equivalent	Foreign Currency	USD Equivalent
85 CAD	a.	130 CHF	d.
1000 EUR	b.	222 ZAR	e.
500 AUD	c.	36 JPY	f.

19. Reid will be driving through Spain this summer. He did some research and knows that the average price of gas in Spain is approximately 1.12 euros per liter.
   a. What is this amount equivalent to in U.S. dollars? approx $1.62 USD/L
   b. What is this rate equivalent to in U.S. dollars per gallon? approx $6.16/gal

20. Shyla will be driving through South Africa. She has found that the average price of gas in Johannesburg is about 19.24 ZAR per liter.
   a. What is this amount equivalent to in U.S. dollars? approx $2.62 USD/L
   b. What is this rate equivalent to in U.S. dollars per gallon? approx $9.96/gal

21. Brenda will be driving through Europe. She plans to pay an average price of h euros per liter for gasoline.
   a. What is this amount equivalent to in U.S. dollars? $\frac{h}{0.69}$ USD/L
   b. What is this rate equivalent to in U.S. dollars per gallon? $\frac{3.8h}{0.69}$ per gal

22. While Willie traveled in India, he paid an average of 87.42 Indian rupees for a liter of gas.
   a. What expression represents the price of this gas in U.S. dollars if the exchange rate was x? $\frac{87.42}{x}$ USD/L
   b. What is this rate equivalent to in U.S. dollars per gallon? $\frac{3.8(87.42)}{x}$ per gal
   c. If Willie spent about $115, how many gallons of gas did he buy? $115 \div \frac{3.8(87.42)}{x}$
   d. If Willie spent about $115, how many liters of gas did he buy? $115 \div \frac{87.42}{x}$

ANSWERS
17. a. 4.07
    b. 2.62
    c. 4.41
    d. 16.85
    e. 10.87
    f. 18.27
    g. 21.40
    h. 13.80
    i. 23.20
    j. 191.00
    k. 123.17
    l. 207.06
    m. 267.50
    n. 172.50
    p. 290.00
    q. 5,885.00
    r. 3,795.00
    s. 6,380.00
18. a. 79.44
    b. 1,449.28
    c. 431.03
    d. 130.00
    e. 30.25
    f. 0.40

Nowhere in this country should we have laws that permit drinking and driving or drinking in vehicles that are on American highways. This is not rocket science. We know how to prevent this. . . .

Byron Dorgan, United States Senator

5-8 Driving Safety Data

Objectives
- Calculate reaction time and distance in the English Standard System.

- Calculate and use the braking distance in both the English Standard and Metric Systems.

- Calculate and use the total stopping distance in both the English Standard and Metric Systems.

Key Terms
- reaction time
- thinking time
- reaction distance
- braking distance
- total stopping distance

HOW CAN YOU USE MATHEMATICS TO BECOME A SAFER DRIVER?

Although a dashboard can give you much information about the car's ability to *go*, it gives little or no information about the car's ability to *stop*. It takes time to stop a moving car safely. Even during the time your foot switches from the gas pedal to the brake pedal, the car continues to travel.

The average, alert driver takes from approximately three-quarters of a second to one and a half seconds to switch from the gas pedal to the brake pedal. This time is the **reaction time** or **thinking time**. During the reaction time, the car travels a greater distance than most people realize. That distance is the **reaction distance**. The distance a car travels while braking to a complete stop is the **braking distance**. Most people think they can stop on a dime. In reality, that is far from the truth. Take a look at these facts.

- There are 5,280 feet in a mile.
- A car traveling 55 mi/h covers 55 miles in one hour.
- A car traveling 55 mi/h covers 55 × 5,280 or 290,400 feet in one hour.
- A car traveling 55 mi/h covers 290,400 ÷ 60 or 4,840 feet in one minute.
- A car traveling 55 mi/h covers 4,840 ÷ 60 or 80.67 feet in one second.

Suppose that your reaction time is one second. That is, it takes you one second from the time you realize that you have to brake to the time you actually apply your foot to the brake pedal. When traveling at 55 mi/h, in that one second of time, you travel about 81 feet.

By thinking about these facts, you can understand how speeding, tailgating, texting while driving, and driving while intoxicated can cost you in damages or even your life!

Skills and Strategies

Here you will learn how to make driving decisions based upon reaction and braking distances.

EXAMPLE 1

What is the reaction distance for a car traveling approximately 48 miles per hour?

SOLUTION 1   The reaction distance is the approximate distance covered in the time it takes an average driver to switch from the gas pedal to the brake pedal.

Research has determined that the average driver takes from 0.75 to 1.5 seconds to react.

A car traveling at 55 mi/h travels about 81 feet per second.

Let x = the distance traveled when the reaction time is 0.75 seconds.

Write a proportion.	$\dfrac{81}{1} = \dfrac{x}{0.75}$
Multiply each side by 0.75.	$\dfrac{81}{1} \times 0.75 = \dfrac{x}{0.75} \times 0.75$
Simplify.	$81 \times 0.75 = x$
Calculate.	$60.75 = x$

Let x = the distance traveled when the reaction time is 1.5 seconds.

Write a proportion.	$\dfrac{81}{1} = \dfrac{x}{1.5}$
Multiply each side by 1.5.	$\dfrac{81}{1} \times 1.5 = \dfrac{x}{1.5} \times 1.5$
Simplify.	$81 \times 1.5 = x$
Calculate.	$121.5 = x$

If the average person's reaction time ranges from 0.75 to 1.5 seconds, the average person's reaction distance when traveling at 55 mi/h ranges from 60.75 to 121.5 feet. That's quite a span in the short time it takes for a person to apply the brakes.

The reaction distances and times are used to give you a sense of how far the car will go. *A conservative rule of thumb for the reaction distance is that a car travels about one foot for each mile per hour of speed.*

Therefore, a car traveling at 48 mi/h has a reaction distance of approximately 48 feet.

> ### ■ CHECK YOUR UNDERSTANDING
>
> A car is traveling at 65 mi/h. Approximately how far will it travel during the average reaction time?

EXAMPLE 2

What is the approximate braking distance for a car traveling at 48 mi/h?

