1.9 Composition of Transformations

Here you'll learn how to perform a composition of transformations. You'll also learn several theorems related to composing transformations.

What if you were given the coordinates of a quadrilateral and you were asked to reflect the quadrilateral and then translate it? What would its new coordinates be? After completing this Concept, you'll be able to perform a series of transformations on a figure like this one in the coordinate plane.

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Composing Transformations CK-12

Guidance

Transformations Summary

A **transformation** *is an operation that moves, flips, or otherwise changes a figure to create a new figure. A* **rigid transformation** *(also known as an* **isometry** *or* **congruence transformation***) is a transformation that does not change the size or shape of a figure. The new figure created by a transformation is called the* **image***. The original figure is called the* **preimage***.*

There are three rigid transformations: translations, rotations and reflections. A **translation** *is a transformation that moves every point in a figure the same distance in the same direction. A* **rotation** *is a transformation where a figure is turned around a fixed point to create an image. A* **reflection** *is a transformation that turns a figure into its mirror image by flipping it over a line.*

Composition of Transformations

A **composition (of transformations)** is when more than one transformation is performed on a figure. Compositions can always be written as one rule. You can compose any transformations, but here are some of the most common compositions:
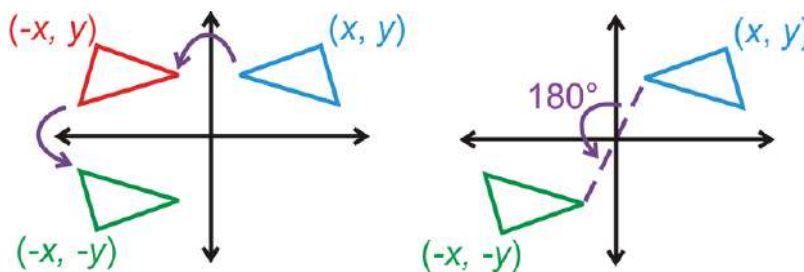
1) A **glide reflection** is a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.
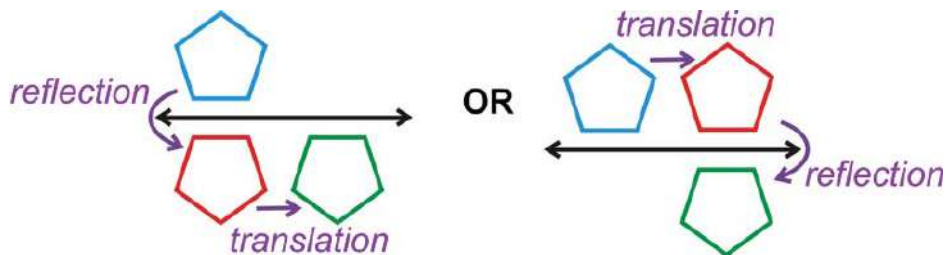
2) The composition of two reflections over parallel lines that are h units apart is the same as a translation of $2h$ units (**Reflections over Parallel Lines Theorem**).
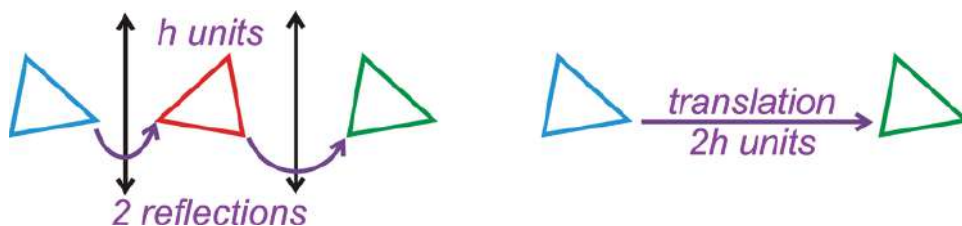
3) If you compose two reflections over each axis, then the final image is a rotation of $180°$ around the origin of the original (**Reflection over the Axes Theorem**).

