Practice Handout #2 - Definition of the Derivative / Tangent Lines

1.) Given $f(x) = x^3$.

(a) Write the equation of the tangent line to f at $x = 1$
(b) Use your answer to (a) to find an approximation for $f(1.1)$.

(c) Find the actual value of $f(1.1)$.

(d) Find the amount of error of your approximation.

2.) Let f be a function with $f'(x) = \sin(x^2)$ and $f(0) = -1$.

(a) Write the equation of the tangent line to f at $x = 0$.
(b) Use your answer to (a) to find an approximation for $f(0.1)$.

(c) Find $f''(x)$ and $f''(0.1)$. Is the graph of f concave up or concave down around $x = 0.1$? Use your answer to determine whether your approximation for $f(0.1)$ is greater than or less than the actual value of $f(0.1)$.

3.) (a) For $f(x) = 2x^3$, find an equation of the linear function that best fits f at $x = 1$.

(b) Use the tangent line equation you found in (a) to approximate $f(1.1)$.

(c) Find the actual value of $f(1.1)$ by using the function $f(x)$. What is the error in your linear approximation?

(d) Fill in the blank with < or >. Tangent line approx. _______ Actual value What does this tell you about the concavity of $f(x)$? Explain.

4.) Let $f(x) = 3(4x-5)^{-2}$

(a) What is the slope of the graph at x=2? Justify your answer.

(b) Is the function value of f changing more rapidly at x=3 or at x=1? Justify your answer.

(c) At what value of x is the slope of the graph undefined? Justify your answer.

5.) Let $f(x) = \dfrac{x^2-1}{x^2+1}$

(a) Determine $f'(x)$

(b) Is the function value of f changing more rapidly at x=0 or at x=-2? Justify your answer.

(c) What is the equation of the line tangent to the graph of f at x=1?

5.) Let $f(x) = \dfrac{x^2+1}{x}$ and $g(x) = 4x+7$

(a) Determine $h(x) = f(x)g(x)$. Write $h'(x)$ as a function of x.

(b) Is f, g, or h changing more rapidly at x=1? Justify your answer.

(c) What is the equation of the line tangent to the graph of h at x=1? Show the work that leads to your conclusion.

Multiple Choice Practice

1.) Let $f(x) = |x-3|$, then $f'(1)$ is

a.) -1	b.) 0	c.) 1	d.) 2	e.) DNE

2.) Let $f(x) = |x-3|$, then $f'(3)$ is

a.) -1	b.) 0	c.) 1	d.) 2	e.) DNE

4.) $\lim_{h\to 0}\frac{(10+h)^3-1000}{h}=$

a.) 0	b.) 1	c.) 30	d.) 300	e.) 3000

5.) $\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}=$

a.) 0	b.) $\frac{1}{12}$	c.) $\frac{1}{3}$	d.) $\frac{4}{3}$	e.) DNE

6.) $\lim_{h\to 0}\frac{\cos\left(\frac{\pi}{3}+h\right)-\frac{1}{2}}{h}=$

a.) -1	b.) $-\frac{\sqrt{3}}{2}$	c.) $-\frac{1}{2}$	d.) $\frac{1}{2}$	e.) $\frac{\sqrt{3}}{2}$

7.) $\lim_{x\to a}\frac{\ln(x)-\ln(a)}{x-a}=$

a.) $\frac{1}{x}$	b.) $\frac{1}{\ln(x)}$	c.) $\frac{1}{\ln(a)}$	d.) $\frac{1}{a}$	e.) $\frac{1}{x}-\frac{1}{a}$

8.) What does the limit statement $\lim_{x\to 1}\frac{\ln(x+1)-\ln(2)}{x-1}=$ represent?

a.) 0	c.) $f'(1)$ if $f(x)=\ln(x+1)$	d.) 1
b.) $\frac{d}{dx}[\ln(x+1)]$	e.) The limit does not exist.	

9.) Find the second derivative of $f(x)$ if.

$$f(x)=(2x+3)^4$$

a.) $4(2x+3)^3$	b.) $8(2x+3)^3$
c.) $12(2x+3)^2$	d.) $24(2x+3)^2$
e.) $48(2x+3)^2$	

Derivatives Practice

1.) $y=(x^4+5)^3$

2.) $f(x)=\sqrt{7x-3}$

3.) $g(x)=(x^2+9x)^{-2}$

4.) $h(x)=(x^3+3x+9)^{-\frac{4}{3}}$

5.) $y=(4x+9)^{\frac{1}{2}}$

6.) $f(x)=(4-2x-3x^2)^5$

7.) $h(x)=(\sqrt{x+1}-1)^{\frac{3}{2}}$

8.) $g(x)=(x+1)^4(2x-1)^3$

9.) $y=\left(9-\left(5-2x^4\right)^7\right)^3$

10.) $f(x)=\frac{(x+1)^{\frac{1}{2}}}{x+2}$

11.) $f(x)=\sqrt{\frac{x+1}{x-1}}$

12.) $f(x)=\left(\sqrt{x^5+4x^3}\right)^7$