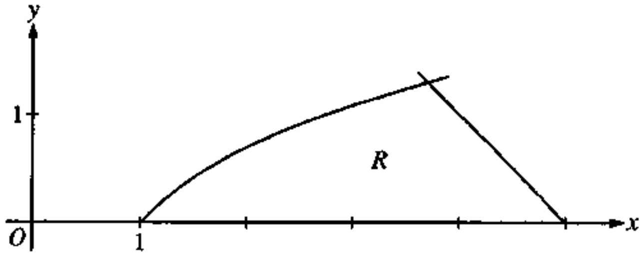
## Question 2

Let $R$ be the region in the first quadrant bounded by the $x$-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.




(a) Find the area of $R$.

(b) Region $R$ is the base of a solid. For the solid, each cross section perpendicular to the $x$-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides $R$ into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of $k$.

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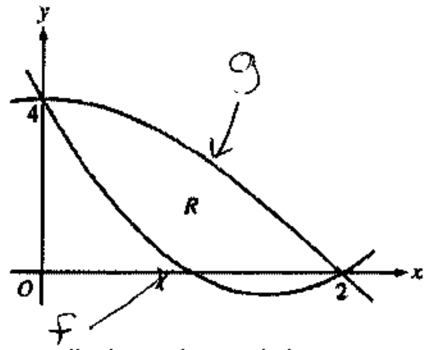
$\ln x = 5 - x \quad \Rightarrow \quad x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

| | |
|---|---|
| (a) $\text{Area} = \int_0^B \left(5 - y - e^y\right) dy = 2.986 \text{ (or } 2.985)$ <br><br> OR <br><br> $\text{Area} = \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx = 2.986 \text{ (or } 2.985)$ | 3 : { 1 : integrand<br>1 : limits<br>1 : answer |
| (b) $\text{Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$ | 3 : { 2 : integrands<br>1 : expression for total volume |
| (c) $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$ | 3 : { 1 : integrand<br>1 : limits<br>1 : equation |

## Question 5

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let $R$ be the region bounded by the graphs of $f$ and $g$, as shown in the figure above.




(a) Find the area of $R$.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 4$.

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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(a)
$$\text{Area} = \int_0^2 [g(x) - f(x)]\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$$
$$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$$
$$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$$

4 : { 1 : integrand; 2 : antiderivative; 1 : answer }

(b)
$$\text{Volume} = \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$$
$$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

3 : { 2 : integrand; 1 : limits and constant }

(c)
$$\text{Volume} = \int_0^2 [g(x) - f(x)]^2\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$$

2 : { 1 : integrand; 1 : limits and constant }

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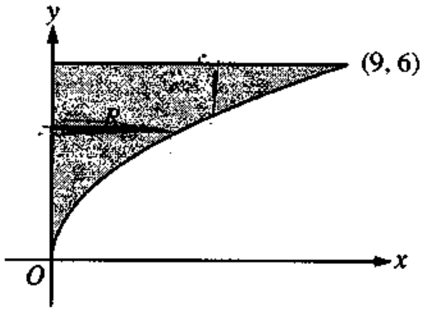
## Question 4




Let $R$ be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the $y$-axis, as shown in the figure above.

(a) Find the area of $R$.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 7$.

(c) Region $R$ is the base of a solid. For each $y$, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the $y$-axis is a rectangle whose height is 3 times the length of its base in region $R$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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(a) Area $= \int_0^9 \left(6 - 2\sqrt{x}\right) dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big|_{x=0}^{x=9} = 18$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

(b) Volume $= \pi \int_0^9 \left(\left(7 - 2\sqrt{x}\right)^2 - (7-6)^2\right) dx$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

(c) Solving $y = 2\sqrt{x}$ for $x$ yields $x = \frac{y^2}{4}$.

