AP CALCULUS AB

SYLLABUS

COURSE OVERVIEW

The purpose of this course is for student's to gain an understanding of calculus concepts including its methods and applications. Students will learn to solve problems graphically, numerically, analytically, and verbally. Students should be able to communicate the interpretations of problems so that they not only understand how problems are worked but why they are worked in that manner.

TECHNOLOGY REQUIREMENT [CR3a,3b,3c]

Students will be using a graphing calculator throughout various topics during the year. Students will have access to a TI-84 Plus in the classroom, and they are encouraged to purchase a graphing calculator to use at home. During some units, such as applications of derivatives and applications of integrals, students will use a graphing calculator frequently so that they can learn how to be most effective with this technology in the areas of graphing, finding zeros, calculating complex derivatives, and calculating integrals for which they do not have the integration techniques to solve. For other units, the use of calculators will be used minimally so that students can learn to find limits, derivatives, and integrals without the aid of technology. The students will also have limited calculator accessibility during these times to ensure that they become proficient in their computational and algebraic skills to be better prepared for the non-calculator section of the AP exam. Some assessments will be given with calculators prohibited while other assessments will require the use of a calculator and without the knowledge of the calculator these problems could not be solve.

EVALUATION:

Throughout this course, students have homework assignments and quizzes that are a minimal percentage of their grade. Students also work collaboratively on daily assignments of problems so that they are communicating mathematically. **[CR2f]** The majority of the students' grades are derived from tests which are influenced by previous AP exams. As the time for the AP exam draws near, students are provided more examples of previous AP exams with a strong emphasis on Free-Response Questions so that students have even more practice justifying, explaining, and interpreting **[CR2f]**their results as well as utilizing the graphing calculator in the most efficient and effective manner. **[CR3b]** Students are also quizzed on problems similar to previous AP exams so that they become more familiar with understanding and identifying the language of Calculus on an individual and independent basis. **[CR2e,f]** Students are allowed to use their calculators on about half of their assessments.

COURSE OUTLINE:

LIMITS AND CONTINUTIY (6 WEEKS)

Activity One: We begin this topic by understanding the meaning and process of limits. We explore the intuitive idea of a limit by using the graphing calculator to graph various functions and zoom in on points to understand that as we approach certain x-values, then the limit describes the y-values. We also use the calculator to create a table of values that demonstrates numerically how a limit is found. The calculator is also used to explore oscillating functions. **[CR2d,3c]**

Activity Two: We define a limit formally with epsilon-delta notation and use graphs to understand this definition. With the aid of a graph students will begin to understand this formal definition to begin an understanding of mathematics vocabulary, symbols, and various notations. We also discuss the various sum, difference, constant multiple, product, and quotient properties of limits. **[CR2a,CR2e]**

Activity Three: Students will be given common algebra errors to correct as they begin to prepare for evaluating limits algebraically. Students will also practice review problems involving factoring, simplifying complex fractions, and rationalizing as well review the unit circle from trig.
We also explore the idea of limits to infinity by noticing asymptotic behavior on our graphs, one-sided limits, and limits that do not exist. We also use the table of values of these graphs to discuss limits and the existence and non-existence of a limit. **[CR2c,3c]**

Activity Four: The Squeeze/Sandwich Theorem will be stated and explained by means of a graph. Students will notice (by a table of values from Activity one) that the $\lim_{\theta} \frac{sin\theta}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{1-cos\theta}{\theta} = 0$. Students will research the proof of $\lim_{\theta} \frac{sin\theta}{\theta} = 1$ as an application to the theorem. **[CR2a]**

Activity Five: To introduce the concept of continuity, students will draw two functions (one they believe is continuous and one that they believe is not continuous). Students will write an explanation as to why they believe their continuous function is continuous. They may use technology/internet resources to discover what makes a function continuous. Several students will be asked to draw their graphs on the board (for both). Using these graphs, we will establish the characteristics of a continuous function and define continuity at a point using three criteria. In pairs students will discuss why each function drawn is or is not a function. From the pair, students will be asked to communicate with the class what they discussed. Students will prove or disprove continuity of the graphs presented based on the three criteria. **[CR2a,CR2f]**

