AP Calculus Unit 2 Quiz 3 (*with Trig & Implicit*) - KEY

1.) Prove that $\frac{d}{dx}(\csc x) = -\csc x \cot x$.

$\frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{\sin x(0) - (1)(\cos x)}{\sin^2 x}$

$= \frac{0 - \cos x}{\sin^2 x}$

$= \frac{-\cos x}{\sin x \sin x} \cdot \frac{1}{\sin x}$

$= -\csc x \cot x$ ✓

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2.) $\lim_{\Delta x \to 0} \frac{\cos 3(x+\Delta x) - \cos(3x)}{\Delta x} =$ _____ *Circle your answer.*

(a) 0 (b) $-\sin(3x)$ (c) $\cos(3x)$ (d) $-3\sin(3x)$ (e) nonexistent

$\frac{f(x+h) - f(x)}{h}$

This is Def. of Derivative of WHAT function??

So question is asking ...

what's the derivative of $\cos(3x)$?

$-\sin(3x) \cdot 3$

$= D: -3\sin 3x$.

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For # 3-6, find the derivatives of the following functions. *Write final answer in the space provided.*

3.) $f(x) = \pi^2 - 3\pi + 1$ 3.) $f'(x) = 0$.

you're KILLING me!! These are constants!!!

4.) $f(x) = \frac{3x^3}{4} - \frac{15x^2}{2} + \frac{7x}{4} - \frac{4}{11}$

$f'(x) = 3\left(\frac{9}{4}\right)x^2 - 2\left(\frac{15}{2}\right)x + \frac{7}{4}$

$= 9x^2 + 15x + \frac{7}{4}$

constant multiple w/ power...duh!!

5.) $y = (3x^2 - 5x)(2x - 9)$ 5.)

product Rule.

$y' = (3x^2 - 5x)(2) + (2x-9)(6x-5)$

$= 6x^2 - 10x + 12x^2 - 10x - 54x + 45$

$y' = 18x^2 - 74x + 45$

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6.) $g(x) = \frac{5x+1}{8x-2}$ 6.) $\frac{-18}{(8x-2)^2}$

$g'(x) = \frac{(8x-2)(5) - (5x+1)(8)}{(8x-2)^2}$

$= \frac{40x - 10 - (40x + 8)}{(8x-2)^2}$

$= \frac{-10-8}{(8x-2)^2} =$

AP Calculus Unit 2 Quiz 3 (*with Trig & Implicit*) - KEY

7.) Find **the equation of the line** that is tangent to $f(x) = x^2 - 4x - 7$ and parallel to $2x + y = 4$. *Show all work; write final answer in the space provided.*

$y = -2x + 4$, $m_{//} = -2$

tangent line requires what??

- Slope (comes from $f'(x)$)
  - ↳ to make it //
  - * $m_{//} = -2$
  - So $f'(x) = 2x - 4 = -2$
    $2x = 2$
    $x = 1$
- pt $(1, f(1))$
  $(1, -10)$

7) _____

ans: $y + 10 = -2(x - 1)$

AP Calculus Unit 2 Quiz 3 (*with Trig & Implicit*) - KEY

8.) If $y = \cos^2 x - \sin^2 x$, then $y' =$ _____ *Circle final answer.*

(a) -1 (b) 0 (c) $-2\sin(2x)$ (d) $-2(\cos x + \sin x)$ (e) $2(\cos x - \sin x)$

Hard way (PTA):

$y' = 2\cos x \cdot (-\sin x)(1) - 2\sin x \cdot (\cos x)(1)$

$= -2\cos x \sin x - 2\sin x \cos x$

$= -4\sin x \cos x$

$= -2(2\sin x \cos x)$

$\sin \sin 2x = 2\sin x \cos x$,

ans $= -2\sin 2x$

Easy way (Use an Identity):

Rewrite $y = \cos 2x$

$y' = -2\sin 2x$

DONE! Boom. Pow.

AP Calculus Unit 2 Quiz 3 (*with Trig & Implicit*) - KEY

9.) Find the value of the derivative $f(t) = \frac{t^3+2}{t}$ at the point $(-2,3)$. *Show all work; write final answer in the space provided.*

Quotient Rule!

$$f'(t) = \frac{t(3t^2) - (t^3+2)(1)}{t^2}$$

$$= \frac{3t^3 - t^3 - 2}{t^2}$$

$$f'(t) = \frac{2t^3-2}{t^2}$$

$$f'(-2) = \frac{2(-2)^3 - 2}{(-2)^2}$$

$$= \frac{2(-8)-2}{4} = \frac{-16-2}{4} = \frac{-18}{4}$$

9.) $\frac{-9}{2}$

AP Calculus Unit 2 Quiz 3 (*with Trig & Implicit*) - KEY

10.) Consider $x \sin y + x^3 = 2y - 7$. Find $\frac{dy}{dx}$ in terms of x and y. *Show all work; write final answer in the space provided.*

product!

$$x \cdot \cos y \cdot \frac{dy}{dx} + (\sin y \cdot (1) + 3x^2) = 2\frac{dy}{dx}$$

$$x\cos y \frac{dy}{dx} - 2\frac{dy}{dx} = -(\sin y + 3x^2)$$

$$\frac{dy}{dx}(x\cos y - 2) = -(\sin y + 3x^2)$$

$$\frac{dy}{dx} = -\frac{\sin y + 3x^2}{x\cos y - 2}$$

10.) _____

AP Calculus Unit 2 Quiz 3 (*with Trig & Implicit*) - KEY

11.) Consider $\frac{d}{dx}\left[x^2 + xy = 10\right]$. Find $\frac{dy}{dx}$. *Show all work; write final answer in the space provided.*

product rule!!

$$2x\frac{dx}{dx} + \left(x\frac{dy}{dx} + y(1)\right) = 0$$

$$2x + x\frac{dy}{dx} + y = 0$$

$$x\frac{dy}{dx} = -(2x+y)$$

$$\frac{dy}{dx} = -\frac{2x+y}{x} \quad \text{OR} \quad \frac{-2x-y}{x}$$

$-2-\frac{y}{x}$

AP Calculus Unit 2 Quiz 3 (*with Trig & Implicit*) - KEY

12.) Find $\frac{dr}{d\theta}$ if $\left[r = \cos(1+\sin\theta)\right]$

RTA

$$\frac{dr}{d\theta} = -\sin(1+\sin\theta)\cdot(\cos\theta)$$

$$\frac{dr}{d\theta} = -\cos\theta\sin(1+\sin\theta)$$

DUE TOMORROW!!!!

Review Exercises, p.157-158

#6,8,16,20,32,34,36,38,70,78,80,82

Unit 2 TEST is THURSDAY!!