Math IV – UNIT 6 QUIZ 3: Verifying Trigonometric Identities

Seigle

1.) $\frac{1+\tan x}{\sin x+\cos x} = \sec x$

1.) Common denominator first, then dividing fraction by fraction.

$$\frac{\frac{\cos x}{\cos x}1 + \frac{\sin x}{\cos x}}{\sin x + \cos x}$$

$$\frac{\frac{\cancel{\cos x + \sin x}}{\cos x}}{\cancel{\sin x + \cos x}} = \frac{1}{\cancel{\sin x + \cos x}} = \frac{1}{\cos x} = \sec x$$

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Katie B

2.) $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$

2.) Common denominator first, then factor.

$$\frac{(1-\cos x)\sin x}{(1-\cos x)(1+\cos x)} + \frac{1+\cos x}{\sin x}$$

$$\frac{(1-\cos x)\sin x}{1-\cos^2 x} + \frac{1+\cos x}{\sin x}$$

$$\frac{(1-\cos x)\cancel{\sin x}}{\sin^{\cancel{2}} x} + \frac{1+\cos x}{\sin x}$$

$$\frac{1-\cancel{\cos x} + 1+\cancel{\cos x}}{\sin x}$$

$$\frac{2}{\sin x}$$

$$\boxed{2\csc x}$$

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Cody G.

2.) $\frac{\sin x}{1+\cos x}\cdot\frac{(\sin x)}{(\sin x)} + \frac{1+\cos x}{\sin x}\cdot\frac{(1+\cos x)}{(1+\cos x)} = 2\csc x$

2.) Common denominator first, then factor.

$$\frac{\sin^2 x}{(1+\cos x)(\sin x)} + \frac{\cancel{1+\cos x+\cos x+\cos^2 x}}{(1+\cos x)(\sin x)} \qquad 1+2\cos x+\cos^2 x$$

$\sin^2+\cos^2=1$

$$\frac{\cancel{\sin^2 x}+1+2\cos x+\cancel{\cos^2 x}}{(1+\cos x)(\sin x)}$$

$$\frac{2+2\cos x}{(1+\cos x)(\sin x)} = \frac{2\cancel{(1+\cos x)}}{\cancel{(1+\cos x)}(\sin x)}$$

$$\frac{2}{\sin x} = 2\csc x \checkmark$$

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Amber

3.) $\frac{\cos x}{1-\sin x}\cdot\frac{(\cos x)}{(\cos x)} - \frac{1+\sin x}{\cos x}\cdot\frac{(1-\sin x)}{(1-\sin x)} = 0$

3.) Common denominator first, then look for an identity.

$$\frac{\cos^2 x}{(1-\sin x)(\cos x)} - \frac{1-\sin^2 x}{(1-\sin x)(\cos x)} \qquad 1-\sin^2 x = \cos^2 x$$

$$\frac{\cos^2 x}{(\cos x)(1-\sin x)} - \frac{\cos^2 x}{(\cos x)(1-\sin x)} = \frac{0}{\cancel{(\cos x)(1-\sin x)}}$$

$$= 0$$

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Nikki

3.) $\frac{\cos x}{1-\sin x} - \frac{1+\sin x}{\cos x} = 0$

3.) Common denominator first, then look for an identity.

$$\frac{\cos x(\cos x)}{1-\sin x(\cos x)} - \frac{1+\sin x(1-\sin x)}{\cos x(1-\sin x)}$$

$$\frac{\cos^2 x - (1-\sin^2 x)}{(1-\sin x)\cos x} \to \frac{\cancel{\cos^2 x - \cos^2 x}}{(1-\sin x)\cos x} = 0$$

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3.) $\frac{\cos x}{1-\sin x} - \frac{1+\sin x}{\cos x} = 0$

3.) Common denominator first, then look for an identity.

$$\frac{\cos^2 x}{(1-\sin x)\cos x} - \frac{\overset{\cos^2 x}{1-\sin^2 x}}{(1-\sin x)\cos x}$$

$$\frac{\cos^2 x}{(1-\sin x)\cos x} - \frac{\cos^2 x}{(1-\sin x)\cos x} = \frac{0}{(1-\sin x)\cos x}$$

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Callie

4.) $\frac{1}{\sin x-1}-\frac{1}{\sin x+1}=-2sec^2x$

4.) Common denominator first, then combine like terms and look for an identity (or close to one).

$$\frac{\sin x+1}{\sin^2 x-1}-\frac{\sin x-1}{\sin^2 x-1}$$

$$\frac{(\sin x+1)-(\sin x-1)}{\sin^2 x-1}$$

$$\frac{2}{\sin^2 x-1}=\frac{2}{-\cos^2 x}=\boxed{-2\sec^2 x}$$

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Ryan D.