SOLUTION   The general formula for the braking distance is

$$\frac{s^2}{20}$$

where s represents the speed of the car. Because this formula is not accessible without a calculator, an equivalent is often used.

$$(0.1 \times s)^2 \times 5$$

Notice the four expressions below are equivalent.

$$(0.1 \times s)^2 \times 5 \qquad \left(\frac{1}{10} \times s\right)^2 \times 5 \qquad \left(\frac{s}{10}\right)^2 \times 5 \qquad \frac{5s^2}{100} = \frac{s^2}{20}$$

Each of the expressions yields the braking distance when $s = 48$.

$$\frac{s^2}{20} = \frac{48^2}{20} = 115.2 \quad \text{or} \quad (0.1 \times s)^2 \times 5 = (0.1 \times 48)^2 \times 5 = 115.2$$

Once the brakes are applied, on average, a car traveling at 48 mi/h will come to a complete stop after the car has traveled approximately 115.2 feet.

■ **CHECK YOUR UNDERSTANDING**

What factors also need to be taken into account that might add to or subtract from the braking distance?

EXAMPLE 3

Rachel is driving at 48 mi/h on a one-lane highway. She sees an accident directly ahead of her about 200 feet away. Will she be able to stop in time?

SOLUTION   The **total stopping distance** from the moment a driver realizes the need to stop to the time that the car is no longer moving is the sum of the reaction distance and the braking distance.

Total stopping distance = Reaction distance + Braking distance

Since the reaction distance of a car traveling at s miles per hour is approximated by using a distance of s feet, the formula can be represented by either of the following.

$$s + (0.1 \times s)^2 \times 5 \quad \text{or} \quad s + \frac{s^2}{20}$$

Rachel's total stopping distance is $48 + 115.2 = 163.2$ feet.

The accident is 200 feet away, so she should be able to stop in time.

■ **CHECK YOUR UNDERSTANDING**

What is the total stopping distance for a car traveling at 65 mi/h?

EXAMPLE 4

Desireé is traveling through Canada. The speedometer in her rented car indicates kilometers per hour and all of the road signs give distances in kilometers. She knows that one kilometer is equal to 1,000 meters and one meter is a little more than 3 feet. Determine Desireé's total stopping distance if she is traveling 88 kilometers per hour.

SOLUTION Since 1 kilometer ≈ 0.6213712 miles, 88 kilometers per hour can be expressed in miles per hour by multiplying 88 by the conversion factor.

$$88 \times 0.621371 = 54.680648$$

$$88 \text{ km/h} \approx 54.68 \text{ mi/h}$$

Evaluate the total stopping distance formula $s + (0.1 \times s)^2 \times 5$ when $s = 54.68$.

$$s + (0.1 \times s)^2 \times 5 = 54.68 + (0.1 \times 54.68)^2 \times 5 \approx 204.17512 \text{ feet}$$

There are approximately 0.3048 meters in 1 foot.

Multiply the stopping distance in feet by this conversion factor.

$$204.17512 \times 0.3048 \approx 62.23 \text{ meters}$$

The approximate stopping distance of Desireé's car is 62.23 meters.

Notice that this gives an answer that has been determined through various stages of rounding since you used rounded versions of answers and conversion factors along the way.

There is a formula that can be used to determine the total stopping distance directly. Let s represent the speed in kilometers per hour.

$$\text{Total stopping distance in meters} = \frac{s^2}{170} + \frac{s}{5}$$

Substitute $s = 88$. $= \dfrac{88^2}{170} + \dfrac{88}{5} \approx 63.15 \text{ meters}$

Notice that the two answers, 62.23 meters and 63.15 meters, are very close to each other.

■ CHECK YOUR UNDERSTANDING

A car is traveling at 78 km/h. What is the total stopping distance in meters? Round your answer to the nearest hundredth of a meter.

■ EXTEND YOUR UNDERSTANDING

Toni's car is traveling 75 km/h. Randy's car is behind Toni's car and is traveling 72 km/h. Toni notices a family of ducks crossing the road 50 meters ahead of her. Will she be able to stop before she reaches the ducks? What is the least distance that Randy's car can be from Toni's car to avoid hitting her car, if he reacts as soon as he sees her brakes?

EXAMPLE 4

The reaction and braking distance formulas used up to this point in this lesson require that the variables be measured in the customary system of measurement. Therefore, before using the formulas, it is necessary for the students to use the conversion factor to change metric measures into customary measures. The solution also shows students the formula that can be used when the units of measure are in the metric system. It compares the results when both methods are employed to solve the problem.

CHECK YOUR UNDERSTANDING

Answer approximately 51.39 meters

EXTEND YOUR UNDERSTANDING

Answer yes; 3.2 meters

Applications

Nowhere in this country should we have laws that permit drinking and driving or drinking in vehicles that are on American highways. This is not rocket science. We know how to prevent this. . . .

Byron Dorgan, United States Senator

ANSWERS

1. The total braking distance is substantial for a normal, non-impaired driver. People driving under the influence of alcohol or drugs significantly increase their reaction time which increases the total stopping distance.

3g. $\dfrac{5,280x}{60}$

3h. $\dfrac{5,280x}{3,600}$

6c. $\dfrac{1,000x}{60}$

6d. $\dfrac{1,000x}{3,600}$

1. Explain how the quote can be interpreted from what you have learned. See margin.

2. There are 5,280 feet in a mile. Round answers to the nearest unit.
   a. How many miles does a car traveling at 65 mi/h go in one hour? 65
   b. How many feet does a car traveling at 65 mi/h go in one hour? 343,200
   c. How many feet does a car traveling at 65 mi/h go in one minute? 5,720
   d. How many feet does a car traveling at 65 mi/h go in one second? 95

3. There are 5,280 feet in a mile. Round answers to the nearest unit.
   a. How many miles does a car traveling at 42 mi/h go in one hour? 42
   b. How many feet does a car traveling at 42 mi/h go in one hour? 221,760
   c. How many feet does a car traveling at 42 mi/h go in one minute? 3,696
   d. How many feet does a car traveling at 42 mi/h go in one second? 62
   e. How many miles does a car traveling at x mi/h go in one hour? x
   f. How many feet does a car traveling at x mi/h go in one hour? 5,280x
   g. How many feet does a car traveling at x mi/h go in one minute? See margin.
   h. How many feet does a car traveling at x mi/h go in one second? See margin.

