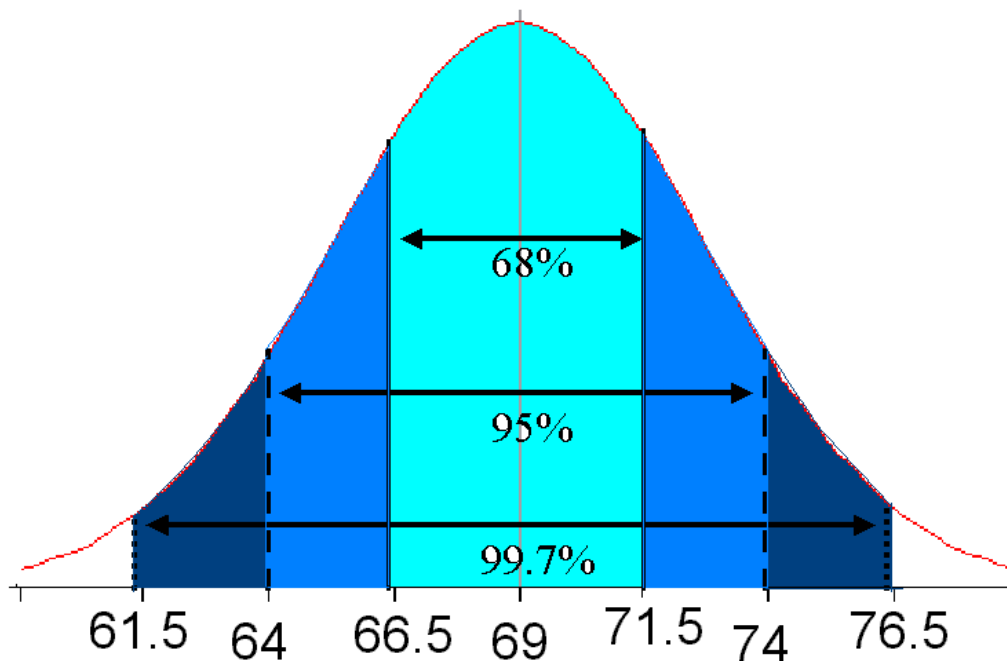


Unit 6: Data Analysis

Z-SCORE

Z-Scores are measurements of how far from the center (mean) a data value falls.



Ex: A man who stands 71.5 inches tall is **1** standardized standard deviation from the mean.

Ex: A man who stands 64 inches tall is **-2** standardized standard deviations from the mean.

Standardized Z-Score

To get a **Z-score**, you need to have 3 things

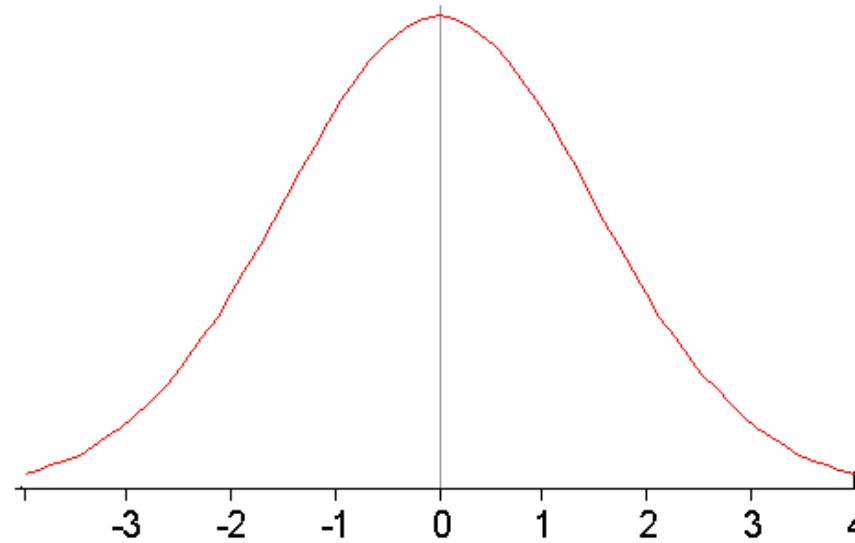
- 1) **Observed actual data value** of random variable x
- 2) **Population mean, μ** also known as expected outcome/value/center
- 3) **Population standard deviation, σ**

Then follow the formula.

