AP Calculus – Free Response Solutions File **Unit 1 – Limits & Continuity**

#1 2008 AB6d	
AP Scoring Rubric	
(d) $\lim_{x \to 0^+} \frac{\ln x}{x} = -\infty$ or Does Not Exist	1 : answer
Actual Solution Receiving Full Credit	
Work for problem 6(d) $\lim_{x \to 0^+} \frac{\ln x}{x} = \frac{\ln(\text{really small pos \#})}{\text{really small pos \#}}$ $= \frac{\text{really Big neg \#}}{\text{really small pos \#}}$ $=$ really, really big neg # $\therefore \lim_{x \to 0^+} f(x) = -\infty$	
Actual Solution Receiving no points	
Work for problem 6(d) $f(x) = \frac{\ln x}{x}$ L'Hopitals rule $\rightarrow \lim_{x \to 0} \frac{1}{x} = \frac{1}{0}$ $\boxed{\text{undefined}}$	

#2 2007B- AB6d

AP Scoring Rubric

(d) Let $h(x) = f(x) - x$.

$h(2) = f(2) - 2 = 5 - 2 = 3$

$h(5) = f(5) - 5 = 2 - 5 = -3$

Since $h(2) > 0 > h(5)$, the Intermediate Value Theorem guarantees that there is a value r, with $2 < r < 5$, such that $h(r) = 0$.

$2 : \begin{cases} 1 : h(2) \text{ and } h(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 6(d)

$h(x) = f(x) - x$

$(2, 5)$

$h(2) = f(2) - 2$
$= 5 - 2 = \underline{\underline{3}}$

$h(5) = f(5) - 5$
$= 2 - 5 = \underline{\underline{-3}}$

Because the values have opposite signs, according the Intermediate Value Theorem, there must exist some number r such that h(r) = 0

The function is continuous ("twice-differentiable") and because it has coordinates above and below the x-axis, then must exist some r.

Actual Solution Receiving 1 point

Work for problem 6(d) $h(x) = f(x) - x$

$h(5) = f(5) - 5 = 2 - 5 = -3$

$h(2) = f(2) - 2 = 5 - 2 = 3$

from Rolle's Therom, we have two numbers where the function changes its sign so the must be r where $h(r) = 0$

Correct work is presented in part (a). In part (b) the student writes about the function g and not g'. In part (c) the student does not refer to g''. In part (d) 1 point was earned for $h(2)$ and $h(5)$. The student appeals to Rolle's Theorem instead of the Intermediate Value Theorem, and so the second point was not earned.

Actual Solution Receiving no points

Work for problem 6(d)

$h(2) = f(2) - 2 = 3$

$h(5) = f(5) - 2 = 3$

and h is differentiable on $[2, 5]$

therefore, there must be a value r for $2 < r < 5$

such that $h(r) = 0$

point. The second point was not earned since the student concludes that $g''(x)$ does not equal 0. In part (d) the student does not have the correct value for $h(5)$, so the first point was not earned. Since 0 is not between the student's values of $h(2)$ and $h(5)$, the student was not eligible for the second point.

#3 2007- AB3a

AP Scoring Rubric

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$

$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$

Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r, $1 < r < 3$, such that $h(r) = -5$.

$2 : \begin{cases} 1 : h(1) \text{ and } h(3) \\ 1 : \text{conclusion, using IVT} \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 3(a)

Differentiability implies continuity so f and g are also continuos for all real numbers. Because f and g are continuous, h(x) is also continuous for all real number. Because h is continuous and $h(3) = -7$ and $h(1) = 3$, $-7 = h(3) \leq -5 = h(r) \leq 3 = h(1)$, so a value of r where $h(r) = -5$ is gooranteed by the Intermediate Value Theorem.

Actual Solution Receiving 1 point

The student earned 6 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student correctly identifies $h(1)$ and $h(3)$ for the first point but mistakenly identifies Rolle's Theorem and thus did not earn the second point. In part (b) the student calculates the difference quotient and applies the

Work for problem 3(a)

$h(1) = f(g(1)) - 6$
$= f(2) - 6$
$= 9 - 6$
$= 3$

$h(3) = f(g(3)) - 6$
$= f(4) - 6$
$= -1 - 6$
$= -7$

according to Rolle's Theorem, if $h(a) = c$ and $h(b) = d$ and g is on the interval $c < g < d$, then there must be a value r that exists on the interval $a < r < b$ for which $h(r) = g$

$h(1) = 3$ and $h(3) = -7$. -5 is on the interval $-7 < -5 < 3$, ∴ there must be a value r for which $h(r) = -5$.

#4 20011B- AB2a

AP Scoring Rubric

(a) $\lim_{t\to 5^-} r(t) = \lim_{t\to 5^-}\left(\frac{600t}{t+3}\right) = 375 = r(5)$ $\lim_{t\to 5^+} r(t) = \lim_{t\to 5^+}\left(1000e^{-0.2t}\right) = 367.879$ Because the left-hand and right-hand limits are not equal, r is not continuous at $t = 5$.	2 : conclusion with analysis

Actual Solutions Receiving Full Credit

Work for problem 2(a)

r will be continuous if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$, in our case

$\lim_{t\to 5^-} r(t) = \lim_{t\to 5^+} r(t)$

$\lim_{t\to 5^-} \frac{600t}{t+3} = 375$

$\lim_{t\to 5^+} 1000e^{-0,2t} = 367,8794$

$\lim_{t\to 5^-} \neq \lim_{t\to 5^+}$, so r is discontinuous.

Work for problem 2(a)

function is continuous only if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

$\lim_{x\to 5^-} \frac{600t}{t+3} = \left\{ \frac{600\cdot 5}{8} \right\} = 375$

$\lim_{x\to 5^+} 1000e^{-0.2\cdot 5} = 367.879$

$\lim_{x\to 5^-} f(t) \neq \lim_{x\to 5^+} r(t) \Rightarrow$ function is not continious a $t = 5$

#5 2003- AB6a	
AP Scoring Rubric	
(a) f is continuous at $x = 3$ because $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 2$. Therefore, $\lim_{x\to 3} f(x) = 2 = f(3)$.	2 : 1 : answers "yes" and equates the values of the left- and right-hand limits 1 : explanation involving limits
Actual Solution Receiving Full Credit	
yes, f(3) is 2, which is the same as the limits as x approaches 3 from either side.	

#6 1998 AB2a	
(a) $\lim_{x\to -\infty} 2xe^{2x} = 0$ $\lim_{x\to \infty} 2xe^{2x} = \infty$ or DNE	**2** 1: 0 as $x \to -\infty$ 1: ∞ or DNE as $x \to \infty$