4. Determine the distance covered by a car traveling 80 km/h for each unit and time given. Round answers to the nearest unit.
   a. kilometers in one hour 80   c. meters in one minute 1,333
   b. meters in one hour 80,000   d. meters in one second 22

5. Determine the distance covered by a car traveling 55 km/h for each unit and time given. Round answers to the nearest unit.
   a. kilometers in one hour 55   c. meters in one minute 917
   b. meters in one hour 55,000   d. meters in one second 15

6. Determine the distance covered by a car traveling x km/h for each unit and time given.
   a. kilometers in one hour x c. meters in one minute See margin.
   b. meters in one hour 1,000x d. meters in one second See margin.

7. Mindy is driving 32 mi/h as she nears an elementary school. A first-grade student runs into the street after a soccer ball, and Mindy reacts in about three-quarters of a second. What is her approximate reaction distance? 32 feet

8. Determine the distance covered by a car traveling 68 mi/h for each unit and time given. Round answers to the nearest unit.
   a. miles in one hour 68   c. feet in one minute 5,984
   b. feet in one hour 359,040   d. feet in one second 99.73

9. Edward is driving 52 mi/h on a one-lane road. He must make a quick stop because there is a stalled car ahead.
  a. What is his approximate reaction distance? 52 feet
  b. What is his approximate braking distance? 135.2 feet
  c. About how many feet does the car travel from the time he switches pedals until the car has completely stopped? 187.2 feet

10. Complete the chart for entries a–j. See margin.

Speed	Reaction Distance	Braking Distance
40 mi/h	a.	f.
30 mi/h	b.	g.
20 mi/h	c.	h.
15 mi/h	d.	i.
5 mi/h	e.	j.

11. David is driving on the highway at the legal speed limit of 70 mi/h. He notices that there is an accident up ahead approximately 200 feet away. His reaction time is approximately $\frac{3}{4}$ of a second. Is he far enough away to bring the car safely to a complete stop? Explain your answer. See margin.

12. Martine is driving on an interstate at 70 km/h. She sees a traffic jam about 50 meters ahead and needs to bring her car to a complete stop before she reaches that point. Her reaction time is approximately $\frac{3}{4}$ of a second. Is she far enough away from the traffic jam to safely bring the car to a complete stop? Explain. See margin.

13. Model the total stopping distance by the equation $y = \frac{x^2}{20} + x$, where x represents the speed in miles per hour and y represents the total stopping distance in feet.
  a. Graph this equation for the values of x, where $x \leq 70$ mi/h. See additional answers.
  b. Use the graph to approximate the stopping distance for a car traveling at 53 mi/h. about 185 feet
  c. Use the graph to approximate the speed for a car that stops completely after 70 feet. about 28 mi/h

14. Model the total stopping distance by the equation $y = \frac{x^2}{170} + \frac{x}{5}$, where x represents the speed in km/h and y represents the total stopping distance in meters.
  a. Graph this equation for the values of x, where $x \leq 100$ km/h. See additional answers.
  b. Use the graph to approximate the stopping distance for a car traveling at 60 km/h. about 33 m
  c. Use the graph to approximate the speed for a car that stops completely after 60 meters. about 85 km/h

15. A spreadsheet user inputs a speed in miles per hour into cell A1.
  a. Write a formula that would enter the approximate equivalent of that speed in km/h in cell A2. =A1*1.60934
  b. Write a spreadsheet formula that would enter the approximate total stopping distance in feet in cell A3. =A1^2/20 + A1
  c. Write a spreadsheet formula that would enter the approximate total stopping distance in kilometers in cell A4. =A2^2/170 + A2/5

5-9 Accident Investigation Data

Objectives

- Determine the minimum skid speed using the skid mark formula.

- Determine the minimum skid speed using the yaw mark formula.

Key Terms

- accident reconstructionist
- skid mark
- shadow skid mark
- anti-lock braking system (ABS)
- yaw mark
- skid speed formula
- drag factor
- braking efficiency
- skid distance
- chord
- middle ordinate

WHAT DATA MIGHT A CAR LEAVE BEHIND AT THE SCENE OF AN ACCIDENT?

Auto accidents happen. Many times it is clear who is at fault, but that may not always be the case. When fault is uncertain, it is up to the authorities to get detailed and accurate information from witnesses and each of the parties involved. It may be necessary to examine the data that was left behind at the scene. That data is interpreted by **accident reconstructionists**, who have knowledge of both crime scene investigations and mathematics that can help them understand the circumstances surrounding the accident.

Reconstructionists pay very close attention to the marks left on the road by the tires of a car. A **skid mark** is a mark that a tire leaves on the road when it is in a locked mode, that is, when the tire is not turning, but the car is continuing to move. When the driver first applies the brakes, the skid mark is light and is a **shadow skid mark**. This mark darkens until the car comes to a complete stop either on its own or in a collision.

Some cars have an **anti-lock brake system (ABS)**, which does not allow the wheels to continuously lock. In cars equipped with this feature, the driver feels a pulsing vibration on the brake pedal and that pedal moves up and down. The skid marks left by a car with ABS look like uniform dashed lines on the pavement. A driver without ABS may try to simulate that effect by pumping the brakes. The skid marks left by these cars are also dashed, but they are not uniform in length.

When a car enters a skid and the brakes lock (or lock intermittently), the driver cannot control the steering. Therefore, the skid is usually a straight line. The vehicle is continuing to move straight ahead as the wheels lock making the tire marks straight. When the vehicle is slipping sideways while at the same time continuing in a forward motion, the tire marks appear curved. These are called **yaw marks**.

Taking skid and yaw measurements, as well as other information from the scene, can lead reconstructionists to the speed of the car when entering the skid. The formulas used are often presented in court and are recognized for their strength in modeling real world automobile accidents.

Skills and Strategies

Here you will learn how to use the skid and yaw formulas to examine the circumstances surrounding an automobile accident.

The **skid speed formula** is

$$S = \sqrt{30 \cdot D \cdot f \cdot n} = \sqrt{30Dfn}$$

where S is the speed of the car when entering the skid, D is the skid distance, f is the **drag factor**, and n is the **braking efficiency**.

Before using the equation, it is important that you understand its component parts. The number 30 is a constant; it is part of the equation and does not change from situation to situation. Simply put, the drag factor is the pull of the road on the tires. It is a number that represents the amount of friction that the road surface contributes when driving. Many accident reconstructionists perform drag factor tests with a piece of equipment known as a drag sled. The table lists acceptable ranges of drag factors for the road surfaces.

