Interpreting Calculus – What Does it Mean?

1. For t $\geq$ 0 hours, *H* is a differentiable function of t that gives the temperature, in degrees Fahrenheit, of a house in Houston in July, where t is measured in hours since midnight. Using correct units, explain the meaning of each of the following in the context of the problem.
   a. $H(15)$ is the temperature, in ^{o}F, of a house in Houston in July 15 hours after midnight.
   b. $H'(15)$ is the instantaneous rate of change of H, temperature, in of a house in Houston in July, measured in ^{o}F/hour at t= 15 hours after midnight.
   c. $\dfrac{\int_0^8 H(t)dt}{8-0}$ is the average value of H, the temperature of a house in Houston in July, in ^{o}F , over the time interval t=0 to t=8 hours after midnight.

2. A water main has broken and a flow meter gives the rate $R(t)$ at which water is gushing out of the pipe in gallons per minute, where *t* is measured in minutes since the pipe broke. Using correct units, explain the meaning of each of the following in the context of the problem.
   a. $R(5)$ is the rate at which water is gushing out of the pipe in gal/ min at t=5 minutes after the pipe broke.
   b. $R'(5)$ is the instantaneous rate of change of R, the rate at which water is gushing out of the pipe measured in gal/ min^2 at t=5 minutes after the pipe broke.
   c. $\int_0^5 R(t)dt$ is the amount of water that gushed out of the pipe measured in gallons over the time interval t=0 to t=5 minutes after the pipe broke.
   d. $\dfrac{\int_0^5 R(t)dt}{5-0}$ is the average value of R, the rate at which water is gushing out of the pipe in gal/ min, over the time interval t=0 to t=5 minutes after the pipe broke.

3. The cost *C*, in dollars per mile, of digging a 10-mile tunnel through a mountain varies with the distance *x*, in miles, from the opening of the tunnel. Using correct units, explain the meaning of each of the following in the context of the problem.
   a. $\frac{1}{10}\int_0^{10} C(x)dx$ is the average value of *C*, the cost, of digging a 10-mile tunnel through a mountain $/mile, over a distance of 10 miles.
   b. $C'(6)$ is the instantaneous rate of change of *C*, the cost, of digging a 10-mile tunnel through a mountain $/mile/mile, at a distance of 6 miles from the opening of the tunnel.
   c. $\int_8^{10} C(x)dx$ is the total cost of digging a tunnel through a mountain in $ from 8 miles from the opening to 10 miles from the opening.

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4. A piece of wire that is 10 inches long is heated by holding a candle to one end. The temperature $T(x)$, in degrees Celsius, of the wire varies with the distance from the candle, where x is measured in inches from the candle. Using correct units, explain the meaning of each of the following in the context of the problem.
    a. $\int_0^{10} T'(x)dx$ is the net change in T(x), the temperature of a wire being heated by a candle, measured in ^{0}C, from the end of the wire (x=0) to a distance of 10 inches from the candle.
    b. $T'(6)$ is the instantaneous rate of change in T(x), the temperature of a wire being heated by a candle, measured in ^{0}C/inch, at a distance of 6 inches from the candle.
    c. $\frac{1}{10}\int_0^{10} T(x)dx$ is the average value of T(x), the temperature of a wire being heated by a candle, measured in ^{0}C, from the end of the wire (x=0) to a distance of 10 inches from the candle.

5. The rate at which water flows into a tank is given by $R(t)$, measured in gallons per hour, for $t \geq 0$. If the tank initially holds 75 gallons of water, use proper calculus notation to represent each of the following.
    a. The amount of water added to the tank during the second hour of flow
    $\int_1^2 R(t)dt$
    b. The rate at which the rate of water is flowing is changing at t = 6 hours.
    $R'(t)$
    c. The average rate of water flow over the time interval t = 0 to t = 10 hours.
    $\frac{1}{10}\int_0^{10} R(t)dt$
    d. The average rate of increase in the rate of water flow from t = 0 to t = 10 hours.
    $\frac{1}{10}\int_0^{10} R'(t)dt$
    e. The total amount of water in the tank at t = 10 hours.
    $75+\int_0^{10} R(t)dt$

6. At t = 0, a pie is taken out of a 350°F oven and left to cool in a kitchen that is 75°F. The rate at which the temperature changes is given by $T(t)$, measured in °F/minute. Use proper calculus notation to represent each of the following.
    a. The change in pie temperature between 10 and 15 minutes after removal from the oven.
    $\int_{10}^{15} T(t)dt$
    b. How fast the rate of change of the temperature is changing at t = 10 minutes.
    $T'(10)$

c. The temperature of the pie after 15 minutes

$350+\int_0^{15} T(t)dt$

7. **For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station.

   a. The change in temperature during the first day

   $H(24)-H(0)$

   b. The change in temperature during the 24th hour

   $H(25)-H(24)$

   c. The average rate at which the temperature changed during the 24th hour

   $H(25)-H(24)$

   d. The rate at which the temperature is changing during the first day

   $H'(t) \quad \text{for } 0 < t < 24$

   e. The rate at which the temperature is changing at the end of the 24th hour

   $H'(25)$

8. The rate at which people enter Reliant Stadium for the Big Game is modeled by a differentiable function $P(t)$, measured in people per hour. Assume the stadium is empty at $t = 0$, when the gates open at 11 am and will close at 3 pm when the game begins. Use proper calculus notation to represent each of the following.

   a. The average rate at which people enter the stadium from noon to 1 pm

   $\frac{1}{2-1}\int_1^2 P(t)dt$

   b. The number of people who enter the game during the hour before the game begins

   $\int_3^4 P(t)dt$

   c. The total number of people who have entered the stadium when the game begins.

   $\int_0^4 P(t)dt$

   d. The rate at which the rate at which people enter the stadium is changing at noon.

   $P'(1)$

9. The cost, C, in dollars per foot, of a piece of fiber optic cable varies with its length, x. Use proper calculus notation to represent each of the following.

   a. The cost of purchasing a 10-foot length of cable

   $\int_0^{10} C(x)dx$

   b. The average cost per foot of that piece of 10-foot cable

   $\frac{1}{10}\int_0^{10} C(x)dx$

   c. The difference in cost between the 10-foot cable and a 12-foot cable

   $\int_{10}^{12} C(x)dx$

   d. How fast the cost is changing when you are at 10 feet of cable

   $C'(10)$