AP Calculus – Study Guide 1 **Limits & Continuity**

"Working" Definition : We say $\lim_{x \to a} f(x) = L$ if we can make $f(x)$ as close to L as we want by taking x sufficiently close to a (on either side of a) without letting $x=a$.

Properties of Limits:

Limits can be added, subtracted, multiplied, multiplied by a constant, divided, and raised to a power.

Assume $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ both exist and c is any number then,

1. $\lim_{x \to a}\left[cf(x)\right] = c\lim_{x \to a} f(x)$
2. $\lim_{x \to a}\left[f(x) \pm g(x)\right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
3. $\lim_{x \to a}\left[f(x)g(x)\right] = \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right)$
4. $\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ provided that $\lim_{x \to a} g(x) \neq 0$
5. $\lim_{x \to a}\left[f(x)\right]^n = \left[\lim_{x \to a} f(x)\right]^n$
6. $\lim_{x \to a}\left[\sqrt[n]{f(x)}\right] = \sqrt[n]{\lim_{x \to a} f(x)}$

Types of Limits:

Limit at Infinity :

- Limits when $x \to \pm\infty$
- Associated w/ horizontal asymptotes
- Compare degree / dominance of numerator vs. denominator
  1.) BOT – Big on Top - there is no limit.
  2.) BOB – Big on Bottom - the limit is zero
  3.) SOS – Same over Same - the limit is equal to the ratio of the leading coefficients (Box it out!).

Infinite Limit :

- Limits when $x \to$ a number
- Associated w/ vertical asymptotes
- Direct Substitution = $\frac{\#}{0}$
- Examine right and left hand limits to see if they match (test numerically!)

Overall limit and one-sided limits

$\lim_{x \to a} f(x) = L \Leftrightarrow \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$

$\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x) \Rightarrow \lim_{x \to a} f(x)$ DNE

No match = No Limit!!!!

Continuous Functions

If $f(x)$ is continuous at a then

i.) $f(a)$ exists

ii) $\lim_{x \to a} f(x)$ exists (show one sided limits!)

iii.) $\lim_{x \to a} f(x) = f(a)$

$\therefore f$ is continuous at $x = a$

Intermediate Value Theorem:

For f continuous on $[a,b]$, and $f(a) < k < f(b)$, then c exists, $a < c < b$ where $f(c) = k$.

1.) Find f(a) and f(b)
2.) Show that $f(a) < k < f(b)$
3.) Conclude "Therefore, by IVT, $a < c < b$

Evaluation Techniques for Limits

i.) Direct Substitution
ii.) Factor and Cancel
iii.) Piecewise Function – 2 one-sided limits

Horizontal Asymptotes

If $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$,then $f(x)$ has a horizontal asymptote at $y = L$