Road Surface	Drag Factor Range
Cement	0.55–1.20
Asphalt	0.50–0.90
Gravel	0.40–0.80
Snow	0.10–0.55
Ice	0.10–0.25

The **skid distance** is a function of the number and lengths of the skid marks left at the scene. If there are four marks of equal length, then that amount is used. But, if the lengths are different or there are fewer than four skid marks, then the average of the lengths is used in the formula. If there is only one skid mark, that length is used.

Finally, you need to know about the braking efficiency of the car. This number is determined by an examination of the rear and front wheel brakes. It can run from 0% efficiency (no brakes at all) to 100% efficiency (brakes are in excellent condition). The braking efficiency number is expressed as a decimal when used in the formula.

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TEACH

This lesson will introduce students to formulas involving square roots. It is important that they have a working understanding of the domain and the nature of the square root function. Here, they will see use the fact that the square root and the square functions are inverses.

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EXAMPLE 1

A car is traveling on an asphalt road with a drag factor of 0.78. The speed limit on this portion of the road is 35 mi/h. The driver just had his car in the shop and his mechanic informed him that the brakes were operating at 100% efficiency. The driver must make an emergency stop, when he sees an obstruction in the road ahead of him. His car leaves four distinct skid marks each 80 feet in length. What is the minimum speed the car was traveling when it entered the skid? Round your answer to the nearest tenth. Was the driver exceeding the speed limit when entering the skid?

SOLUTION   Determine the car speed.

Use the skid speed formula.	$S = \sqrt{30Dfn}$
Substitute 80 for D, 0.78 for f, and 1.0 for n.	$S = \sqrt{30 \times 80 \times 0.78 \times 1.0}$
Simplify	$S = \sqrt{1,872}$
Take the square root.	$S \approx 43.3$

The car was traveling at approximately 43.3 miles per hour. The driver was exceeding the speed limit and could be fined.

■ **CHECK YOUR UNDERSTANDING**

A portion of road has a drag factor of x. A car with a y percent braking efficiency is approaching a traffic jam ahead, causing the driver to apply the brakes for an immediate stop. The car leaves four distinct skid marks of z feet each. Write an expression for determining the minimum speed of the car when entering into the skid.

EXAMPLE 2

Melissa was traveling at 50 mi/h on a concrete road with a drag factor of 1.2. Her brakes were working at 90% efficiency. To the nearest tenth of a foot, what would you expect the average length of the skid marks to be if she applied her brakes in order to come to an immediate stop?

SOLUTION   You are asked to find the skid distance given the speed, the drag factor, and the braking efficiency.

Use the skid speed formula.	$S = \sqrt{30Dfn}$
Substitute 50 for S, 1.2 for f, and 0.9 for n.	$50 = \sqrt{30 \times D \times 1.2 \times 0.9}$
Simplify the expression under the radical.	$50 = \sqrt{32.4D}$

It is necessary to solve for a variable that is under a radical sign. To undo the square root, square both sides.

$$(50)^2 = \left(\sqrt{32.4D}\right)^2$$

Simplify.

$$2{,}500 = 32.4D$$

Divide each side by 32.4.

$$\frac{2{,}500}{32.4} = \frac{32.4D}{32.4}$$

Simplify.

$$\frac{2{,}500}{32.4} = D$$

Round your answer to the nearest tenth.

$$77.2 \approx D$$

Under the given conditions, you would expect the average of the skid marks to be approximately 77.2 feet.

CHECK YOUR UNDERSTANDING

Answer $\dfrac{M^2}{\left(30A\dfrac{(x + y + z)}{3}\right)}$

■ **CHECK YOUR UNDERSTANDING**

Neil is traveling on a road at M miles per hour when he slams his foot on the brake pedal in order to avoid hitting a car up ahead. He is traveling on a gravel road with a drag factor of A and his brakes are operating at 100% efficiency. His car leaves three skid marks of length x, y, and z, respectively. Write an algebraic expression that represents the drag factor, A.

Yaw Marks

Examine how the minimum speed can be determined from the data available by measuring the yaw marks. If S is the minimum speed, f is the drag factor, and r is the radius of the arc of the yaw mark, the most basic formula is

$$S = \sqrt{15fr}$$

To identify a radius, you must be able to pinpoint the center of the circle of which the arc is part. Here is how reconstructionists do just that. First, they select two points on the outer rim of the arc and connect them with a chord. A **chord** is the line segment that connects two points on an arc or circle as shown.

The center of the chord is located and a perpendicular line segment is drawn from that center to the arc. A line is perpendicular to another line if it meets at a right angle. This short line segment is the **middle ordinate**.

Reconstructionists use the following formula to determine the radius.

$$r = \frac{C^2}{8M} + \frac{M}{2}$$
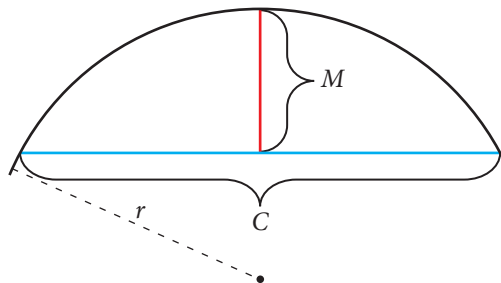
where r is the radius of the yaw arc, C is the length of the chord, and M is the length of the middle ordinate.

EXAMPLE 3

An accident reconstructionist took measurements from yaw marks left at a scene. Using a 43-foot length chord, she determined that the middle ordinate measured approximately 4 feet. The drag factor for the road surface was determined to be 0.8. Determine the radius of the curved yaw mark to the nearest tenth of a foot. Determine the minimum speed that the car was going when the skid occurred to the nearest tenth.

SOLUTION Solve for r by substituting 43 for C and 4 for M in the equation.

$$r = \frac{C^2}{8M} + \frac{M}{2}$$

$$r = \frac{43^2}{8 \cdot 4} + \frac{4}{2}$$

$$r \approx 59.8$$

The radius of the curve is approximately 59.8 feet.

Solve for S by substituting $r = 59.8$ and $f = 0.8$ in the equation.

$$S = \sqrt{15fr}$$

$$S = \sqrt{15 \cdot 0.8 \cdot 59.8}$$

$$S \approx 26.8$$

The car entered the skid with an approximate minimum speed of 26.8 miles per hour.

■ CHECK YOUR UNDERSTANDING

Determine the minimum speed of a car at the point the brakes are immediately applied to avoid a collision based upon a yaw mark chord measuring 62.4 feet and a middle ordinate measuring 5 feet. The drag factor of the road surface is 1.2. Round your answer to the nearest tenth.