$$Z = \frac{x - \mu}{\sigma}$$

Empirical Rule & Z-Score

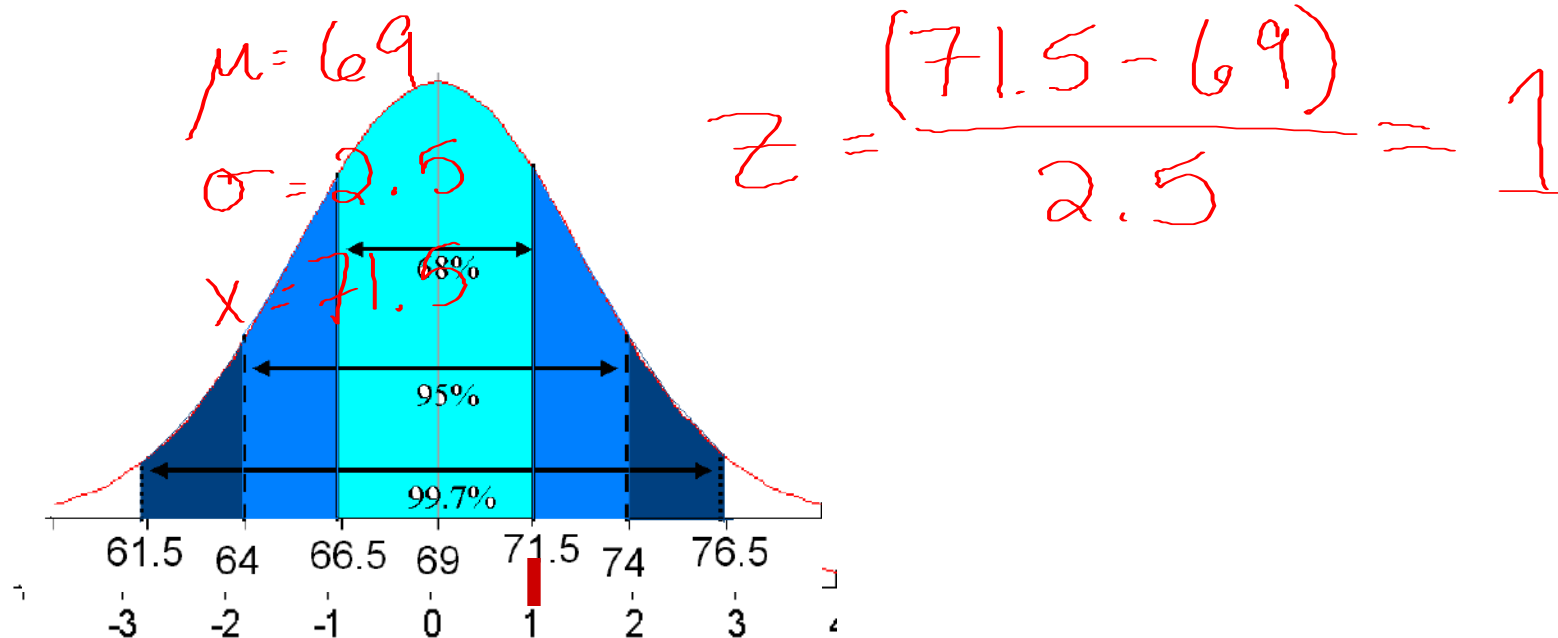
About 68% of data values in a normally distributed data set have z-scores between **-1 and 1**; approximately 95% of the values have z-scores between **-2 and 2**; and about 99.7% of the values have z-scores between **-3 and 3**.



Z-Score & Let $H \sim N(69, 2.5)$

mean *s.d.*

What would be the **standardized score** for an adult male who stood 71.5 inches?



$$H \sim N(69, 2.5) \quad Z \sim N(0, 1)$$

Z-Score & Let $H \sim N(69, 2.5)$

What would be the **standardized score** for an adult male who stood 65.25 inches?

$$\frac{65.25 - 69}{2.5} = -1.5$$



Comparing Z-Scores

Suppose Bubba's score on exam A was 65, where Exam A $\sim N(50, 10)$ and Bubbette's score was an 88 on exam B, where Exam B $\sim N(74, 12)$.

Who outscored who? Use Z-score to compare.

$$\text{Bubba: } \frac{65 - 50}{10} = 1.5$$

$$\text{Bubbette: } \frac{88 - 74}{12} = 1.17$$

Comparing Z-Scores

Heights for traditional college-age students in the US have means and standard deviations of approximately 70 inches and 3 inches for males and 165.1 cm and 6.35 cm for females. If a male college student were 68 inches tall and a female college student was 160 cm tall, who is relatively shorter in their respected gender groups?

$$\text{Male } z = (68 - 70)/3 = -.667$$

$$\text{Female } z = (160 - 165.1)/6.35 = -.803$$

What if I want to know the
PROBABILITY of a certain
z-score?

Use the calculator! Normcdf!!!

2nd Vars

2: normcdf(

normcdf(lower, upper, mean(0), std. dev(1))

Find $P(z < 1.85)$

Find $P(z > 1.85)$

Find $P(-.79 < z < 1.85)$

What if I know the probability that an event will happen, how do I find the corresponding z-score?

- 1) Use the z-score formula and work backwards!
- 2) Use the InvNorm command on your TI by entering in the probability value (representing the area shaded to the left of the desired z-score), then 0 (for population mean), and 1 (for population standard deviation).

$$P(Z < z^*) = .8289$$

What is the value of z^* ?

Using TI-84

```
0:QUIT DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:tpdf(
5:tcdf(
6:x²pdf(
7↓x²cdf(
```

```
invNorm(.77,0,1)
```

```
(area[, $\mu$ , $\sigma$ ])
```

```
| PASTE | ESC
```

```
invNorm(.77,0,1)
```

```
.7388468537
```


$$P(Z < x) = .80$$

What is the value of x ?

$$P(Z < z^*) = .77$$

What is the value of z^* ?

