3.2 Implicit Differentiation

Notes

Write your questions and thoughts here!

Recall:

Explicit equation

Implicit equation

Chain Rule and Implicit Differentiation

In terms of x	In terms of <i>y</i>		
$\frac{d}{dx}x =$	$\frac{d}{dx}y =$		
$\frac{d}{dx}x^2 =$	$\frac{d}{dx}y^2 =$		
$\frac{d}{dx}e^{5x} =$	$\frac{d}{dx}e^{5y} =$		

Implicit Differentiation Example: Find $\frac{dy}{dx}$ for $y^2 - 5x^3 = 3y$

Step 1: Take the derivative. Each time the derivative of "y" is involved, include a $\frac{dy}{dx}$.

Step 2: Gather all terms with $\frac{dy}{dx}$ on the left side, everything else on the right.

Step 3: Factor out the $\frac{dy}{dx}$ if necessary, to create only one $\frac{dy}{dx}$ term.

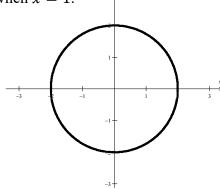
Step 4. Solve for $\frac{dy}{dx}$.

1.
$$y^3 - 2x = x^4 + 2y$$

$$2. \sin(xy) = 10x$$

Derivative at a point – implicit differentiation.

3. Find the equation of all tangent lines for $x^2 + y^2 = 4$ when x = 1.



Horizontal and Vertical Tangent Lines

Horizontal tangent lines exist when the slope, $\frac{dy}{dx}$ =

Vertical tangent lines exist when the slope, $\frac{dy}{dx}$ is

- 4. Find all *horizontal* tangent lines of the graph $3x^2 + 2y^2 = 16$.
- 5. Find all *vertical* tangent lines of the graph $3x^2 + 2y^2 = 16$.

3.2 Implicit Differentiation

Calculus

Practice

Find
$$\frac{dy}{dx}$$
.

Find
$$\frac{dy}{dx}$$
.

1. $5x^2 + 2y^3 = 4$

$$2. \ 5y^2 + 3 = x^2$$

$$3. \sin(x+y) = 2x$$

4. $4x + 1 = \cos y^2$	$5. \ 5x^2 - e^{4y^2} = -6$		6. $\ln(y^3) = 5x + 3$		
7 2 4 3 . 5 2	0.53.2.5	3			
$7. \ x^2 = 4y^3 + 5y^2$	$8. \ 5x^3 - 2y = 5y^3$		9. $\ln y^2 + \cos^2 x = 1 - y$		
$10. \sin\left(\frac{y}{2}\right) + e^y = 4x$		$11. \ x^3 + y^3 = 6.$	xy		
$12. \ \frac{x}{\sin y} = 5$		13. $\ln x e^{3y} = 2$	y^2		

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Find the	sione ot the	tangent line	at the given	noint.	Show work.
I III W CIIC	stope of the		ar the Siven	POILE	DIIO II II III

Find the slope of the tangent 1
14.
$$2 = 3x^4 + xy^4$$
 at $(-1, 1)$

15.
$$x \ln y = 4 - 2x$$
 at (2, 1)

Find the equation of the tangent line at the given point. 16. $x^2 + y^2 + 19 = 2x + 12y$ at (4, 3)

16.
$$x^2 + y^2 + 19 = 2x + 12y$$
 at (4, 3)

17.
$$x \sin 2y = y \cos 2x$$
 at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Find the equations of all horizontal and vertical tangent lines. Calculator allowed. Round to three decimals. 18. $x^2 + x + 2y^2 = 8$ 19. $x + y = y^2$

$$18. \ x^2 + x + 2y^2 = 8$$

19.
$$x + y = y^2$$

Horizontal:

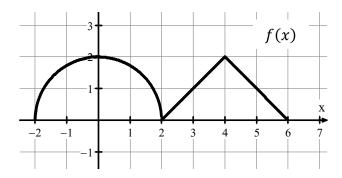
Horizontal:

Vertical:___

20. Find the slope of the normal line to $y = x + \cos(xy)$ at (0,1).

- (A) 1
- (B) -1
- (C) 0
- (D) 2
- (E) Undefined

21. The graph of f(x), shown below, consists of a semicircle and two-line segments. f'(1) =



- (A) -1
- (B) $-\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$
- (D) 1
- (E) $\sqrt{3}$

22. Find the value(s) of $\frac{dy}{dx}$ of $x^2y + y^2 = 5$ at y = 1.

- (A) $-\frac{3}{2}$ only (B) $-\frac{2}{3}$ only (C) $\frac{2}{3}$ only (D) $\pm \frac{2}{3}$ (E) $\pm \frac{3}{2}$