Applications

It takes 8,460 bolts to assemble an automobile, and one nut to scatter it all over the road.

Author Unknown

1. Explain how the quote can be interpreted from what you have learned. See margin.

2. Ron's car left four skid marks on the road after he slammed his foot on the brake pedal to make an emergency stop. The police measured them to be 55 ft, 55 ft, 62 ft, and 62 ft. What skid distance will be used when calculating the skid speed formula? 58.5 ft

3. Jennie's car left three skid marks on the road surface in her highway accident. They measured 35 ft, 38 ft, and 47 ft. What skid distance will be used when calculating the skid speed formula? 40 ft

4. Kate's car left two skid marks each A feet long and two skid marks each B feet long after she had to immediately apply the brakes to avoid hitting a car. Write the algebraic expression that represents the skid distance that will be used in the skid speed formula. $\dfrac{A+B}{2}$

5. Rona was driving on an asphalt road that had a 35 mi/h speed limit posted. A deer jumped out from the side of the road causing Rona to slam on her brakes. Her tires left three skid marks of lengths 70 ft, 72 ft, and 71 ft. The road had a drag factor of 0.78. Her brakes were operating at 95% efficiency. The police gave Rona a ticket for speeding. Rona insisted that she was driving under the limit. Who is correct (the police or Rona)? Show your work. See margin.

6. In the spreadsheet, the prompts for entering data are in column A. The user enters the data in column B.
   a. Write the spreadsheet formula that will calculate the skid distance in cell B9. =sum(B5:B8)/B3
   b. Write the spreadsheet formula that will calculate the minimum skid speed in cell B11. The format for finding a square root in a spreadsheet is SQRT(number or expression). =sqrt(30*B1*B2*B9)
   c. Verify the accuracy of your formula for the following input values: drag factor, 0.6; braking efficiency, 0.8; and two skid marks, 45.3 ft and 48.2 ft. 25.9 mi/h

7. Ravi was driving on an asphalt road with a drag factor of 0.75. His brakes were working at 85% efficiency. He hit the brakes in order to avoid a dog that ran out in front of his car. Two of his tires made skid marks of 36 ft and 45 ft respectively. What was the minimum speed Ravi was going at the time he went into the skid? 27.8 mi/h

	A	B	C
1	Enter the road surface drag factor in B1.		
2	Enter the braking efficiency as a decimal in B2.		
3	Enter the number of skid marks on the road in B3.		
4	Enter lengths of skid marks. If fewer than 4 skid marks, enter measures and zero in the remaining cell(s).		
5	Skid mark #1 – cell B5		
6	Skid mark #2 – cell B6		
7	Skid mark #3 – cell B7		
8	Skid mark #4 – cell B8		
9	Calculated skid distance		
10	Minimum skid speed		

8. A car leaves four skid marks each 50 feet in length. The drag factor for the road is 0.9. Let x represent the braking efficiency.
   a. What is the range of values that can be substituted for x? 0 – 1 (0% – 100%)
   b. Let the speed be represented by the variable y and x represent the braking efficiency. Write the skid speed equation in terms of x and y. $y = \sqrt{30 \cdot 50 \cdot 0.9 \cdot x} = \sqrt{1,350x}$
   c. Graph the skid speed equation using the braking efficiency as the independent variable and the skid speed as the dependent variable. See additional answers.
   d. Use your graph to estimate the skid speed for braking efficiencies of 20%, 40%, 60%, 80%, and 100%. 20%, 16 mi/h; 40%; 22 mi/h; 60%, 29 mi/h; 80%, 33 mi/h; 100%, 37 mi/h

9. A car is traveling at 57 mi/h before it enters into a skid. The drag factor of the road surface is 1.1, and the braking efficiency is 100%. How long might the average skid mark be to the nearest tenth of a foot? 98.5 ft

10. Steve is driving at 35 mi/h when he makes an emergency stop. His wheels lock and leave four skid marks of equal length. The drag factor for the road surface was 0.97 and his brakes were operating at 90% efficiency. How long might the skid marks be to the nearest foot? 47 ft

11. Marielle was in an accident. She was traveling down a road at 36 mi/h when she slammed on her brakes. Her car left two skid marks that averaged 50 ft in length with a difference of 4 ft between them. Her brakes were operating at 80% efficiency at the time of the accident.
   a. What was the possible drag factor of this road surface? 1.08
   b. What were the lengths of each skid mark? 52 ft and 48 ft

12. An accident reconstructionist takes measurements of the yaw marks at the scene of an accident. What is the radius of the curve if the middle ordinate measures 4.8 feet when using a chord with a length of 42 ft? Round your answer to the nearest tenth of a foot. 48.3 ft

13. The measure of the middle ordinate of a yaw mark is 6 ft. The radius of the arc is 70 ft. What was the length of the chord used in this situation? Round the answer to the nearest tenth of a foot. 56.7 ft

14. The following measurements from yaw marks left at the scene of an accident were taken by the authorities. Using a 31-ft length chord, the middle ordinate measured approximately 3 ft. The drag factor for the road surface is 1.02.
   a. Determine the radius of the yaw mark to the nearest tenth of a foot. 41.5 ft
   b. Determine the minimum speed that the car was going when the skid occurred to the nearest tenth. 25.2 mi/h

15. Juanita is an accident reconstruction expert. She measured a 70-ft chord from the outer rim of the yaw mark on the road surface. The middle ordinate measured 9 ft in length. The drag factor of the road surface was determined to be 1.13.
   a. Determine the radius of the yaw mark to the nearest tenth of a foot. 72.6 ft
   b. Determine the minimum speed that the car was going when the skid occurred to the nearest tenth. 35.1 mi/h

