POD 2/9/15:

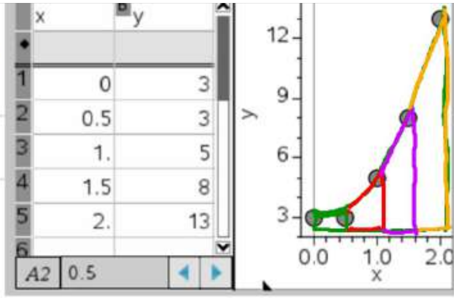
1997 AP Calculus AB: Section I, Part B

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

POD 2/9/15:



AP Calculus AB: Section I, Part B

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$$\frac{1}{2}\left[\left(\frac{3+3}{2}\right)+\left(\frac{3+5}{2}\right)+\left(\frac{5+8}{2}\right)+\left(\frac{8+13}{2}\right)\right]$$

$$\frac{1}{2}\cdot\frac{1}{2}\left[6+8+13+21\right]$$

$$\frac{1}{4}(48) =$$

THEOREM 4.16 The Trapezoidal Rule

Let f be continuous on $[a, b]$. The Trapezoidal Rule for approximating $\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right].$$

Moreover, as $n \to \infty$, the right-hand side approaches $\int_a^b f(x)\,dx$.

#12 Find the average value of $f(x) = \sin x$ on the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

NOTE: ***think about what "average value" would mean!!*** *(see pg. 286)*

Sum of values: $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x\,dx$

$\left(\frac{1}{b-a}\right) \quad -\cos x\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$\left(\frac{1}{\frac{\pi}{2}-\frac{\pi}{4}}\right) \quad 0 - -\frac{\sqrt{2}}{2}$

$\left(\frac{1}{\frac{\pi}{4}}\right) \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot \frac{4}{\pi} =$

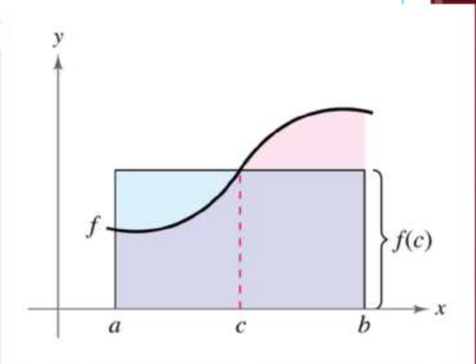
ANSWER: $\frac{2\sqrt{2}}{\pi}$

Theorem 4.10 Mean Value Theorem for Integrals and Figure 4.30

THEOREM 4.10 Mean Value Theorem for Integrals

If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b - a).$$

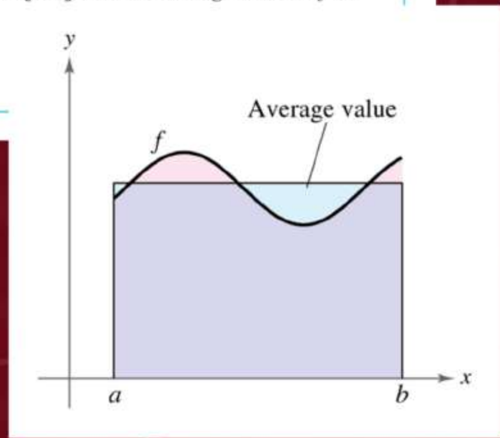


Definition of the Average Value of a Function on an Interval and Figure 4.32

Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b - a}\int_a^b f(x)\,dx.$$



Additional Problems from book:

- **p.280 #47, 48, 49 (using graphs to integrate)**
- **p.291 #48, 50 (average value)**
- **p.292 #54-60 (average value using graph)**

50.) $f(x) = \cos x \quad [0, \frac{\pi}{2}]$

p.291

48) $f(x) = \frac{4(x^2+1)}{x^2} \quad [1,3]$

$$\text{Avg. Value} = \frac{1}{3-1}\int_1^3 (4 + 4x^{-2})\,dx$$

$$= \frac{1}{2}\left[4x - 4x^{-1}\right]_1^3 = \frac{1}{2}\left[\left(12 - \frac{4}{3}\right) - (4-4)\right]$$

$$= \frac{1}{2}\left(\frac{32}{3}\right) = \frac{16}{3}$$