Activity Six: Points of discontinuity: Students explore limits at discontinuities in four ways: first, using the table feature on their calculators with decreasing increments; second, using algebraic techniques to "simplify" the expressions given as formulas; third, using the graph trace feature on their calculators; and fourth, using verbal descriptions of functions written in words to create graphs that match the verbal descriptions. Student work for the activity includes a written summary, using complete sentences, of their findings that compares and contrasts jump, removable, and asymptotic discontinuities. They give an oral presentation describing how the different representations reveal the discontinuities in different ways. **[CR2c,CR2d,CR2f,CR3c]**

TOPICS: [CR1a]

1. Evaluate limits numerically
2. Evaluate limits graphically
3. Evaluate limits algebraically
    A. Substituting
    B. Factoring and simplifying
    C. Simplifying complex fractions
    D. Rationalizing the denominator
    E. Using special trig limits (Application of Sandwich/Squeeze Theorem)
    F. One-sided limits (graphically, numerically, and algebraically)
    G. Limits to infinity (Application for horizontal asymptotes)
4. L'Hospital's Rule
5. Definition and intuitive understanding (no breaks, no gaps, no holes) of continuity
6. Types of discontinuity (recognizing algebraically and graphically)
7. Criteria for proving continuity at a point
8. Intermediate Value Theorem and its application (Before introducing the theorem use the idea the website below to gain a better understanding.) **[CR2a]**

https://bowmandickson.com/2012/10/20/teaching-through-concrete-examples-the-intermediate-value-theorem/

DERIVATIVES (8 WEEKS)

Activity One: We begin this topic by verbally discussing how we might find the slope the tangent to a curve given that tangents only involve one point on the curve whereas slope normally involves two points. Relate the entire idea to the average rate of change they have discussed in Algebra and Advanced Algebra. We draw a curve and begin by drawing a secant. We observe as one point approaches the other, the distance between x-values get smaller, approaching zero. Students are able to visually see and discuss how a derivative at a point is derived from a limit. Through this process we are able to establish the definition of a derivative. Once this definition is established students will use textbooks and/or web resources to investigate the various notations that are used for a derivative. **[CR2a,2b,2d,2e]**

Activity Two: Differentiablity and Continuity Activity. Given a set of graphs, students will determine where the limit exists, where the graph is continuous, and where the graph is differentiable. They will establish a relationship amongst the three concepts. Students will also match the graph of function to the graph of its derivative based solely on the idea that a derivative represents the slope of the tangents of the original function. **[CR2b]**

Activity Three: For power rule and chain rule, the presentation of the topics will begin with a "What's My Rule?" strategy. Students will be given the original function and the derivative for 2-3 problems. They will then be asked to notice a pattern and use the pattern to find the derivative of another problem. Ask for a volunteer to present the answer without explanation. Continue to do this until all students understand the power rule and chain rule (polynomial raised to a power only).

Within this unit, student practice and assessments will contain questioning similar to that of the multiple choice questions on the AP exam. These questions will include, but not limited to, finding derivatives with generic functions involving a table of values, finding derivatives at a point (including computing fractional powers), finding horizontal tangents, finding equations of tangents and normals, finding points where a tangent to a graph is parallel to a given line, finding values that make a piecewise function differentiable (a system of equations results from the function also having to be continuous), finding tangents, normals, and linear approximations of g(x) given only information about f(x) and g(x) as a function of f(x), and students will be presented derivative questions all notations possible, including a second derivative. **[CR2b,2c,2e]**

TOPICS: [CR1b]

1. Average rate of change of a function
2. Definition of a derivative (including limit of difference quotient and graphical representation) and <u>all notations</u> possible for a derivative
3. Interpreting derivative as slope of tangent to a curve or instantaneous rate of change
4. Recognizing average rate of change from a table and understanding instantaneous rate of change as the limit of the average rate of change. Estimating derivatives in a table of values using the idea of average rate of change.
5. Differentiability of functions including where derivatives fail to exist
6. Relationship between differentiability and continuity
7. Basic derivative rules (power, product, and quotient)
8. Finding equations of tangent and normal equations to curves and using the tangent equation to approximate values for curve at a point. Also, finding equations of a horizontal tangent and discussing what happens at a vertical tangent.
9. Using the concept of a derivative to find the velocity and acceleration, given the position function.
10. Interpreting position, speed, velocity graphs.
11. Derivatives of 6 trig functions
12. Chain rule for derivatives
13. Implicit differentiation (as an extension of chain rule)
14. Derivatives of inverse trig functions and basic inverse functions.
15. Derivatives of exponential and logarithmic functions
16. Finding higher order derivatives and understanding the notation for higher order derivatives

