AP Calculus – Free Response Post Exam Set #2

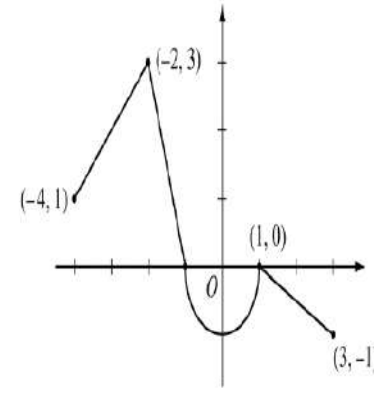
Exam Problem #3.)
Let *f* be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let *g* be the function given by $g(x) = \int_1^x f(t)\, dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

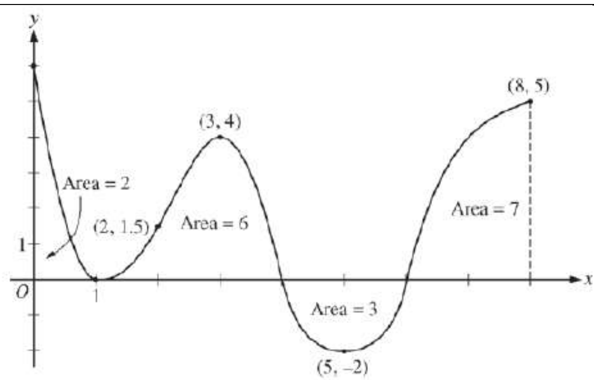
(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the *x*-coordinate of each point at which the graph of *g* has a horizontal tangent line. For each of these points, determine whether *g* has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of *x* for which the graph of *g* has a point of inflection. Explain your reasoning.



Graph of *f*



Graph of *f* ′

Practice #1 The figure above shows the graph of f', the derivative of a twice-differentiable function *f*, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at *x* = 1, *x* = 3, and *x* = 5. The areas of the regions between the graph of f' and the *x*-axis are labeled in the figure. The function *f* is defined for all real numbers and satisfies $f(8) = 4$.

(2013 AB4)

(a) Find all values of *x* on the open interval $0 < x < 8$ for which the function *f* has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of *f* on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of *f* both concave down and increasing?
Explain your reasoning.

(d) The function *g* is defined by $g(x) = f(x)^3$. If $f(3) = -\frac{5}{2}$ find the slope of the line tangent to the graph of *g* at *x* = 3.

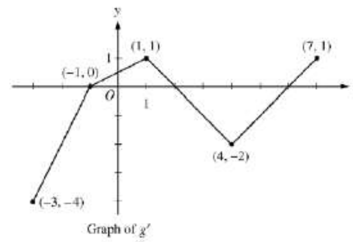
Practice #2 - Let g be a continuous function with $g(2)=5$. The graph of the piecewise-linear function g', the derivative of g, is shown for $-3 \le x \le 7$. (2008 ABB5)

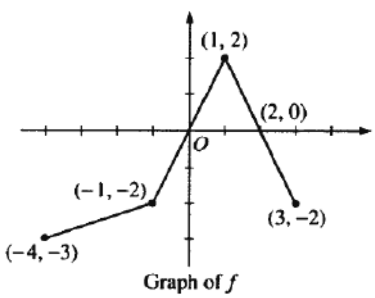
(a) Find the *x*-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 \le x \le 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that g"(c) is equal to the average rate? Why or why not?





Practice #3 – The graph of the function f consists of three line segments. (2005 ABB4)

(a) Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\, dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does nor exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function defined by $h(x) = \int_{x}^{3} f(t)\, dt$. Find all of the values of x on the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.