16. The formula used to determine the radius of the yaw mark arc is derived from a geometric relationship about two intersecting chords in a circle. In the figure, chords $\overline{AB}$ and $\overline{CD}$ intersect at point E in the circle. The product of the two segment lengths making up chord $\overline{AB}$, $AE \times EB$, is equal to the product of the two segment lengths making up chord $\overline{CD}$, $CE \times ED$.
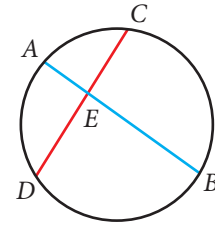
In the next figure, the yaw mark is darkened and it is continued to form a complete circle. A chord is drawn connecting two points on the yaw mark. The middle ordinate is also drawn. The length of the middle ordinate is M and the length of the chord is CD. The middle ordinate cuts the chord into two equal pieces with each half of the chord $\dfrac{CD}{2}$ units in length. The radius of the circle has length r as shown in the diagram. Applying the property to the two intersecting chords in this diagram, you get $AE \times EB = CE \times ED$.
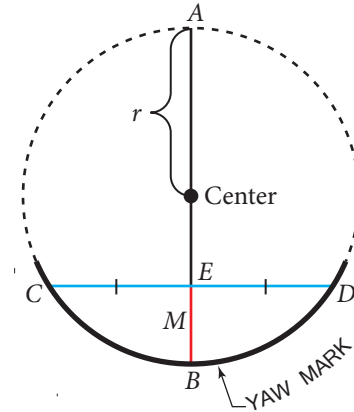
a. From the diagram, $CE = \dfrac{CD}{2}$, $ED = \dfrac{CD}{2}$, and $EB = M$. You need to determine the length of the segment AE. Notice that $AB = 2r$. (It is a diameter, which equals the length of two radii.) Also notice that $AE = AB - EB$. Write an algebraic expression that represents the length of AE. **$2r - M$**

b. Write the algebraic expression for the product of the segments of a chord that applies to this situation. Do not simplify. **$(2r - M)M = \left(\dfrac{CD}{2}\right)\left(\dfrac{CD}{2}\right)$**

c. Simplify the side of the equation that represents the product of the segments of chord $\overline{CD}$. Write the new equation. **$(2r - M)M = \dfrac{(CD)^2}{4}$**

d. Solve the equation for r by isolating the variable r on one side of the equation. Show your work. Compare your answer with the radius formula. **$r = \dfrac{(CD)^2}{8M} + \dfrac{M}{2}$**

17. In the spreadsheet, the prompts for entering data are in column A. The user enters the data in column B.

a. Write the spreadsheet formula that will calculate the radius in cell B4. **=(B2^2/(8*B3)+B3/2)**

b. Write the spreadsheet formula that will calculate the minimum skid speed in cell B5. The formula for finding that speed is found by taking the $\sqrt{15}$ times the product of the drag factor and the radius. **=sqrt(15*B1*B4)**

c. Verify the accuracy of your formula for the following input values: drag factor, 0.97; chord length, 47 ft; and middle ordinate, 5 feet. **approximately 29 mi/h**

	A	B	C
1	Enter the road surface drag factor in B1.		
2	Enter the length of the chord connecting two points on the yaw mark in B2.		
3	Enter the length of the middle ordinate in B3.		
4	Calculated radius		
5	Minimum skid speed		

18. Ghada works for an insurance company as an accident reconstruction expert. She measured a 52-ft chord from the outer rim of the yaw mark on the road surface. The middle ordinate measured x ft in length. The drag factor of the road surface was determined to be 1.05.

a. What is the expression for the radius of the yaw mark? **$\dfrac{x^2 + 676}{2x}$**

b. Determine the expression for the minimum speed that the car was going when the skid occurred. **$\sqrt{\dfrac{2x^2 + 31.5x + 676}{2x}}$**

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Real Numbers

You Write the Story!!

The graph below is a scatterplot and its regression curve. It gives actual hybrid car sales for the years 2000–2005 and then predictions for hybrid sales up until 2015. Write a short newspaper-type article centered on this graph. You can find an electronic copy at www.cengage.com/school/math/financialalgebra. Copy it and paste it into your article.
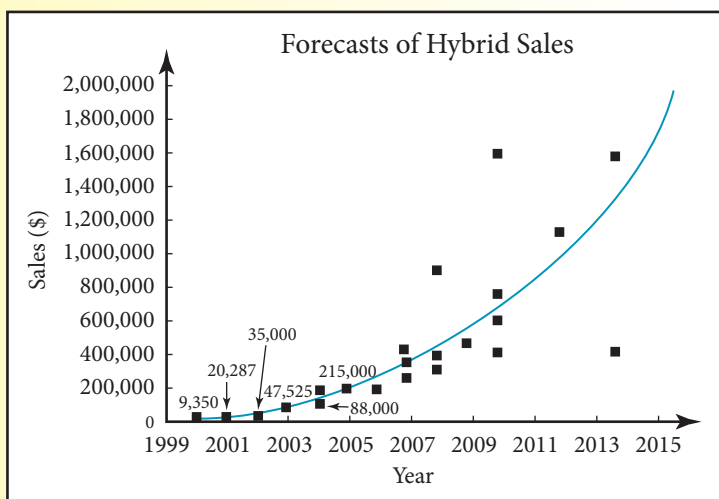
Source: hybridcars.com

Write a short newspaper-type article centered on this graph. You can find an electronic copy at www.cengage.com/school/math/financialalgebra. Copy it and paste it into your article.

Reality Check

1. Go to a new car dealership. Pick out a car and make a list of the options you would order. Find the price of the car, the price of each option, and the total cost. Compute the sales tax and make a complete list of any extra charges for new car delivery. Report your findings to the class.

2. Pick any new or used car you would like to own. Make a list of the options you would like the car to have. Search for used car price on the Internet and find out what the car is worth. Print pages that summarize the car and its value. Visit a local insurance agent, and find out the cost of insurance for the car. Display your findings on a poster.

3. Visit your local motor vehicle department or their website. Make a list of the forms needed to register a car and get license plates. If possible, get sample copies of each form. Show and explain each form to the class.

4. Pick a road trip you would like to make. Estimate the gas cost. Get hotel prices for any overnight stays. Get the full cost of staying at your destination. Approximate food expenses. Interview a local travel agent. Before the interview, list questions you want to ask. Include the expenses, questions, and answers from the interview in a report.

5. Talk to your teacher about having an insurance agent speak to your class. Have the class submit questions about automobile insurance. Copy the questions neatly on a sheet of paper and give it to the agent before the talk.

6. Write an ad to sell a used car. Contact several newspapers online to find the price of both a print and online ad for one week. Report your findings to the class.

7. Pick out a new or used car that you would like to own. Choose one of the following repair jobs: complete brake job, complete tune-up, or complete exhaust system replacement. Go to a garage or repair shop and get a price estimate for the job. Be sure to include parts and labor. Then go to an auto supply store and find out what each of the parts would cost. Compare the garage or repair shop's estimate of parts and labor to the cost of repairing the car yourself.