4.) $\frac{1}{\sin x-1}-\frac{1}{\sin x+1}=-2sec^2x$

4.) Common denominator first, then combine like terms and look for an identity (or close to one).

$$\frac{1}{\sin x-1}-\frac{1}{\sin x+1}=\quad -2\sec^2 x$$

$\sin x+1$ $\quad$ $\sin x-1$

$$\frac{\sin x+1-(\sin x-1)}{\sin^2 x-1}=-2\sec^2 x$$

$$\frac{\cancel{\sin x}+1-\cancel{\sin x}+1}{\sin^2 x-1}=\frac{2}{\sin^2 x-1}=\frac{2}{-\cos^2 x}$$

$$=-2\sec^2 x$$

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Savannah

5.) $\frac{\cos x}{1-\sin x} = \sec x + \tan x$

5.) Conjugate, then identity, then simplify.

$$\frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$\frac{\cos x(1+\sin x)}{1-\sin^2 x}$$

$$\frac{\cancel{\cos x}(1+\sin x)}{\cos^{\cancel{2}} x}$$

$$\frac{1+\sin x}{\cos x} = \frac{1+\sin x}{\cos x} \checkmark$$

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Emilio

6.) $(\sec x - \tan x)^2 = \frac{1-\sin x}{1+\sin x}$

6.) Reciprocal Identity, then square, then look for Pythagorean Identity.

$$\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$$

$$\left(\frac{1-\sin x}{\cos x}\right)^2$$

$$\frac{(1-\sin x)^2}{\cos^2 x}$$

$$\frac{(1-\sin x)\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}(1+\sin x)}$$

$$\frac{1-\sin x}{1+\sin x} = \frac{1-\sin x}{1+\sin x}$$

$$\frac{1-\sin x}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x}$$

$$\frac{(1-\sin x)^2}{1-\sin^2 x}$$

$$= \frac{(1-\sin x)^2}{\cos^2 x}$$

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Blake

7.) $\cos x\, sec^2 x \tan x - \cos x\, tan^3 x = \sin x$

7.) Factor first.

$$\cos x \tan x\,(\sec^2 x - \tan^2 x) = \sin x$$

$$\cos x \tan x(1) = \sin x$$

$$\frac{\cancel{\cos x}}{1}\cdot\frac{\sin x}{\cancel{\cos x}} = \sin x$$

$$\sin x = \sin x \checkmark$$

Math IV – UNIT 6 QUIZ 3: Verifying Trigonometric Identities

Emilio

7.) $\cos x\, sec^2 x \tan x - \cos x\, tan^3 x = \sin x$

7.) Factor first.

$$\frac{\cancel{\cos x}}{1}\cdot\frac{1}{\cos^2 x}\cdot\frac{\sin x}{\cancel{\cos x}} - \frac{\cancel{\cos x}}{1}\cdot\frac{\sin^3 x}{\cos^{\cancel{3}} x^2}$$

$$\frac{\sin x}{\cos^2 x} - \frac{\sin^3 x}{\cos^2 x}$$

$$\frac{\sin x - \sin^3 x}{\cos^2 x}$$

$$\frac{\sin x(\cancel{1-\sin^2 x})}{\cancel{1-\sin^2 x}}$$

$$\sin x \checkmark$$

Unit 6: Sum & Difference and Double-Angle Identities TASK 2

Sum & Difference Identities

Sum and difference identities can be derived from other identities. These four identities can be summarized with the following two statements.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \textit{*same signs*}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \qquad \textit{*opposite signs*}$$

Recall that so far, you can only calculate the exact values of the sines and cosines of multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$. These identities will allow you to calculate the exact value of the sine and cosine *of many more angles*.

EXAMPLES:

1. Evaluate $\sin 75°$ by applying the angle addition identity for sine and evaluating each trigonometric function:

$$\sin(30° + 45°) = \sin 30°\cos 45° + \cos 30°\sin 45°$$
$$= (\tfrac{1}{2})(\tfrac{\sqrt{2}}{2}) + (\tfrac{\sqrt{3}}{2})(\tfrac{\sqrt{2}}{2})$$
$$= (\tfrac{\sqrt{2}}{4}) + (\tfrac{\sqrt{6}}{4})$$

$$\text{Sin}(75°) = (\tfrac{\sqrt{2}+\sqrt{6}}{4})$$

Check with calculator ~.9659(in degree mode)

$x \cdot y = xy$

$\sqrt{3}\cdot\sqrt{2} = \sqrt{6}$

$x + y = x + y$

$\sqrt{3}+\sqrt{2} = \sqrt{3}+\sqrt{2}$

2. Evaluate cos 30° by applying the angle subtraction identity for cosine and evaluating each trigonometric function:

$$\cos(90° - 60°) = \cos 90°\cos 60° + \sin 90°\sin 60°$$
$$= (0)(\tfrac{1}{2}) + (1)(\tfrac{\sqrt{3}}{2})$$

$$\text{Cos } 30° = \tfrac{\sqrt{3}}{2}$$

Makes sense with what we already know!

NOW YOU TRY:

3. Similarly, find the exact value of the following trigonometric expressions:

a. $\cos(15°) = \cos(45° - 30°)$

$\cos 45° \cos 30° + \sin 45° \sin 30°$

b. $\sin\left(\frac{\pi}{12}\right)$

$\sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

c. $\cos(345°) = \cos(300° + 45°)$

d. $\sin\left(\frac{19\pi}{12}\right)$

$\sin\left(\frac{10\pi}{12} + \frac{9\pi}{12}\right)$

$\sin\left(\frac{5\pi}{6} + \frac{3\pi}{4}\right)$