**AP Exam Review**

**Sunday, 4/22/18 4:30 - 6:30**

## 2016 AP Calculus AB Scoring Worksheet

AP Score Conversion Chart
Calculus AB

| Composite Score Range | AP Score |
|---|---|
| 67-108 | 5 |
| 55-66 | 4 |
| 42-54 | 3 |
| 35-41 | 2 |
| 0-34 | 1 |

**We had FOUR people earn a 5 in 2016!!**

throughout the United States. In general, the AP composite score points are set so that the lowest raw score needed to earn an AP score of 5 is equivalent to the average score among college students earning grades of A in the college course. Similarly, AP Exam scores of 4 are equivalent to college grades of A–, B+, and B. AP Exam scores of 3 are equivalent to college grades of B–, C+, and C.

### Using and Interpreting AP Scores

The extensive work done by college faculty and AP teachers in the development of the course and the exam and throughout the scoring process ensures that AP Exam scores accurately represent students' achievement in the equivalent college course. While colleges and universities are responsible for setting their own credit and placement policies, AP scores signify how qualified students are to receive college credit and placement:

| AP Score | Recommendation |
|---|---|
| 5 | Extremely well qualified |
| 4 | Well qualified |
| 3 | Qualified |
| 2 | Possibly qualified |
| 1 | No recommendation |

## Exam Content and Format

The 2017 AP Calculus AB Exam is 3 hours and 15 minutes in length. There are two sections:

- Section I is 1 hour, 45 minutes and consists of 45 multiple-choice questions in two separately-timed parts, accounting for 50 percent of the final score. Part A consists of 30 questions in 60 minutes and does not allow the use of a calculator. Part B consists of 15 questions in 45 minutes and requires the use of a graphing calculator.
- Section II is 1 hour, 30 minutes and consists of 6 free-response questions in two separately-timed parts, accounting for 50 percent of the final score. Part A consists of 2 questions in 30 minutes and requires the use of a graphing calculator. Part B consists of 4 questions in 60 minutes and does not allow the use of a calculator. During the timed portion for Part B, students are permitted to continue to work on questions in Part A, but they are not allowed to use a calculator during this time.

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

1. If $y = \cos 2x$, then $\frac{dy}{dx} =$

(A) $-2\sin 2x$ (B) $-\sin 2x$ (C) $\sin 2x$ (D) $2\sin 2x$ (E) $2\sin x$

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2. $\int x^2 (x^3 - 1)^{10}\, dx =$

(A) $\frac{x^3}{3}\left(\frac{x^4}{4} - x\right)^{10} + C$

(B) $\frac{(x^3 - 1)^{11}}{11} + C$

(C) $\frac{x^2 (x^3 - 1)^{11}}{11} + C$

(D) $\frac{(x^3 - 1)^{11}}{33} + C$

(E) $\frac{x^3 (x^3 - 1)^{11}}{33} + C$

$u = x^3 - 1$

$du = 3x^2\, dx$

$\frac{1}{3} du = x^2\, dx$

$\int u^{10} \cdot \frac{1}{3} du$

$= \frac{1}{3} \cdot \frac{1}{11} u^{11} + C \rightarrow \boxed{\frac{1}{33}(x^3 - 1)^{11} + C}$

end behavior (HA)

3. $\lim_{x \to \infty} \frac{\sqrt{9x^4 + 1}}{4x^2 + 3}$ is $\approx \lim_{x \to \infty} \frac{3x^2}{4x^2}$

same degree = Ratio of Leading Coefficients.

(A) $\frac{1}{3}$ (B) $\frac{3}{4}$ (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) infinite

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4. If $y = \left(\frac{x}{x+1}\right)^5$, then $\frac{dy}{dx} =$ $5\left(\frac{x}{x+1}\right)^4 \cdot \boxed{\frac{(x+1)(1) - (x)(1)}{(x+1)^2}}$

(A) $5(1 + x)^4$ (B) $\frac{x^4}{(x+1)^4}$ (C) $\frac{5x^4}{(x+1)^4}$ (D) $\frac{5x^4}{(x+1)^6}$ (E) $\frac{5x^4 (2x + 1)}{(x+1)^6}$

$= 5\left(\frac{x^4}{(x+1)^4}\right) \cdot \left(\frac{1}{(x+1)^2}\right)$

$= 5\left(\frac{x^4}{(x+1)^6}\right)$

| $t$ (minutes) | 0 | 4 | 7 | 9 |
|---|---|---|---|---|
| $r(t)$ (gallons per minute) | 9 | 6 | 4 | 3 |

5. Water is flowing into a tank at the rate $r(t)$, where $r(t)$ is measured in gallons per minute and $t$ is measured in minutes. The tank contains 15 gallons of water at time $t = 0$. Values of $r(t)$ for selected values of $t$ are given in the table above. Using a trapezoidal sum with the three intervals indicated by the table, what is the approximation of the number of gallons of water in the tank at time $t = 9$?