8. Interview a local insurance agent. Find out when premiums must be paid, types of discounts offered, insurance that is mandatory in your state, optional insurance that is available, and any other questions you can think of. Summarize your interview in a report.

9. Go online and find the cost of renting a car of your choice for two weeks. Pay particular attention to the limited damage waiver they offer. If you rent a car, you will be asked if you want to pay a limited damage waiver. This will reduce your liability for physical damage to the car. This type of insurance is expensive. Certain credit cards provide this coverage as a service. Go online and get contact information for two credit card companies. Contact them and ask which of their cards includes coverage for the limited damage waiver for a rented car. Give a report.

10. Flamboyant cars have graced movie and television screens for decades. Go online and/or to the library and make a list of famous cars. Give the make, model, and year of each car. Include information on where these cars are now and the highest price paid for each car as it changed owners. Add photos and other interesting facts about each car. Present your information on a poster.

11. A nomograph or nomogram is a chart that graphs the relationships between three quantities. Nomographs have been used in many fields such as medicine, physics, information technology, geology, and more. One such nomograph charts the fuel economy relationship—distance is equal to the miles per gallon fuel consumption of the car times the number of gallons used. Research the creation and usages of nomographs and find one that relates to fuel economy. Write a short description of this nomograph, explaining how it works and how it can be helpful to drivers. Include an example of the nomograph.

REALITY CHECK

You can offer students extra projects for extra credit.

Students completing Project 1 can also visit a bank and price a loan for their car. They can also combine it with Project 10, and price insurance for that same car. The loan payment and monthly insurance cost can give them a glimpse at the monthly cost of owning the car.

If a student chooses Project 14, you can make copies of the nomograph they downloaded and the student can show the class how to use it.

Go to www.cengage.com/school/math/financialalgebra where you will find a link to a website containing current issues about automobile ownership. Try one of the activities.

Really? Really!   REVISITED
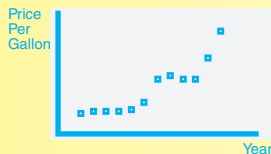
Take another look at the gas price table. As you marvel at how inexpensive gas prices may have seemed, remember one word—*inflation*. If you research the price of gas on the Internet and look at prices adjusted for inflation, you will be surprised. In 1960, the median annual income of United States families was $5,600. Nevertheless, it is always fascinating to look at different prices over the course of history.

Year	Price per Gallon
1950	$0.27
1955	$0.30
1960	$0.31
1965	$0.31
1970	$0.35
1975	$0.53
1980	$1.13
1985	$1.19
1990	$1.13
1995	$1.14
2000	$1.66
2005	$2.33

1. Enter the data from the table of gas prices and draw a scatter plot on a sheet of graph paper. See margin.

2. Go online and find out the average cost of a gallon of gas today. Answers vary.

3. Add today's cost to your scatterplot. Answers vary.

4. Draw a smooth curve that, by eye, looks like the best fit to the points on your scatterplot. Answers vary.

5. Go online and look up the median U.S. income for last year. Also find out the base price of this year's Corvette. Answers vary.

Applications

1. The college newspaper charges by the character for classified ads. Letters, numbers, spaces, and punctuation count as one character. They charge $34 for the first 100 characters, and $0.09 for each additional character. If x represents the number of characters, express the cost $c(x)$ of an ad as a piecewise function. Graph the function. *See margin.*

2. The *Classic Car Monthly* charges $49 for a three-line classified ad. Each additional line costs $9.50. For an extra $30, a seller can include a photo. How much would a five-line ad with a photo cost? *$98*

3. A local newspaper charges d dollars for a three-line classified ad. Each additional line costs a dollars. Express the cost of a six-line ad algebraically. *$d + 3a$*

4. The straight line depreciation equation for a car is $y = -2{,}400x + 36{,}000$.
   a. What is the original price of the car? *$36,000*
   b. How much value does the car lose per year? *$2,400*
   c. How many years will it take for the car to totally depreciate? *15 years*

5. A car is originally worth $43,500. It takes 12 years for this car to totally depreciate. *$y = -3{,}625x + 43{,}500$*
   a. Write the straight-line depreciation equation for this situation.
   b. How long will it take for the car to be worth one quarter of its original price? *9 years*
   c. How long will it take for the car to be worth $20,000? Round your answer to the nearest tenth of a year. *6.5 years*

6. Prices for used stainless steel side trim for a 1957 Chevrolet convertible are $350, $350, $390, $400, $500, $500, $500, $600, $650, $725, $800, $850, $900, and $1,700. The prices vary depending on condition.
   a. Find the mean of the trim prices to the nearest dollar. *$658*
   b. Find the median of the trim prices. *$550*
   c. Find the mode of the trim prices. *$500*
   d. Find the four quartiles for this data. *Q_1 = $400; Q_2 = $550; Q_3 = $800; Q_4 = $1,700*
   e. Find the interquartile range for this data. *$400*
   f. Find the boundary for the lower outliers. Are there any lower outliers? *–$200; there are no lower outliers.*
   g. Find the boundary for the upper outliers. Are there any upper outliers? *$1,400; yes, there is one upper outlier: $1,700.*
   h. Draw a modified box-and-whisker plot. *See additional answers.*

7. Kathy purchased a new car for $37,800. From her research she has determined that it straight-line depreciates over 14 years. She made a $7,000 down payment and pays $710 per month for her car loan.
   a. Create an expense and depreciation function where x represents the number of months. *depreciation: $y = -225x + 37{,}800$; expense: $y = 710x + 7{,}000$*
   b. Graph these functions on the same axes. *See additional answers.*
   c. Interpret the region before, at, and after the intersection point in the context of this situation. *See margin.*

ANSWERS

1. $c(x) =$
$\begin{cases} 34 & \text{when } x \le 100 \\ 34 + 0.09(x - 100) & \text{when } x > 100 \end{cases}$
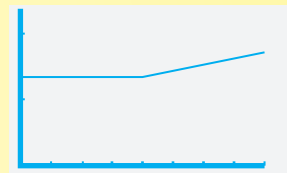
7c. Using a graphing tool, the coordinates of the intersection point, rounded to the nearest hundredth, are (31.87, 30628.88). This means that after a little less than 31 months, both the expenses-to-date and the car's value are the same. In the region before the intersection point, the expenses are lower than the value of the car. But, the region after the intersection point indicates a period of time that the value of the car is less than what was invested in it.

