Estimating Square Roots

Warm Up

Find the two square roots of each number.

1. 144 ±12

2. 256 ±16

Evaluate each expression.

3. $8+\sqrt{144}$ 20

4. $7\sqrt{289}$ 119

Estimating Square Roots

Essential Question:
How do you solve problems using square roots?

Standards:

MCC8.NS.2:
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.

EXPLORE Estimating Irrational Numbers

Estimate the value of $\sqrt{2}$.

WB: pg. 93

A Since 2 is not a perfect square, $\sqrt{2}$ is irrational.

To estimate $\sqrt{2}$, first find two consecutive perfect squares that 2 is between. Complete the inequality by writing these perfect squares in the boxes.

$$\square < 2 < \square$$

Now take the square root of each number.

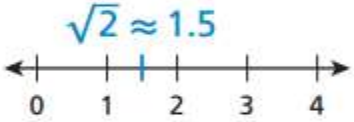
$$\sqrt{\square} < \sqrt{2} < \sqrt{\square}$$

Simplify the square roots of perfect squares.

$$\square < \sqrt{2} < \square$$

$\sqrt{2}$ is between _____ and _____.

Estimate that $\sqrt{2} \approx 1.5$.



B To find a better estimate, first choose some numbers between 1 and 2 and square them. For example, choose 1.3, 1.4, and 1.5.

$1.3^2 =$ _____ $1.4^2 =$ _____ $1.5^2 =$ _____

Is $\sqrt{2}$ between 1.3 and 1.4? How do you know?

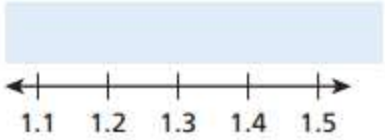
_____

Is $\sqrt{2}$ between 1.4 and 1.5? How do you know?

_____

$\sqrt{2}$ is between _____ and _____, so $\sqrt{2} \approx$ _____.

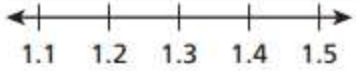
Locate and label this value on the number line.



REFLECT

$\sqrt{2}$ is between _____ and _____, so $\sqrt{2} \approx$ _____.

Locate and label this value on the number line.



REFLECT

1a. How could you find an even better estimate of $\sqrt{2}$?

1b. Use your method from **1a** to find a better estimate of $\sqrt{2}$.
Draw a number line and locate and label your estimate.

$\sqrt{2}$ is between _____ and _____, so $\sqrt{2} \approx$ _____.

TRY THIS!

1c. Estimate the value of $\sqrt{7}$ to the nearest hundredth.
Draw a number line and locate and label your estimate.

$\sqrt{7}$ is between _____ and _____, so $\sqrt{7} \approx$ _____.

MCC8.NS.2

2 EXAMPLE

Estimating Square Roots

Additional Example 1A: Estimating Square Roots of Numbers

The square root is between two integers. Name the integers. Explain your answer.

$\sqrt{55}$ *Think: What are perfect squares close to 55?*

$7^2 = 49$ *49 < 55*

$8^2 = 64$ *64 > 55*

$\sqrt{55}$ is between 7 and 8 because 55 is between 49 and 64.

Estimating Square Roots

Additional Example 1B: Estimating Square Roots of Numbers Continued

The square root is between two integers. Name the integers. Explain your answer.

$-\sqrt{90}$ *Think: What are perfect squares close to 90?*

$-9^2 = 81$ *81 < 90*

$-10^2 = 100$ *100 > 90*

$-\sqrt{90}$ is between −9 and −10 because 90 is between 81 and 100.

Estimating Square Roots

Check It Out: Example 1A

The square root is between two integers. Name the integers. Explain your answer.

$\sqrt{80}$ *Think: What are perfect squares close to 80?*

$8^2 = 64$ *64 < 80*

$9^2 = 81$ *81 > 80*

$\sqrt{80}$ is between 8 and 9 because 80 is between 64 and 81.

Estimating Square Roots

Check It Out: Example 1B

The square root is between two integers. Name the integers.

$-\sqrt{45}$ *Think: What are perfect squares close to 45?*

$(-6)^2 = 36$ *36 < 45*

$(-7)^2 = 49$ *49 > 45*

$-\sqrt{45}$ is between –6 and –7 because 45 is between 36 and 49.