(A) 52 (B) 57 (C) 67 (D) 77 (E) 79

$15 + A_1 = \frac{1}{2}(9+6)(4) + A_2 = \frac{1}{2}(6+4)(3) + A_3 = \frac{1}{2}(4+3)(2)$

$15 + \frac{60}{2} + \frac{30}{2} + \frac{14}{2}$

$15 + 30 + 15 + 7 = 67$

---

6. The slope of the line tangent to the graph of $y = \ln(1 - x)$ at $x = -1$ is

(A) $-1$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\ln 2$ (E) 1

$y' = \frac{1}{1-x} \cdot (-1)$

$y' = \frac{-1}{1-x} \Big|_{x=-1}$

$= \frac{-1}{1-(-1)} = \frac{-1}{2}$

7. For which of the following pairs of functions $f$ and $g$ is $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ infinite?

(A) $f(x) = x^2 + 2x$ and $g(x) = x^2 + \ln x$ same deg.

(B) $f(x) = 3x^3$ and $g(x) = x^4$ bottom

(C) $f(x) = 3^x$ and $g(x) = x^3$

(D) $f(x) = 3e^x + x^3$ and $g(x) = 2e^x + x^2$ $\frac{3}{2}$.

(E) $f(x) = \ln(3x)$ and $g(x) = \ln(2x)$ ? same

f(x) must get bigger faster than g(x).

8. $\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx =$ $u = x^2+9$

$du = 2x\,dx \rightarrow \frac{1}{2}du = x\,dx$

(A) $-2$ (B) $-\frac{2}{15}$ (C) 1 (D) 2 (E) 5

$\int u^{-1/2} \cdot \frac{1}{2} du$

$= \frac{1}{2} \cdot 2u^{1/2} + C$

$= u^{1/2} + C$

So, $\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx = \sqrt{x^2+9}\Big|_0^4$

$= \sqrt{25} - \sqrt{9}$

$= 5 - 3$

$= 2$

---

9. Let $f$ be the function with derivative given by $f'(x) = \frac{-2x}{\left(1+x^2\right)^2}$. On what interval is $f$ decreasing?

(A) $[0, \infty)$ only

(B) $(-\infty, 0]$ only

(C) $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$ only

(D) $(-\infty, \infty)$

(E) There is no such interval.

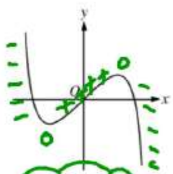
10. $\int (e^x + e)\,dx =$ $e^x + ex + C$

(A) $e^x + C$ (B) $2e^x + C$ (C) $e^x + e + C$ (D) $e^{x+1} + ex + C$ (E) $e^x + ex + C$

$\int 2\,dx = 2x + C$

So

$\int e\,dx = ex + C$

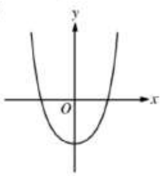


Graph of $f$

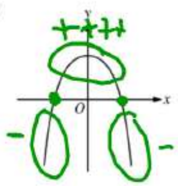
slopes of tangent!

11. The graph of the function $f$ is shown in the figure above. Which of the following could be the graph of $f'$, the derivative of $f$?

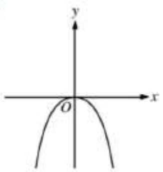
(A)



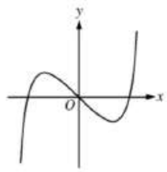
(B)



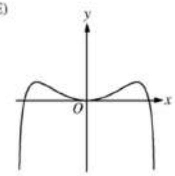
(C)



(D)

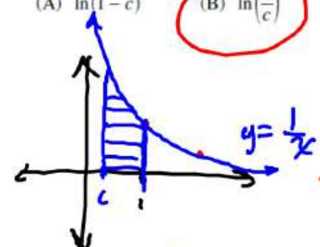


(E)



12. If $0 < c < 1$, what is the area of the region enclosed by the graphs of $y = 0$, $y = \frac{1}{x}$, $x = c$, and $x = 1$?

