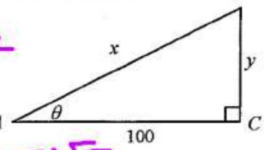
RELATED RATES PACKET:

1.)

$100^2 + 50^2 = x^2$

$\sqrt{12{,}500} = x$

$10\sqrt{125} \Rightarrow x = 50\sqrt{5}$



$\leftarrow \frac{dy}{dt} = 3\ m/s$

The figure above represents an observer at point A watching balloon B as it rises from point C. The balloon is rising at a constant rate of 3 meters per second and the observer is 100 meters from point C.

(a) Find the rate of change in x at the instant when $y = 50$.

(b) Find the rate of change in the area of right triangle BCA at the instant when $y = 50$.

(c) Find the rate of change in θ at the instant when $y = 50$.

$\frac{dx}{dt}$

(a) $\frac{d}{dt}\left[100^2 + y^2 = x^2\right]$

$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

$2(50)(3) = 2(\ )\frac{dx}{dt}$

$300 = 2(50\sqrt{5})\frac{dx}{dt}$

$\frac{3}{\sqrt{5}}\ m/sec = \frac{dx}{dt}$

(b) $\frac{d}{dt}\left[\frac{1}{2}(100)y = A\right]$

$50\frac{dy}{dt} = \frac{dA}{dt}$

$50(3) = \frac{dA}{dt} = 150\ m^2/sec$

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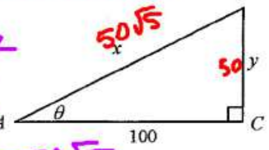

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$\frac{1}{100}y$

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$\tan\theta = \frac{50}{100}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan\theta = \frac{y}{100}$ $\frac{d}{dt}$

$\sec^2\theta\ \frac{d\theta}{dt} = \frac{1}{100}\frac{dy}{dt}$

$\cancel{\left(\frac{\sqrt{5}}{2}\right)^2}\ \frac{d\theta}{dt} = \frac{3}{100} \cdot \frac{4}{5}$

$\frac{d\theta}{dt} = \frac{3}{125}\ rad/sec$

2.)

The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second. $\frac{dr}{dt} = .04$

(Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

ⓐ $\frac{d}{dt}\left[V = \frac{4}{3}\pi r^3\right]$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$= 4\pi(10)^2(.04)$

$\boxed{\frac{dV}{dt} = 16\pi \text{ cm}^3/\text{sec}}$

$\frac{4}{3}\pi r^3 = 36\pi$

$r^3 = \cancel{36}^{9} \cdot \frac{3}{\cancel{4}}$

$r^3 = 27$

$r = 3$

ⓑ $\frac{d}{dt}\left[A_{cross} = \pi r^2\right]$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(3)(.04)$

$\boxed{\frac{dA_{cross}}{dt} = .24\pi \text{ cm}^2/\text{sec}}$

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ⓒ $\frac{dV}{dt} = \frac{dr}{dt}$

$4\pi r^2 \frac{dr}{dt} = 0.04$

$4\pi r^2(\cancel{.04}) = \cancel{.04}$

$4\pi r^2 = 1$

$r^2 = \frac{1}{4\pi}$

$r^2 = \frac{1}{4\pi}$

$r = \sqrt{\frac{1}{4\pi}}$

$\boxed{r = \frac{1}{2\sqrt{\pi}} \text{ cm}}$

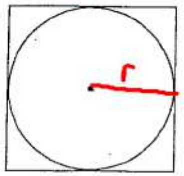
3.) A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$.)

$\frac{dC}{dt} = 6$ in/sec.

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

2r



ⓐ $\left[P = 8r\right] \frac{d}{dt}$

$\frac{d}{dt}\left[C = 2\pi r\right]$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dP}{dt} = 8\frac{dr}{dt}$

$6 = 2\pi \frac{dr}{dt}$

$\frac{3}{\pi} = \frac{dr}{dt}$

$\frac{dP}{dt} = 8\left(\frac{3}{\pi}\right)$

$\frac{dP}{dt} = \frac{24}{\pi}$ in./sec

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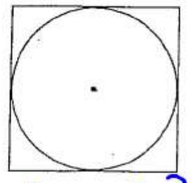

ⓑ $\frac{d}{dt}\left[A_e = A_\square - A_\circ\right]$ $(4-\pi)r^2$

$\frac{dA_E}{dt} = \frac{dA_\square}{dt} - \frac{dA_\circ}{dt}$

$= \frac{d}{dt}\left[4r^2\right] - \frac{d}{dt}\left[\pi r^2\right]$

$= 8r\frac{dr}{dt} - 2\pi r\frac{dr}{dt}$

$= 8(5)\left(\frac{3}{\pi}\right) - 2\pi(5)\left(\frac{3}{\pi}\right) = \frac{120}{\pi} - 30 = \frac{dA}{dt}$ in²/sec.

$A_\square = 4r^2 = 10$

$A_\circ = \pi r^2 = 25$

$r = 5$