APPLICATIONS OF DERIVATIVES (6 WEEKS)

<u>Activity One</u>: We begin this topic by sketching a curve with endpoints, maximums, minimums, different concavities, and non-differentiable points. We discuss the basic terminology of increasing, decreasing, absolute extrema, relative or local extrema, concavity, and points of inflection. We repeatedly refer to this sketch throughout the unit in order to explore how these ideas are derived from the relationship between the graph of a function and its first derivative and/or second derivative. From this activity students will notice make a connection between derivatives = 0, derivatives not existing, and extreme points. They will also see a relationship

between a function increasing or decreasing and the sign of the derivative. They will communicate with a partner the difference absolute and relative extrema as well as the effect that endpoints have on analyzing a graph. **[CR2b,2f]**

Activity Two: Students will be presented with the graph of a derivative and they will be asked questions concerning f′(x) (equal to 0, less than 0, and greater than 0). They will also be asked questions about the slope of f′(x). From this information, students will do a guided discovery in understanding how the graph of the derivative can tell us about the original function as well as the second derivative. Based on these pieces of information, students will be asked to sketch f(x). Students will practice several problems given f′(x). Students will be able to continually compare f(x), f′(x), and f″(x) given and f(x) or f′(x) graph. **[CR2b]**

Activity Three: Demonstration: With the aid of another student, the teacher will demonstrate the concept of speeding up and slowing down based on the positive and negative signs of the velocity and acceleration. The students will summarize the demonstration using correct vocabulary as well as correct notation and symbols. The students will be required to write the derivatives using multiple notations. **[CR2b,2e,2f]**

Activity Four: To get prepared for applications involving optimization, students will complete two tasks. First, they will represent the sides of a rectangle as a single variable based on the given information (either involving area or perimeter) and based on one side being bounded or divided into multiple sections. This activity will remind students of algebraic concepts from prior years. Second, each student will be given a square piece of construction paper (9″ x 9″). They will be asked to create a box by cutting out uniform squares from each corner. The students should cut out the square to maximize the volume. Each student will explain mathematically why their box will have the maximum volume. Students will fill the box to see whose box will hold the most. As a class, we will then use derivatives to see what dimensions a box with maximum volume should have. Through this visual representation, students will be able to see the height of the box is the square length they cut out. **[CR2c,2f]**

TOPICS: [CR1b]

1. Critical points (determining domain and derivative)

2. The Mean Value Theorem and Extreme Value Theorem and its verbal and graphical interpretation
3. The First Derivative Test to determine increasing and decreasing functions
4. Extreme points found from the first derivative, increasing or decreasing, and endpoints
5. Concavity and points of inflection of a function based on the second derivative
6. The Second Derivative Test to determine extrema
7. Analyzing the characteristics of a function based on the graph of its derivative and vice versa and analyzing the graphs of curves (inc, dec, concavity, extrema, ect…)
8. Analyzing the characteristics of the derivative of a function based on either the graph of the function or the graph of the second derivative
9. Optimizing distance, perimeter, area, volume, ect…
10. Solving rates of change problems, including related rates

INTEGRALS (4 WEEKS)

Activity One: Students will be given a list of basic polynomial and trig functions and be asked to find the derivative of each. At least two of the functions should have the same derivative. Students will the be asked to look at the derivative and discuss with a partner how they might find the original function given the derivative. Students should be sure to address the problems where two different functions have the same derivative. Now, students will be given a list of functions that represent the derivative, and they will be asked to find the original function (the antiderivative). Students will be asked to create a rule that will work every time for polynomial functions. The teacher will summarize the activity by relating the antiderivative to the indefinite integral and showing appropriate notation. **[CR2b,2e,2f]**

TOPICS: [CR1c]