(A) $\ln(1 - c)$ (B) $\ln\left(\frac{1}{c}\right)$ (C) $\ln c$ (D) $\frac{1}{c^2} - 1$ (E) $1 - \frac{1}{c^2}$




$$\text{Area} = \int_c^1 \frac{1}{x}\,dx$$

$$= \ln|x|\Big]_c^1$$

$$= \ln x\Big|_c^1$$

$$= \ln 1 - \ln c = \ln\left(\frac{1}{c}\right)$$

or

$\ln 1 - \ln c$

$0 - \ln c$

$-\ln c$

$\ln c^{-1}$

$\ln\left(\frac{1}{c}\right)$

13. $\frac{d}{dx}\left(\tan^{-1}x + 2\sqrt{x}\right) = \boxed{\phantom{xx}} + x^{-\frac{1}{2}}$

(A) $-\frac{1}{\cancel{\sin^2 x}} + \frac{1}{\sqrt{x}}$

(B) $\cancel{\frac{1}{\sqrt{1-x^2}} - 4\sqrt[3]{x}}$

(C) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x}}$

(D) $\cancel{\frac{1}{1+x^2} - 4\sqrt[3]{x}}$

(E) $\frac{1}{1+x^2} + \frac{1}{\sqrt{x}}$

★ see inverse trig derivatives & antiderivatives on pg. 378.

14. If $y = f(x)$ is a solution to the differential equation $\frac{dy}{dx} = e^{x^2}$ with the initial condition $f(0) = 2$, which of the following is true?

(A) $f(x) = 1 + e^{x^2}$ ✗

(B) $f(x) = 2xe^{x^2}$ ✗

(C) $f(x) = \int_1^x e^{t^2}\,dt$

(D) $f(x) = 2 + \int_0^x e^{t^2}\,dt$

(E) $f(x) = 2 + \int_2^x e^{t^2}\,dt$

Why?? Initial Condition!

$dy = e^{x^2}dx$

$\int dy = \int e^{x^2}dx$

$y = \int e^{x^2}dx$

$y = 2 + \int_0^x e^{t^2}\,dt$

15. A function $f(t)$ gives the rate of evaporation of water, in liters per hour, from a pond, where $t$ is measured in hours since 12 noon. Which of the following gives the meaning of $\int_4^{10} f(t)\,dt$ in the context described?

(A) The total volume of water, in liters, that evaporated from the pond during the first 10 hours after 12 noon

(B) The total volume of water, in liters, that evaporated from the pond between 4 P.M. and 10 P.M.

(C) The net change in the rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(D) The average rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(E) The average rate of change in the rate of evaporation, in liters per hour per hour, from the pond between 4 P.M. and 10 P.M.

anti derivative of Rate of Evap. of water is just water

* think from V(t) (rate of chg. in position) goes backwards to X(t) (position.

16. The first derivative of the function $f$ is given by $f'(x) = 3x^4 - 12x^3$. What are the $x$-coordinates of the points of inflection of the graph of $f$ ?

(A) $x = 3$ only

(B) $x = 4$ only ✗

(C) $x = 0$ and $x = 2$ ✗

(D) $x = 0$ and $x = 3$

(E) $x = 0$ and $x = 4$ ✗

POIs ⇒ 2nd deriv.

$$f''(x) = 12x^3 - 36x^2$$
$$= 12x^2(x-3)$$

$f''$: − (0) − (3) +

POI!!

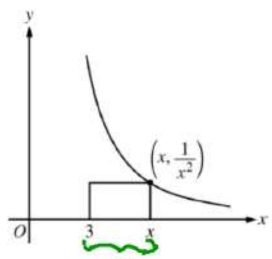
$\frac{1}{b-a}\int_a^b f(x)\,dx$

17. Let $f$ be the function defined by $f(x) = \frac{1}{x}$. What is the average value of $f$ on the interval $[4, 6]$ ?

(A) $-\frac{1}{24}$ (B) $\frac{5}{24}$ (C) $\frac{1}{2}\ln\frac{3}{2}$ (D) $\ln\frac{3}{2}$ (E) $\frac{1}{2}\ln 2$

$$\frac{1}{6-4}\int_4^6 \frac{1}{x}\,dx = \frac{1}{2}\ln|x|\Big|_4^6 = \frac{1}{2}\left[\ln 6 - \ln 4\right] = \frac{1}{2}\ln\left(\frac{6}{4}\right) = \frac{1}{2}\ln\frac{3}{2}$$

or $\ln\sqrt{\frac{3}{2}}$




$\underbrace{\quad}_{6}$

18. The points $(3, 0)$, $(x, 0)$, $\left(x, \frac{1}{x^2}\right)$, and $\left(3, \frac{1}{x^2}\right)$ are the vertices of a rectangle, where $x \geq 3$, as shown in the figure above. For what value of $x$ does the rectangle have a $\boxed{\text{maximum area?}}$ optimization!!

