Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

1. If $f(x) = 2x^2 + 4$, which of the following will calculate the derivative of $f(x)$?

(a) $\dfrac{[2(x + \Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

(b) $\lim\limits_{\Delta x \to 0} \dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

(c) $\lim\limits_{\Delta x \to 0} \dfrac{[2(x + \Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

(d) $\dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

(e) None of these

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

2. Differentiate: $y = \dfrac{1 + \cos x}{1 - \cos x}$

(a) -1

(b) $-2 \csc x$

(c) $2 \csc x$

(d) $\dfrac{-2 \sin x}{(1 - \cos x)^2}$

(e) None of these

2	$y' = \dfrac{(1-\cos x)(-\sin x) - (1+\cos x)(\sin x)}{(1-\cos x)^2} = \dfrac{\sin x \cos x - \sin x - \sin x - \sin x \cos x}{(1-\cos x)^2} \Rightarrow y' = \dfrac{-2\sin x}{(1-\cos x)^2}$	D

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

3. Find dy/dx for $y = (x^3)\sqrt{x+1}$.

(a) $\frac{3x^2}{2\sqrt{x+1}}$ (b) $\frac{x^2(7x+6)}{2\sqrt{x+1}}$ (c) $3x^2\sqrt{x+1}$

(d) $\frac{7x^3+x^2}{2\sqrt{x+1}}$ (e) None of these

$y = x^3(x+1)^{1/2}$

3

$y = x^3\sqrt{x+1}$

$y' = x^3\left(\frac{1}{2}(x+1)^{-1/2}\right) + \sqrt{x+1}\,(3x^2)$

$= \frac{x^3}{2\sqrt{x+1}} + 3x^2\sqrt{x+1}\left(\frac{2\sqrt{x+1}}{2\sqrt{x+1}}\right) = \frac{x^3+6x^2(x+1)}{2\sqrt{x+1}}$

$= \frac{7x^3+6x^2}{2\sqrt{x+1}} = \frac{x^2(7x+6)}{2\sqrt{x+1}}$

B

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

4. Find $f'(x)$ for $f(x) = (2x^2+5)^7$.

(a) $7(4x)^6$ (b) $(4x)^7$ (c) $28x(2x^2+5)^6$

(d) $7(2x^2+5)^6$ (e) None of these

4

$f(x) = (2x^2+5)^7$

$f'(x) = 7(2x^2+5)^6 \cdot 4x$

$f'(x) = 28x(2x^2+5)^6$

C

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

5. Differentiate: $y = \sec^2 x + \tan^2 x$.

(a) 0 (b) $\tan x + \sec^4 x$ (c) $\sec^2 x(\sec^2 x + \tan^2 x)$

(d) $4 \sec^2 x \tan x$ (e) None of these

5		
	$\frac{d}{dx}\sec^2 x \quad \frac{d}{dx}\tan^2 x$ $y' = \overbrace{2\sec x \cdot \sec x \tan x}^{} + \overbrace{2\tan x \cdot \sec^2 x}^{}$ $= 2\sec^2 x \tan x + 2\sec^2 x \tan x$ $= 4\sec^2 x \tan x$	D

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

6. Find the derivative: $f(\theta) = \sqrt{\sin 2\theta}$.

(a) $\dfrac{\cos 2\theta}{\sqrt{\sin 2\theta}}$ (b) $\sqrt{\sec 2\theta}$ (c) $\dfrac{\cos 2\theta}{2\sqrt{\sin 2\theta}}$

(d) $\cos\theta$ (e) None of these

6		
	$f'(\theta) = \cancel{\tfrac{1}{2}}(\sin 2\theta)^{-1/2} \cdot \cos 2\theta \cdot \cancel{2}$ $= \dfrac{\cos 2\theta}{\sqrt{\sin 2\theta}}$	A

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

7. Find the derivative: $s(t) = \csc\frac{t}{2}$ → $\csc\left(\frac{1}{2}t\right)$

(a) $-\csc\frac{t}{2}\cot\frac{t}{2}$ (b) $-\frac{1}{2}\cot^2\frac{t}{2}$ (c) $\frac{1}{2}\csc\frac{t}{2}\cot\frac{t}{2}$

(d) $\frac{1}{2}\cot^2\frac{t}{2}$ (e) None of these

7 $s'(t) = -\csc\frac{t}{2}\cot\frac{t}{2}\cdot\frac{1}{2}$

$-\frac{1}{2}\csc\frac{t}{2}\cot\frac{t}{2}$

E

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

8. Find an equation for the tangent line to the graph of $f(x) = 2x^2 - 2x + 3$ at the point where $x = 1$.

(a) $y = 2x - 2$ (b) $y = 4x^2 - 6x + 5$ (c) $y = 2x + 1$

(d) $y = 4x^2 - 6x + 2$ (e) None of these

8 $f'(x) = 4x - 2 \;\big|_{x=1}$

$f'(1) = 2$

$m = 2$

pt $(1, f(1)) \to (1, 3)$

$y - 3 = 2(x - 1)$

$y = 2x - 2 + 3$

$y = 2x + 1$

C

FC

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

coordinate!

