AP Calculus – Free Response Solutions File **Unit 6 Free Response #2**

#1a (2004 ABB3)

AP Scoring Rubric

(a) Midpoint Riemann sum is

$10 \cdot [v(5) + v(15) + v(25) + v(35)]$

$= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$

The integral gives the total distance in miles that the plane flies during the 40 minutes.

$3 : \begin{cases} 1 : v(5) + v(15) + v(25) + v(35) \\ 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 3(a)

~~Area = 10(9.2) + 10(7.5)~~ ~~= 10f~~

area = $10(f(5) + f(15) + f(25) + f(35))$

$= 10(9.2 + 7 + 2.4 + 4.3)$

$\boxed{\text{area} = 229 \text{ miles}}$

$\int_0^{40} v(t)dt$ is the total distance traveled between $t = 0$ and $t = 40$ minutes

#1b

AP Scoring Rubric

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t.

$2 : \begin{cases} 1 : \text{two instances} \\ 1 : \text{justification} \end{cases}$

Actual Solution Receiving Full Credit

$a(t)=0$

~~between (0, 15)~~

on the intervals $[0,15]$ and $[25,30]$

the smallest number of instances the acceleration can equal zero is 2 by MVT and Rolle's Theorem

#1c

AP Scoring Rubric

(c) $f'(23) = -0.407$ or -0.408 miles per minute^2	1 : answer with units

Actual Solution Receiving Full Credit

$f'(t) = \frac{-1}{10}\sin t/10 + 3 \cdot \frac{7}{40}\cos 7t/40$

$f'(t) = -1/10 \sin t/10 + 21/40 \cos 7t/40$

$f'(23) = -1/10 \sin 23/10 + 21/40 \cos 161/40$

$f'(23) = \boxed{-.408 \text{ miles per minute}^2}$

#1d

AP Scoring Rubric

(d) Average velocity $= \frac{1}{40}\int_0^{40} f(t)\,dt$ $= 5.916$ miles per minute	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 3(d)

~~average v = $\frac{f(40)-f(0)}{40-0}$~~

~~$= \frac{7.317-6}{40}$~~

~~= .033 miles per minute~~

$$\frac{1}{40-0}\int_0^{40} 6+\cos(t/10)+3\sin(7t/40)\,dt$$

average velocity $= \frac{1}{40} \cdot 236.65079$

$\boxed{= 5.916 \text{ miles per minute}}$

#2a (2009 AB1)

AP Scoring Rubric

(a) $a(7.5) = v'(7.5) = \dfrac{v(8)-v(7)}{8-7} = -0.1$ miles/minute^2	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{units} \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 1(a)

$$a(7.5) = v'(7.5) = \frac{v(8)-v(7)}{8-7} = 0.2 - 0.3 = -0.1$$

$$\therefore a(7.5) = -0.1 \ \text{mi}/\text{min}^2$$

#2b

AP Scoring Rubric

(b) $\int_0^{12} \lvert v(t) \rvert \, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t=0$ to $t=12$. $\int_0^{12} \lvert v(t) \rvert \, dt = \int_0^2 v(t)\,dt - \int_2^4 v(t)\,dt + \int_4^{12} v(t)\,dt$ $= 0.2 + 0.2 + 1.4 = 1.8$ miles	$2 : \begin{cases} 1 : \text{meaning of integral} \\ 1 : \text{value of integral} \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 1(b)

$\int_0^{12} |v(t)|\,dt = 0.2+0.2+0.15+0.3+0.2+0.05+6(0.1)+0.1$

$= 1.8$ miles

$\therefore \int_0^{12} |v(t)|\,dt$ is the total distance that Caren traveled from time $t=0$ min to $t=12$ min to arrive to the school, which is 1.8 mi.

#2c

AP Scoring Rubric

(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

Actual Solution Receiving Full Credit

She turns around at $t=2$ minutes because that is when her velocity changes from positive to negative.

#2d

AP Scoring Rubric

(d) $\int_0^{12} w(t)\,dt = 1.6$; Larry lives 1.6 miles from school.

$\int_0^{12} v(t)\,dt = 1.4$; Caren lives 1.4 miles from school.

Therefore, Caren lives closer to school.

$3 : \begin{cases} 2 : \text{Larry's distance from school} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{value} \\ 1 : \text{Caren's distance from school} \\ \quad \text{and conclusion} \end{cases}$

Actual Solution Receiving Full Credit

$\int_0^{12} w(t)\,dt = \int_0^{12} \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)dt = 1.6$ mi

The distance from Larry's house to school : 1.6mi

$\int_0^{12} v(t)\,dt = 0.15+0.3+0.2+0.05+0.6+0.1 = 1.4$mi

The distance from Caren's house to school : 1.4mi

$\therefore$ Caren lives closer to school because the distance from school to her house is smaller than that to Larry's house.