Estimating Square Roots

Additional Example 2: *Application*

You want to sew a fringe on a square tablecloth with an area of 500 square inches. Calculate the length of each side of the tablecloth and the length of fringe you will need to the nearest tenth of an inch.

The length of each side of the square is $\sqrt{500}$.

$484 < 500 < 529$ *List the perfect squares nearest 500.*

$\sqrt{484} < \sqrt{500} < \sqrt{529}$ *Find the square roots of the perfect squares.*

$22 < \sqrt{500} < 23$ *The number will be between 22 and 23.*

$\sqrt{500} \approx 22.4$

The length of each side of the table is about 22.4 in., and you will need about 89.6 in. of fringe.

Estimating Square Roots

Check It Out: Example 2

A tent was advertised in the newspaper as having an enclosed square area of 168 ft². What is the approximate length of the sides of the square area? Round your answer to the nearest foot.

The length of each side of the square is $\sqrt{168}$

$144 < 168 < 169$ *List the perfect squares nearest 168.*

$\sqrt{144} < \sqrt{168} < \sqrt{169}$

$12 < \sqrt{168} < 13$ *Find the square roots of 144 and 169.*

$\sqrt{168} \approx 13$ *168 is closer to 169 than to 144.*

Each side of the tent is about 13 feet long.

Estimating Square Roots

Additional Example 3: Approximating Square Roots to the Nearest Hundredth

Approximate $\sqrt{141}$ to the nearest hundredth.

Step 1: Find the value of the whole number.

$121 < 141 < 144$ *Find the perfect squares nearest 141.*

$\sqrt{121} < \sqrt{141} < \sqrt{144}$ *Find the square roots of the perfect squares.*

$11 < \sqrt{141} < 12$ *The number will be between 11 and 12.*

The whole number part of the answer is 11.

Estimating Square Roots

Additional Example 3 Continued

Approximate $\sqrt{141}$ to the nearest hundredth.

Step 2: Find the value of the decimal.

$141 - 121 = 20$ *Find the difference between the given number, 141, and the lower perfect square.*

$144 - 121 = 23$ *Find the difference between the greater perfect square and the lower perfect square.*

$\frac{20}{23}$ *Write the difference as a ratio.*

$20 \div 3 \approx 0.869$ *Divide to find the approximate decimal value.*

Estimating Square Roots

Additional Example 3 Continued

Approximate $\sqrt{141}$ to the nearest hundredth.

Step 3: Find the approximate value.

$11 + 0.869 = 11.869$ *Combine the whole number and decimal.*

$11.869 \approx 11.87$ *Round to the nearest hundredth.*

The approximate value of $\sqrt{141}$ to the nearest hundredth is 11.87.

Estimating Square Roots

Check It Out: Example 3

Approximate $\sqrt{240}$ to the nearest hundredth.

Step 1: Find the value of the whole number.

$225 < 240 < 256$ *Find the perfect squares nearest 240.*

$\sqrt{225} < \sqrt{240} < \sqrt{256}$ *Find the square roots of the perfect squares.*

$15 < \sqrt{240} < 16$ *The number will be between 15 and 16.*

The whole number part of the answer is 15.

Estimating Square Roots

Check It Out: Example 3 Continued

Approximate $\sqrt{240}$ to the nearest hundredth.

Step 2: Find the value of the decimal.

$240 - 225 = 15$ *Find the difference between the given number, 240, and the lower perfect square.*

$256 - 225 = 31$ *Find the difference between the greater perfect square and the lower perfect square.*

$\frac{15}{31}$ *Write the difference as a ratio.*

$15 \div 31 \approx 0.484$ *Divide to find the approximate decimal value.*

Estimating Square Roots

Check It Out: Example 3 Continued

Approximate $\sqrt{240}$ to the nearest hundredth.

Step 3: Find the approximate value.

$15 + 0.484 = 15.484$ *Combine the whole number and decimal.*

$15.484 \approx 15.48$ *Round to the nearest hundredth.*

The approximate value of $\sqrt{240}$ to the nearest hundredth is 15.48.

MCC8.NS.2

2 EXAMPLE Comparing Irrational Numbers

Compare. Write <, >, or =.

A $\sqrt{3} + 5 \bigcirc 3 + \sqrt{5}$

First approximate $\sqrt{3}$ to the nearest tenth.

