AP Calculus – Free Response   Unit 1 – Limits & Continuity

Instructions – Follow these directions carefully. Work each of the following on notebook paper. You should use at least ~1/2 page per problem. Your solutions should be in pencil. After completing your solution, use the online file to score your solution with the AP rubric. Make comments or corrections that you find helpful. Your comments should contain information that will help you score 100% in the future. If you do score 100%, you still need to comment. Grading and comments should be in ink. Your grade on this assignment will be determines as follows – Your solutions – 60 points (10 points each – they must be readable), AP Grading – 15 points, Comments – 15 points, Instructions followed – 10 points.)

1.) Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all x>0. Find $\lim\limits_{x \to 0^+} f(x)$. (2008 AB6d)

2.) Let f be a function such that $f(2) = 5$ and $f(5) = 2$. Let $h(x) = f(x) - x$. Explain why there must be a value r for 2<r<5 such that $h(r) = 0$. (2007B- AB6d)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3.) The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$. Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(2007 AB3a)

4.) A 12,000 liter tank of water is filled to capacity. At time t=0, water begins to drain out of the tank at a rate modeled by r(t), measured in liters per hour, where r is the piecewise –defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\[2mm] 1000e^{-0.2t} & \text{for } t>5 \end{cases}$$

Is r continuous at t=5? Show the work that leads to your answer.
(2011B-AB2a)

5.) Let f be a function defined by $f(x) = \begin{cases} \sqrt{x+1} & \text{for} & 0 \le x \le 3 \\ 5-x & \text{for} & 3 < x \le 5 \end{cases}$. (2003 AB6a)

Is f continuous at $x = 3$. Explain why or why not?

6.) Let f be a function given by $f(x) = 2xe^{2x}$. Find $\lim\limits_{x\to\infty} f(x)$ & $\lim\limits_{x\to-\infty} f(x)$.(1998 AB2 a)