#3a (2006 AB3)

AP Scoring Rubric

(a) $g(4) = \int_0^4 f(t)\,dt = 3$

$g'(4) = f(4) = 0$

$g''(4) = f'(4) = -2$

$$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$$

Actual Solution Receiving Full Credit

Work for problem 3(a)

$g(4) = \int_0^4 f(t)\,dt = 3$

$\boxed{g(4) = 3}$

$\boxed{g'(4) = f(4) = 0}$

$g''(4) = f'(4) = \frac{2+2}{3-5} = \frac{4}{-2} = -2$

$\boxed{g''(4) = f'(4) = -2}$

#3b

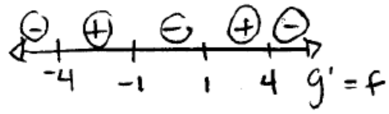
AP Scoring Rubric

(b) g has a relative minimum at $x = 1$ because $g' = f$ changes from negative to positive at $x = 1$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

Actual Solution Receiving Full Credit

Work for problem 3(b)



g has a relative minimum at x = 1 because g'(x) = f(x) changes from negative to positive at x = 1

#3c

AP Scoring Rubric

(c) $g(0) = 0$ and the function values of g increase by 2 for every increase of 5 in x.

$$g(10) = 2g(5) = 4$$

$$g(108) = \int_0^{105} f(t)\,dt + \int_{105}^{108} f(t)\,dt$$
$$= 21g(5) + g(3) = 44$$

$$g'(108) = f(108) = f(3) = 2$$

An equation for the line tangent to the graph of g at $x = 108$ is $y - 44 = 2(x - 108)$.

$$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$

Actual Solution Receiving Full Credit

Work for problem 3(c)

$g'(x) = f(x)$
$g(x) = \int f(x)\,dx$

- if $g(5) = 2$ and f is periodic w/ a period length of 5, the $\boxed{g(10) = 4}$
- $g(108) = ?$

$g(108) = \int_0^{108} f(x)\,dx = \boxed{44}$

$g'(108) = f(108) = 2$

$\boxed{(y - 44) = 2(x - 108)}$

$y = 2x - 72$

#4a . (2012 AB6)

AP Scoring Rubric

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \Rightarrow t = 3, 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

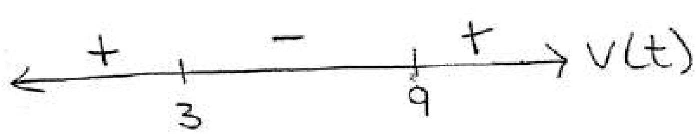
$2 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

Actual Solution Receiving Full Credit

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$. $x(0) = -2$

(a) For $0 \le t \le 12$, when is the particle moving to the left?

$\cos(\frac{\pi}{6}t) = 0$ when $x = 3, x = 9$



The particle is moving lest on $(3, 9)$

#4b

AP Scoring Rubric

(b) $\int_0^6 \lvert v(t) \rvert \, dt$	1 : answer

Actual Solution Receiving Full Credit

$$D = \int_0^6 \left| \cos\left(\frac{\pi}{6} t\right) \right| dt$$

#4c

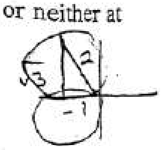
AP Scoring Rubric

(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$ $a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$ $v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$ The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.	$3 : \begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

Actual Solution Receiving Full Credit

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

$a(t) = v'(t) = -\sin\left(\frac{\pi}{6}t\right) \cdot \frac{\pi}{6} = \boxed{-\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)}$



$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\pi}{6}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi\sqrt{3}}{12}$

$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

The speed is increasing because $a(t)$ and $v(t)$ have the same sign $(-)$ at $t=4$

#4d

AP Scoring Rubric

(d) $x(4) = -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt$

$= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4$

$= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right]$

$= -2 + \frac{6}{\pi} \cdot \frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi}$

$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

Actual Solution Receiving Full Credit

(d) Find the position of the particle at time $t = 4$.

$x(t) = \int \cos(\frac{\pi}{6}t)\,dt$ $\quad u = \frac{\pi}{6}t$, $du = \frac{\pi}{6}dt$

$x(t) = \frac{6}{\pi}\int \cos u\,du$

$x(t) = \frac{6}{\pi}\sin(\frac{\pi}{6}t) + C$

$\frac{6}{\pi}\sin(\frac{\pi}{6}t) + C = -2$ when $x = 0$

$\frac{6}{\pi}\sin(0) + C = -2$

$C = -2$

$x(t) = \frac{6}{\pi}\sin(\frac{\pi}{6}t) - 2$

$x(4) = \frac{6}{\pi}\sin(\frac{2\pi}{3}) - 2$

$x(4) = \frac{6}{\pi}(\frac{\sqrt{3}}{2}) - 2$

$\boxed{x(4) = \frac{3\sqrt{3}}{\pi} - 2}$