$\sqrt{3}$ is between _____ and _____, so $\sqrt{3} \approx$ _____.

Next approximate $\sqrt{5}$ to the nearest tenth.

$\sqrt{5}$ is between _____ and _____, so $\sqrt{5} \approx$ _____.

Then use your approximations to simplify the expressions.

$\sqrt{3} + 5 \bigcirc 3 + \sqrt{5}$

$\square + 5 \bigcirc 3 + \square$ *Substitute your approximations.*

$\square \bigcirc \square$ *Simplify, then compare.*

So, $\sqrt{3} + 5 \bigcirc 3 + \sqrt{5}$

B $\sqrt{10} + 2 \bigcirc 10 + \sqrt{2}$

First approximate $\sqrt{10}$ to the nearest tenth.

Simplify, then compare.

So, $\sqrt{3} + 5 \quad 3 + \sqrt{5}$

B $\sqrt{10} + 2 \quad 10 + \sqrt{2}$

First approximate $\sqrt{10}$ to the nearest tenth.

$\sqrt{10}$ is between _____ and _____, so $\sqrt{10} \approx$ _____.

Next approximate $\sqrt{2}$ to the nearest tenth.

$\sqrt{2}$ is between _____ and _____, so $\sqrt{2} \approx$ _____.

Then use your approximations to simplify the expressions.

$\sqrt{10} + 2 \quad 10 + \sqrt{2}$

$\quad + 2 \quad 10 +$ *Substitute your approximations.*

Simplify, then compare.

So, $\sqrt{10} + 2 \quad 10 + \sqrt{2}$

Simplify, then compare.

So, $\sqrt{10} + 2 \quad 10 + \sqrt{2}$

TRY THIS!

Compare. Write <, >, or =.

2a. $\sqrt{2} + 4 \quad 2 + \sqrt{4}$

2b. $\sqrt{12} + 6 \quad 12 + \sqrt{6}$

MCC8.NS.2

3 EXAMPLE Ordering Irrational Numbers

Order $\sqrt{3}$, π, and 1.5 from least to greatest.

First approximate $\sqrt{3}$ to the nearest tenth.

$\sqrt{3}$ is between _____ and _____, so $\sqrt{3} \approx$ _____.

You need to find a better estimate for $\sqrt{3}$ so you can compare it to 1.5.

Approximate $\sqrt{3}$ to the nearest hundredth.

2a. $\sqrt{2} + 4$ $\quad 2 + \sqrt{4}$ **2b.** $\sqrt{12} + 6$ $\quad 12 + \sqrt{6}$

MCC8.NS.2

3 EXAMPLE Ordering Irrational Numbers

Order $\sqrt{3}$, π, and 1.5 from least to greatest.

First approximate $\sqrt{3}$ to the nearest tenth.

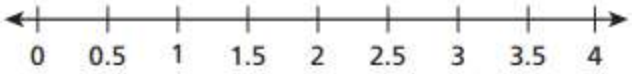
$\sqrt{3}$ is between _____ and _____, so $\sqrt{3} \approx$ _____.

You need to find a better estimate for $\sqrt{3}$ so you can compare it to 1.5.
Approximate $\sqrt{3}$ to the nearest hundredth.

$\sqrt{3}$ is between _____ and _____, so $\sqrt{3} \approx$ _____.

An approximate value of π is 3.14.

Plot $\sqrt{3}$, π, and 1.5 on a number line.



Read the numbers from left to right to place them in order from least to greatest.

From least to greatest, the numbers are _____, _____, _____.

TRY THIS!

Order the numbers from least to greatest.

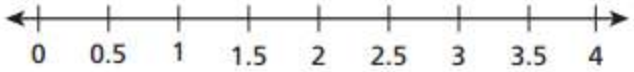
Read the numbers from left to right to place them in order from least to greatest.

From least to greatest, the numbers are _____, _____, _____.

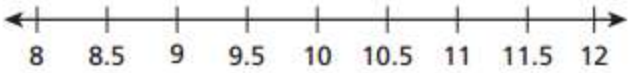
TRY THIS!

Order the numbers from least to greatest.

3a. $\sqrt{5}, 2.5, \sqrt{3}$ _____



3b. $\pi^2, 10, \sqrt{75}$ _____



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Homework:

Workbook Pg. 96