

Practice

Common Logarithms

Given that $\log 3 = 0.4771$, $\log 5 = 0.6990$, and $\log 9 = 0.9542$, evaluate each logarithm.

1. $\log 300,000$

5.477

2. $\log 0.0005$

-3.301

3. $\log 9000$

3.954

4. $\log 27$

1.431

5. $\log 75$

1.875

6. $\log 81$

1.908

Evaluate each expression.

7. $\log 66.3$

1.822

8. $\log \frac{17^4}{5}$

$4 \log 17 - \log 5 = 4.223$

9. $\log 7(4^3)$

$\log 7 + 3 \log 4 = 2.651$

Find the value of each logarithm using the change of base formula.

10. $\log_6 832$

$\frac{\log 832}{\log 6} = 3.753$

11. $\log_{11} 47$

$\frac{\log 47}{\log 11} = 1.606$

12. $\log_3 9$

$\frac{\log 9}{\log 3} = 2$

Solve each equation or inequality.

13. $8^x = 10$

$x = \frac{\log 10}{\log 8} = 1.107$

14. $2.4^x \leq 20$

$x \leq \frac{\log 20}{\log 2.4} = 3.422$

15. $1.8^{2x-6} = 19.8$

$x - 3 = \frac{\log 19.8}{\log 1.8} + 3$

16. $3^{5x} = 85$

$\frac{5x}{5} = \frac{\log 85}{\log 3} \div 5$
 $x = 0.809$

17. $4^{2x} \geq 25$

$\frac{2x}{2} \geq \frac{\log 25}{\log 4} \div 2$
 $x \geq 1.161$

18. $3^{2x-2} = 2^x$

$\log 3^{2x-2} = \log 2^x$
 $(2x-2) \log 3 = x \log 2$
 $-2 \log 3 = x \log 2 - 2x \log 3$
 $-2 \log 3 = x(\log 2 - 2 \log 3)$
 $x = 1.461$

19. **Seismology** The intensity of a shock wave from an earthquake

is given by the formula $R = \log_{10} \frac{I}{I_0}$, where R is the magnitude, I is a measure of wave energy, and $I_0 = 1$. Find the intensity per unit of area for the following earthquakes.

a. Northridge, California, in 1994, $R = 6.7$

5011872.336

$6.7 = \log_{10} I$
 $10^{6.7} = I$

b. Hector Mine, California, in 1999, $R = 7.1$

12589254.12

$7.1 = \log_{10} I$
 $10^{7.1} = I$

Study Guide

Common Logarithms

Logarithms with base 10 are called **common logarithms**.

The change of base formula, $\log_a n = \frac{\log_b n}{\log_b a}$, where a , b , and n

are positive numbers and neither a nor b is 1, allows you to evaluate logarithms in other bases with a calculator.

Logarithms can be used to solve **exponential equations**.

Example 1 Evaluate each expression.

a. $\log 8(3)^2$

$$\begin{aligned} \log 8(3)^2 &= \log 8 + 2 \log 3 && \log ab = \log a + \log b, \log b^n = n \log b \\ &\approx 0.9031 + 2(0.4771) && \text{Use a calculator.} \\ &\approx 0.9031 + 0.9542 \\ &\approx 1.8573 \end{aligned}$$

b. $\log \frac{15^3}{7}$

$$\begin{aligned} \log \frac{15^3}{7} &= 3 \log 15 - \log 7 && \log \frac{a}{b} = \log a - \log b, \log a^m = m \log a \\ &\approx 3(1.1761) - 0.8451 && \text{Use a calculator.} \\ &\approx 3.5283 - 0.8451 \\ &\approx 2.6832 \end{aligned}$$

Example 2 Find the value of $\log_8 2037$ using the change of base formula.

$$\begin{aligned} \log_8 2037 &= \frac{\log_{10} 2037}{\log_{10} 8} && \log_a n = \frac{\log_b n}{\log_b a} \\ &\approx \frac{3.3090}{0.9031} && \text{Use a calculator.} \\ &\approx 3.6641 \end{aligned}$$

Example 3 Solve $7^{2x} = 93$.

$$\begin{aligned} 7^{2x} &= 93 \\ \log 7^{2x} &= \log 93 && \text{Take the logarithm of each side.} \\ 2x \log 7 &= \log 93 && \log_b m^p = p \cdot \log_b m \\ 2x &= \frac{\log 93}{\log 7} && \text{Divide each side by } \log 7. \\ 2x &\approx 2.3293 && \text{Use a calculator.} \\ x &\approx 1.1646 \end{aligned}$$