EXTRA CREDIT (2 pts each):

A. Find all points on the graph of $f(x) = -x^3 + 3x^2 - 2$ at which there is a horizontal tangent line.

(a) $(0, -2), (2, 2)$ (b) $(0, -2)$ (c) $(1, 0), (0, -2)$

(d) $(2, 2)$ (e) None of these

EC A

$f'(x) = -3x^2 + 6x = 0$

$3x(2 - x) = 0$

$x = 0, 2$

$f(0) = -2$ $f(2) = -8 + 12 - 2 = 2$

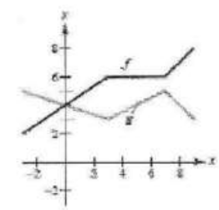
$(0, -2)$ $(2, 2)$

A

EC

Unit 2 Quiz 2: Differentiation Techniques (with Trig) - KEY

B. Let $p(x) = f(x)g(x)$. Use the figure to find $p'(5)$.



(a) 7 (b) 3

(c) 0 (d) 24

(e) None of these

m_{tan} of $g(x)$ at $x=5$

m_{tan} of $f(x)$ at $x=5$

EC B

$p'(x) = f(x)g'(x) + g(x)f'(x)$

$p'(5) = f(5)g'(5) + g(5)f'(5)$

$= (6)\cdot\left(\frac{1}{2}\right) + (4)\cdot(0)$

$= 3 + 0$

$= 3$

$g'(5) = \frac{g(7) - g(3)}{7 - 3}$

$= \frac{5 - 3}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

B

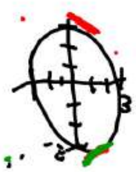
Explicit vs Implicit Differentiation

$y = 3x^3 + 2x - 4$

$\frac{dy}{dx} = 9x^2 + 2$

Find m_{tan} at $x=1$

is $\frac{-1}{2\sqrt{2}}$ or $\frac{1}{2\sqrt{2}}$ $(1, 2\sqrt{2})$ or $(1, -2\sqrt{2})$



$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

$x^2 + y^2 = 9 \Big|_{x=1}$:

$1 + y^2 = 9$

$y^2 = 8$

$y = 2\sqrt{2}$.

or

$-2\sqrt{2}$.

$y = \pm\sqrt{9 - x^2}$ ..

$\frac{dy}{dx} = \frac{??}{}$

$\frac{d}{dx}\left[x^2 + y^2 = 9\right]$

$2x\cancel{\frac{dx}{dx}}^{1} + 2y\frac{dy}{dx} = 0\cancel{\frac{dx}{dx}}^{1}$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y}$

Example 1 – *Differentiating with Respect to x*

a. $\frac{d}{dx}[x^3] = 3x^2 \cancel{\frac{dx}{dx}}^{1}$ Variables agree: use Simple Power Rule.

Variables agree

b. $\frac{d}{dx}[\overbrace{y^3}^{u^n}] = \overbrace{3y^2}^{nu^{n-1}}\overbrace{\frac{dy}{dx}}^{u'}$ Variables disagree: use Chain Rule.

Variables disagree

c. $\frac{d}{dx}[x + 3y] = 1\cancel{\frac{dx}{dx}}^{1} + 3\frac{dy}{dx}$ Chain Rule: $\frac{d}{dx}[3y] = 3y'$

Example 1 – *Differentiating with Respect to x*

cont'd

d. $\frac{d}{dx}[xy^2] = x\frac{d}{dx}[y^2] + y^2\frac{d}{dx}[x]$ Product Rule

$= x\left(2y\frac{dy}{dx}\right) + y^2(1)\cancel{\frac{dx}{dx}}$ 1 Chain Rule

$= 2xy\frac{dy}{dx} + y^2$ Simplify.