1. Antiderivatives and their relationship to indefinite integrals
2. Basic rules for antiderivatives
3. Using initial conditions to determine specific antiderivatives
4. Basic properties for definite integrals
5. Applying the Fundamental Theorem of Calculus for evaluating definite integrals

6. Applying the Fundamental Theorem of Calculus for solving problems where a function is given as an integral
7. Evaluating integrals by substitution of variables (showing change of limits for definite integrals)

APPLICATIONS OF INTEGRALS (8 WEEKS)

Activity One: The teacher will use the process of Riemann sums to define a definite integral. The students will make discoveries about the relationship between the definite integral and the area under a curve. The students will practice in pairs problems involving finding definite integrals that are easily calculated, definite integrals that can only be found by calculating the area under the curve with trapezoids, triangles, rectangles, and circles, and area problems where students must rely on definite integrals. Students will begin to see the differences between area with integrals and simply integrals. With their partner, they will create a summary of the similarities and differences, and certain students will present to the class. **[CR2a,2b,2d,2f]**

Activity Two: Students will use the graphing calculator to compute the integrals of a function. Once again they will use the calculator as a means of seeing the difference between and integral and the area. The students will also be presented with problems that they must rely on the calculator to produce a complex graph, find the zeros and/or intersections of graphs, and calculate an integral in order to find the area under the curve or between curves. Students will practice all of these calculator skills through a variety of AP level problems involving trig functions, exponential functions, rational functions, and logarithmic functions. **[CR3a,3b,3c]**

Activity Three: Students are given a lab of past free-response questions in which they must use the Fundamental Theorem of Calculus. Within these problems, they are often required to calculate a definite integral with their calculators. In addition, they must answer questions about extrema and inflection points of g using calculus if given a function $g(x) = \int_a^x f(t)dt$ and a graph of f. **[CR2e]**

Activity Four: Students are asked on a worksheet how far they travel if they drive at 60 mph for 2 hours. They are then asked to create a velocity graph representing this situation. Students then evaluate the integral of $f(t)$ = 60 from 0 to 2 and discuss any connections they see. Students will analyze functions defined by an integral using both parts of the Fundamental Theorem of Calculus. Part one states if f is continuous on $[a, b]$, then the functions g defined by $g(x) = \int_a^x f(t)dt$ is an antiderivative of f. That is, $g'(x) = f(x)$ for $a < x < b$. Part two states if f is continuous on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$ where $F(x)$ is any antiderivative of $f(x)$. **[CR2e]**

TOPICS: [CR1c]

1. Approximating areas under curves as left, right, and midpoint of Riemann sums (given either an equation and/or graph of a function or a table of values)
2. Approximating the area under curves using the trapezoidal rule
3. Interpreting a definite integral as a limit of Riemann sums
4. Finding the exact area under curves by graphing and using the ideas of semicircles, trapezoids, rectangles and triangles to find the area or using the idea of a definite integral to find the exact area
5. Understanding that the integral of a rate of change represents a "total" change (total feet, total cars, total tons, etc….)
6. Applying the concept of average value to numerous concepts
7. Interpreting total distance as an integral of the absolute value of the function
8. Finding the area between two curves (includes understanding that the graphing calculator must sometimes be used as a tool for finding where the two curves intersect as well as a tool for evaluating a definite integral for seemingly impossible integrals)
9. Solving separable differential equations (some with initial conditions, some without) and applying this idea to various problems including applications to motion along a line, exponential growth, and decay.
10. Using the idea that derivative represents slope and solving separable differential equations to understand the geometric interpretation of differential equations via slope fields and understanding how these slope fields relate to the solutions of the differential equations)
11. Finding the volume of solids of revolution by discs, washers, and cross sections

RESOURCES:

Primary textbook:

Finney, Demana, Waits, Kennedy. *Calculus Graphical, Numerical, and Algebraic,* 1st ed. Menlo Park: Scott Foresman Addison Wesley, 1999.

Other textbooks:

Larson, Hostetler, Edwards. *Calculus of a Single Variable*, 6th ed. Boston: Houghton Mifflin Company, 1998.

Thomas, Finney. *Calculus*, 9th ed. Menlo Park: Addison-Wesley, 1996.

Stewart. *Single Variable Calculus*, 4th ed. Albany: Brooks/Cole Publising, 1999.