(A) 3

(B) 4

(C) 6

(D) 9

(E) There is no such value of $x$.

$A = bh$

$A = (x-3)\left(\frac{1}{x^2}\right)$

$A = \frac{x-3}{x^2}$ $\quad - (2x^2 - 6x)$

$A' = \frac{(x^2)(1) - (x-3)(2x)}{(x^2)^2} = \frac{6x - x^2}{x^4}$

$A' = \frac{(6-x)}{x^3}$

$A'$ + | − (6) max

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19. What are all values of $x$ for which $\int_x^2 t^3\,dt$ is equal to 0 ?

(A) ~~$-2$ only~~ (B) ~~0 only~~ (C) 2 only (D) $-2$ and 2 only (E) $-2$, 0, and 2

FTC:

$= \frac{1}{4} t^4 \Big|_x^2$

$= \frac{1}{4}\left(16 - x^4\right)$

$= \frac{16 - x^4}{4} = 0$

$16 - x^4 = 0$

$16 = x^4 \rightarrow x = 2, -2$

20. Let $h$ be the function defined by $h(x) = \int_{\pi/4}^{x} \sin^2 t \, dt$. Which of the following is an equation for the line tangent to the graph of $h$ at the point where $x = \frac{\pi}{4}$?

(A) $y = \frac{1}{2}$

(B) $y = \sqrt{2}x$

(C) $y = x - \frac{\pi}{4}$

(D) $y = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

(E) $y = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$y - \boxed{0} = \boxed{\frac{1}{2}}\left(x - \boxed{\frac{\pi}{4}}\right)$

$h\left(\frac{\pi}{4}\right)$ $\qquad$ $h'\left(\frac{\pi}{4}\right)$

$\int_{\pi/4}^{\pi/4} \sin^2 t \, dt$

$= 0$

(property!)

2nd FTC

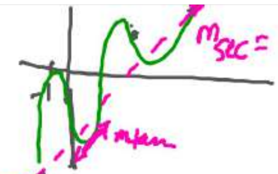
$h'(x) = \frac{d}{dx}\int_{\pi/4}^{x} \sin^2 t \, dt$

$= \sin^2 t \Big|_{t=x} \cdot (1) - 0$

$h'(x) = \sin^2 x$

So $h'\left(\frac{\pi}{4}\right) = \sin^2 \frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

| $x$ | $f(x)$ |
|---|---|
| $-1$ | $-30$ |
| 0 | $-2$ |
| 3 | 10 |
| 5 | 18 |




21. The table above gives selected values for a twice-differentiable function $f$. Which of the following must be true?

(A) $f$ has no critical points in the interval $-1 < x < 5$. no max, no min.

(B) $f'(x) = 8$ for some value of $x$ in the interval $-1 < x < 5$.

(C) $f'(x) > 0$ for all values of $x$ in the interval $-1 < x < 5$. always increasing?

(D) $f''(x) < 0$ for all values of $x$ in the interval $-1 < x < 5$. always concave down??

(E) The graph of $f$ has no points of inflection in the interval $-1 < x < 5$. no chg in concav??

MVT: $m_{sec} = m_{tan}$ at least once if cont $[1,5]$ and diff $(1,5)$

$\frac{f(5)-f(-1)}{5-(-1)} = m_{tan}$

$\frac{18 - -30}{6} = \frac{48}{6} = 8$ ✓

22. A particle moves along the $x$-axis so that at time $t \ge 0$, the acceleration of the particle is $a(t) = 15\sqrt{t}$. The position of the particle is 10 when $t = 0$, and the position of the particle is 20 when $t = 1$. What is the velocity of the particle at time $t = 0$?

(A) $-14$ (B) 0 (C) 5 (D) 6 (E) 10

Sep. variables!!!

$\frac{dy}{y} = \frac{2x}{x^2+1}\,dx$

23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{2xy}{x^2+1}$ whose graph contains the point $(0, 1)$?

(A) $y = e^{x^2}$

(B) $y = x^2 + 1$

(C) $y = \ln(x^2+1)$

(D) $y = 1 + \ln(x^2+1)$

(E) $y = \sqrt{1 + 2\ln(x^2+1)}$

$e^{\ln(x^2+1)} = |y|$

property!