Stem-and-leaf plot:

```
1 | 1 8 9 9
2 | 0 0 3
3 | 4 6 6 7 7 8
4 | 0 1 2
5 | 5 5 6 8 9 9
6 | 1 2 2
      4| 1 = 41
```

ANSWERS

10a.

11c. $50,000 under PIP.

8. Grahamsville High School recently polled its teachers to see how many miles they drive to work each day. At the left is a stem-and-leaf plot of the results.
  a. How many teachers were polled? 25
  b. Find the mean to the nearest mile. 40
  c. Find the median. 38
  d. Find the mode(s). 19, 20, 36, 37, 55, 59, 62
  e. Find the range. 51
  f. Find the four quartiles. $Q_1 = 21.5$; $Q_2 = 38$; $Q_3 = 57$; $Q_4 = 62$
  g. What percent of the teachers travel more than 38 miles to work? 48%
  h. Find the interquartile range. 35.5
  i. What percent of the teachers travel from 38 to 57 miles to work? 28%

9. Stewart has $25,000 worth of property damage insurance and a $1,000 deductible collision insurance policy. He crashed into a fence when his brakes failed and did $7,000 worth of damage to the fence. The crash caused $3,600 in damages to his car.
  a. Which insurance covers the damage to the fence? property damage
  b. How much will the insurance company pay for the fence? $7,000
  c. Stewart's car still was drivable after the accident. On the way home from the accident, he hit an empty school bus and did $20,000 worth of damage to the bus and $2,100 worth of damage to his car. How much will the insurance company pay for this damage to the bus? $20,000
  d. Which insurance covers the damage to Stewart's car? collision
  e. How much will the insurance company pay for the damage to the car? $3,700

Historical Prices	
Age	**Value ($)**
0	32,000
1	29,100
2	26,500
3	24,120
4	21,950
5	20,000
6	18,100
7	16,500
8	15,000
9	13,700
10	12,500

10. The historical prices of a car with the same make, model, and features are recorded for a period of 10 years as shown in the table.
  a. Construct a scatterplot for the data. See margin.
  b. Determine the exponential depreciation formula that models this data. Round all numbers to the nearest hundredth. $y = 31,985.36 \times 0.91^x$
  c. Determine the depreciation rate to the nearest percent. approximately 9%
  d. Use the model equation to predict the value of this car after 66 months. Round to the nearest thousand dollars. $19,000

11. Gina has 250/500/50 liability insurance and $50,000 PIP insurance. She changes lanes too quickly, hits the metal guard rail, and then hits a tour bus. Four people are seriously hurt and sue her. Twenty others have minor injuries. Gina's boyfriend, who was in her car, was also hurt.
  a. The guard rail will cost $2,000 to replace. Gina also did $9,700 worth of damage to the bus. What insurance will cover this, and how much will the company pay? property damage under $11,700
  b. The bus driver severed his hand and cannot drive a bus again. He sues for $2,500,000 and is awarded $1,750,000 in court. What type of insurance covers this? How much will the insurance company pay? $250,000 under BI
  c. The bus driver (from part b) had medical bills totaling $90,000 from an operation after the accident. What type of insurance covers this, and how much will the insurance company pay? See margin.
  d. Gina's boyfriend is hurt and requires $19,000 worth of medical attention. What insurance covers this, and how much will the company pay? $19,000 under PIP

286   **Chapter 5**   **Automobile Ownership**

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12. Joshua just purchased a 4-year-old car for $12,000. He was told that this make and model depreciates exponentially at a rate of 5.8% per year. What was the original price to the nearest hundred dollars? $15,200

13. A graphing calculator has determined the following exponential regression equation: $y = a*b^x$, $a = 28,158.50$, $b = 0.815$.
   a. What is the rate of depreciation for this car? 18.5%
   b. How much is this car worth to the nearest dollar after 6 years? $8,252
   c. How much is this car worth to the nearest hundred dollars after 39 months? $14,500
   d. How much is this car worth after y years? $28,158.50(0.815^y)$

14. Jonathan's car gets approximately 25 miles per gallon. He is planning a 980-mile trip. About how many gallons of gas will his car use for the trip? At an average price of $4.00 per gallon, how much should Jonathan expect to spend for gas? Round to the nearest ten dollars. 39.2 gallons; $160

15. Ann's car gets about 12 kilometers per liter of gas. She is planning a 2,100 kilometer trip. To the nearest liter, how many liters of gas should Ann plan to buy? At an average price of $1.49 per liter, how much should Ann expect to spend for gas? 175 liters; $260.75

16. Max is driving 42 miles per hour. A dog runs into the street and Max reacts in about three-quarters of a second. What is his approximate reaction distance? 42 ft

17. Tricia is driving 64 miles per hour on an interstate highway. She must make a quick stop because there is an emergency vehicle ahead.
   a. What is her approximate reaction distance? 64 ft
   b. What is her approximate braking distance? 204.8 ft
   c. About how many feet does the car travel from the time she starts to switch pedals until the car has completely stopped? 268.8 ft

18. Marlena is driving on an interstate at 65 km/h. She sees a traffic jam about 30 meters ahead and needs to bring her car to a complete stop before she reaches that point. Her reaction time is approximately $\frac{3}{4}$ of a second. Is she far enough away from the traffic jam to safely bring the car to a complete stop? Explain. See margin.

19. Richie was driving on an asphalt road that had a 40 mi/h speed limit. A bicyclist darted out from the side of the road causing him to slam on his brakes. His tires left three skid marks of 69 ft, 70 ft, and 74 ft. The road had a drag factor of 0.95. His brakes were operating at 98% efficiency. The police gave Richie a ticket for speeding. Richie insisted that he was driving under the speed limit. Who is correct? Explain. See margin.

20. A car is traveling at 52 mi/h before it enters into a skid. It has been determined that the drag factor of the road surface is 1.05, and the braking efficiency is 80%. How long might the average skid mark be to the nearest tenth of a foot for this situation? 107.3 ft

21. A reconstructionist took measurements from yaw marks left at the scene of an accident. Using a 46-ft chord, the middle ordinate measured approximately 6 ft. The drag factor for the road surface was 0.95. Determine the radius of the yaw mark to the nearest tenth of a foot. Determine the minimum speed when the skid occurred to the nearest tenth mile. 47.1 ft; 25.9 mi/h