Example 2 – *Implicit Differentiation*

Find *dy*/*dx* given that $y^3 + y^2 - 5y - x^2 = -4$. m_{tan} @ $(2,0)$

Solution:

1. Differentiate both sides of the equation with respect to *x*.

$$\frac{d}{dx}[y^3 + y^2 - 5y - x^2] = \frac{d}{dx}[-4]$$

$$\frac{d}{dx}[y^3] + \frac{d}{dx}[y^2] - \frac{d}{dx}[5y] - \frac{d}{dx}[x^2] = \frac{d}{dx}[-4]$$

$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - 2x\cancel{\frac{dx}{dx}} = 0$$

$\frac{dy}{dx}(3y^2+2y-5) = 2x$

$\boxed{\frac{dy}{dx} = \frac{2x}{3y^2+2y-5}}$ $\Big|_{(2,0)} = \frac{-4}{5}$.

∴ function is decreasing at $(2,0)$

p. 145 #9

Find $\frac{dy}{dx}$: $\frac{d}{dx}\left[x^3 - 3x^2y + 2xy^2 = 12\right]$

$$3x^2 \cancel{\frac{dx}{dx}} - \left[3x^2\frac{dy}{dx} + y\left(6x\cancel{\frac{dx}{dx}}\right)\right] + \left[2x\cdot 2y\frac{dy}{dx} + y^2\cdot 2\cancel{\frac{dx}{dx}}\right] = 0$$

$$3x^2 - 3x^2\frac{dy}{dx} - 6xy + 4xy\frac{dy}{dx} + 2y^2 = 0$$

$$-3x^2\frac{dy}{dx} + 4xy\frac{dy}{dx} = -\left(3x^2 - 6xy + 2y^2\right)$$

$$\frac{dy}{dx}\left(4xy - 3x^2\right) = -\left(3x^2 - 6xy + 2y^2\right)$$

$$\boxed{\frac{dy}{dx} = -\frac{3x^2 - 6xy + 2y^2}{4xy - 3x^2}}$$

p. 145 #5

$\frac{d}{dx}\left[x^3 - xy + y^2 = 7\right]$

$$3x^2 - \left[x\frac{dy}{dx} + y(1)\right] + 2y\frac{dy}{dx} = 0$$

$$3x^2 - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -\left(3x^2 - y\right)$$

$$\frac{dy}{dx}\left(2y - x\right) = -\left(3x^2 - y\right)$$

$$\boxed{\frac{dy}{dx} = -\frac{3x^2 - y}{2y - x}}$$

p. 145 #11

$$\frac{d}{dx}\left[\sin x + 2\cos 2y = 1\right]$$

$$\cos x - 2\sin 2y\left(2\frac{dy}{dx}\right) = 0$$

$$\cos x - 4\frac{dy}{dx}\sin 2y = 0$$

$$-4\frac{dy}{dx}\sin 2y = -\cos x$$

$$\frac{dy}{dx} = \frac{-\cos x}{-4\sin 2y}$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{4\sin 2y} \quad \text{or} \quad = \frac{1}{4}\cos x \csc 2y}$$

p. 145 #26

$$\frac{d}{dx}\left[x^3 + y^3 = 6xy - 1\right] \quad pt\ (2,3)$$

$$3x^2 + 3y^2\frac{dy}{dx} = \left(6x\frac{dy}{dx} + y(6)\right) - 0$$

$$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$$

$$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx}(3y^2 - 6x) = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$= \frac{3(2y - x^2)}{3(y^2 - 2x)}$$

$$\boxed{\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}}$$

So, $\frac{dy}{dx}$ at $(2,3)$

$$= \frac{2(3) - 2^2}{3^2 - 2(2)}$$

$$= \frac{2}{5}$$

p. 145 #27

$$\frac{d}{dx}\left[\tan(x+y)=x\right] \quad pt\ (0,0)$$

$$\sec^2(x+y)\cdot\left(1+\frac{dy}{dx}\right)=1$$

$$1+\frac{dy}{dx}=\frac{1}{\sec^2(x+y)}$$

$$1+\frac{dy}{dx}=\cos^2(x+y)$$

$$\frac{dy}{dx}=\cos^2(x+y)-1\ \Big|_{(0,0)}$$

$$=\cos^2(0)-1$$
$$=1-1=0$$

p. 145 #33

$$\frac{d}{dx}\left[(y-3)^2=4(x-5)\right] \quad pt\ (6,1)$$

Due Thurs 11/9:
2.5 p.145-146
#4-10even.
14,22,26,28,34,
38,57,58

$$2(y-3)\left(\frac{dy}{dx}\right)=4(1)$$

$$\frac{dy}{dx}=\frac{4}{2(y-3)}\ \Big|_{(6,1)}$$

$$m_{tan}=\frac{4}{-4}$$

$$\left.\begin{array}{l} m_{tan}=-1 \\ pt: (6,1) \end{array}\right\}$$

$$\boxed{\text{Tangent Line: } y-1=-(x-6)}$$