$x^2+1 = |y|$

$y = x^2+1$ ✓ or $-(x^2+1)$ ✗

$\int \frac{dy}{y} = \int \frac{2x}{x^2+1}\,dx$

$\ln|y| = \ln|x^2+1| + C$

does not needed!

$(0,1)$

$\ln 1 = \ln|1| + C$

$C = 0$

$\ln|y| = \ln(x^2+1)$

24. Sand is deposited into a pile with a circular base. The volume $V$ of the pile is given by $V = \frac{r^3}{3}$, where $r$ is the radius of the base, in feet. The circumference of the base is increasing at a constant rate of $5\pi$ feet per hour. When the circumference of the base is $8\pi$ feet, what is the rate of change of the volume of the pile, in cubic feet per hour?

$\frac{dC}{dt} = 5\pi$

(A) $\frac{8}{\pi}$ (B) 16 (C) 40 (D) $40\pi$ (E) $80\pi$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi\frac{dr}{dt} = 5\pi$

$\frac{dr}{dt} = \frac{5}{2}$

$2\pi r = 8\pi$

$r = 4$

$\frac{dV}{dt} = \boxed{\phantom{00}}$

$V = \frac{1}{3}r^3$

$\frac{dV}{dt} = r^2\frac{dr}{dt}$

*need $r$ & $\frac{dr}{dt}$ ✓

Some other way!

$\frac{dV}{dt} = 4^2\left(\frac{5}{2}\right) = 40$

25. $\lim\limits_{h \to 0} \frac{e^{-1-h} - e^{-1}}{h}$ is

(A) $-1$ (B) $\frac{-1}{e}$ (C) 0 (D) $\frac{1}{e}$ (E) nonexistent

26. Let $f$ be the function given by $f(x) = x^3 + 5x$. For what value of $x$ in the closed interval $[1,3]$ does the $\boxed{\text{instantaneous rate of change}}$ of $f$ equal the $\boxed{\text{average rate of change of } f}$ on that interval?

$m_{tan} = m_{sec}$:

(A) $\sqrt{\frac{7}{3}}$ (B) $\sqrt{\frac{13}{3}}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$ (E) $\sqrt{\frac{19}{3}}$

$f'(x) = 3x^2 + 5 = 18$

$3x^2 = 13$

$x^2 = \frac{13}{3}$

$x = \sqrt{\frac{13}{3}}$

$m_{sec} = \frac{f(3) - f(1)}{3-1}$

$= \frac{(27+15) - (1+5)}{2}$

$= \frac{36}{2}$

$= 18$

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27. If $e^{xy} - y^2 = e - 4$, then at $x = \frac{1}{2}$ and $y = 2$, $\frac{dy}{dx} =$

$\frac{d}{dx}\left[e^{xy} - y^2 = e - 4\right]$

(A) $\frac{e}{4}$ (B) $\frac{e}{2}$ (C) $\frac{4e}{8-e}$ (D) $\frac{4e}{4-e}$ (E) $\frac{8-4e}{e}$

$e^{xy} \cdot \left(x\frac{dy}{dx} + y(1)\right) - 2y\frac{dy}{dx} = 0$

$e^{xy} \cdot x\frac{dy}{dx} + e^{xy}y - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(xe^{xy} - 2y\right) = -ye^{xy}$

$\frac{dy}{dx} = \frac{-ye^{xy}}{xe^{xy} - 2y}$ $\left(\frac{1}{2}, 2\right)$

$= \frac{-2e}{\frac{1}{2}e - 4} = \frac{-2e}{\frac{e-8}{2}} \cdot \frac{2}{e-8} = \frac{-4e}{e-8} = \frac{4e}{8-e}$

28. Let $f$ be the function defined by $f(x) = x^3 + x^2 + x$. Let $g(x) = f^{-1}(x)$, where $g(3) = 1$. What is the value of $g'(3)$?

(A) $\frac{1}{39}$ (B) $\frac{1}{34}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) 39

$f'(x) = 3x^2 + 2x + 1$

$f'(1) = 3 + 2 + 1$

$= 6$

*Formula: $g'(x) = \frac{1}{f'(g(x))}$

$g'(3) = \frac{1}{f'(g(3))}$

$= \frac{1}{f'(1)}$

$= \frac{1}{6}$

| | # Right | Confident | Narrowed Guess | Total Guess |
|---|---|---|---|---|
| Micah | 6 23 7.2 | 3 | 2 | 1 *not good. |
| Carm | 8 31 =9.6 | 2 | 1 | 5 |
| Tyler | | | | |

/26 312