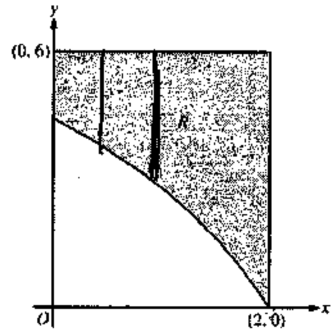
Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$.

Volume $= \int_0^6 \frac{3}{16}y^4 \, dy$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

### Question 1

In the figure above, $R$ is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.




(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the horizontal line $y = 8$.

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a square. Find the volume of the solid.

| | |
|---|---|
| | 1 : Correct limits in an integral in (a), (b), or (c) |
| (a) $\int_0^2 (6 - 4\ln(3 - x))\,dx = 6.816$ or $6.817$ | 2 : { 1 : integrand<br>1 : answer |
| (b) $\pi \int_0^2 \left( (8 - 4\ln(3 - x))^2 - (8 - 6)^2 \right) dx$<br>$= 168.179$ or $168.180$ | 3 : { 2 : integrand<br>1 : answer |
| (c) $\int_0^2 (6 - 4\ln(3 - x))^2\,dx = 26.266$ or $26.267$ | 3 : { 2 : integrand<br>1 : answer |

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## Question 4

Let $R$ be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

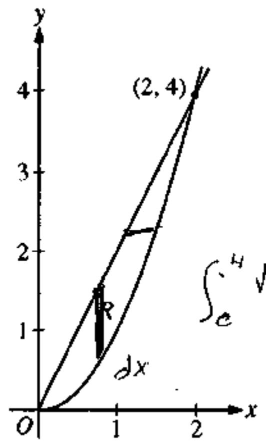
$x = y/2$

$x = \sqrt{y}$

(a) Find the area of $R$.

(b) The region $R$ is the base of a solid. For this solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base $R$. For this solid, the cross sections perpendicular to the $y$-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.




$\int_0^4 \sqrt{y} - \frac{y}{2}\, dy$

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(a) Area $= \int_0^2 \left(2x - x^2\right) dx$

$= x^2 - \frac{1}{3}x^3 \Big|_{x=0}^{x=2}$

$= \frac{4}{3}$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \int_0^2 \sin\left(\frac{\pi}{2}x\right) dx$

$= -\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big|_{x=0}^{x=2}$

$= \frac{4}{\pi}$

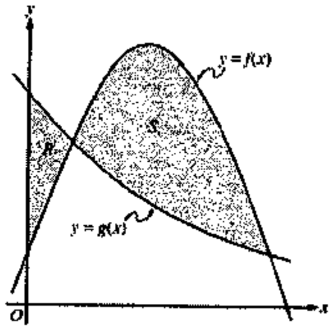
$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

### Question 1

Let $f$ and $g$ be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let $R$ be the shaded region in the first quadrant enclosed by the $y$-axis and the graphs of $f$ and $g$, and let $S$ be the shaded region in the first quadrant enclosed by the graphs of $f$ and $g$, as shown in the figure above.




(a) Find the area of $R$.

(b) Find the area of $S$.

(c) Find the volume of the solid generated when $S$ is revolved about the horizontal line $y = -1$.

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$f(x) = g(x)$ when $\frac{1}{4} + \sin(\pi x) = 4^{-x}$.

$f$ and $g$ intersect when $x = 0.178218$ and when $x = 1$.
Let $a = 0.178218$.

| | |
|---|---|
| (a) $\int_0^a (g(x) - f(x))\,dx = 0.064$ or $0.065$ | 3 : { 1 : limits<br>1 : integrand<br>1 : answer |
| (b) $\int_a^1 (f(x) - g(x))\,dx = 0.410$ | 3 : { 1 : limits<br>1 : integrand<br>1 : answer |
| (c) $\pi\int_a^1 \left((f(x)+1)^2 - (g(x)+1)^2\right)dx = 4.558$ or $4.559$ | 3 : { 2 : integrand<br>1 : limits, constant, and answer |