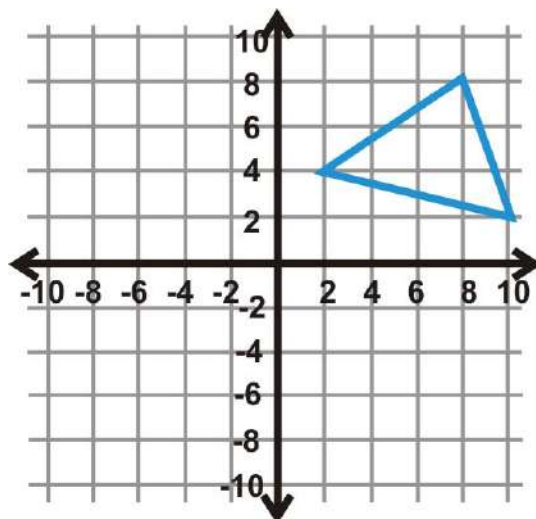4) A composition of two reflections over lines that intersect at $x°$ is the same as a rotation of $2x°$. The center of rotation is the point of intersection of the two lines of reflection (**Reflection over Intersecting Lines Theorem**).
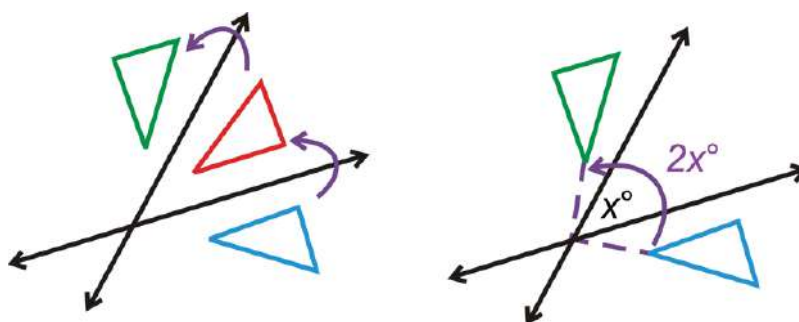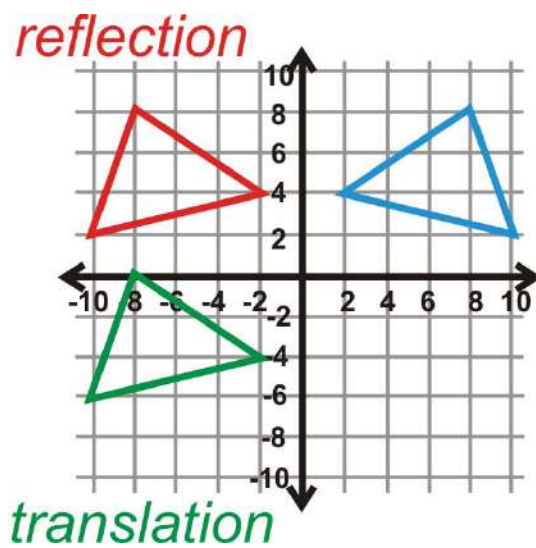

Example A

Reflect $\triangle ABC$ over the $y-$axis and then translate the image 8 units down.

The green image to the right is the final answer.

reflection

translation

$$A(8,8) \rightarrow A''(-8,0)$$
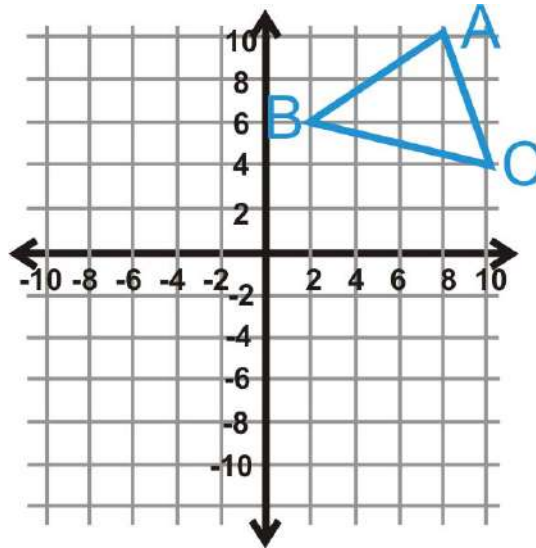$$B(2,4) \rightarrow B''(-2,-4)$$
$$C(10,2) \rightarrow C''(-10,-6)$$

Example B

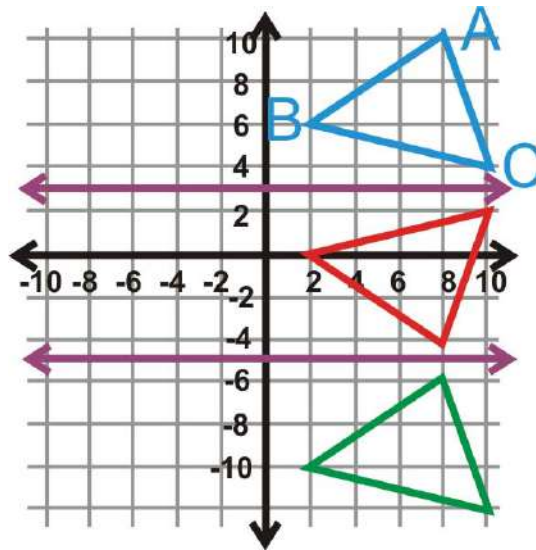Write a single rule for $\triangle ABC$ to $\triangle A''B''C''$ from Example A.

Looking at the coordinates of A to A'', the $x-$value is the opposite sign and the $y-$value is $y-8$. Therefore the rule would be $(x,y) \rightarrow (-x, y-8)$.

Example C

Reflect $\triangle ABC$ over $y = 3$ and then reflect the image over $y = -5$.

Order matters, so you would reflect over $y = 3$ first, (red triangle) then reflect it over $y = -5$ (green triangle).

Example D

A square is reflected over two lines that intersect at a $79°$ angle. What one transformation will this be the same as?

From the Reflection over Intersecting Lines Theorem, this is the same as a rotation of $2 \cdot 79° = 178°$.
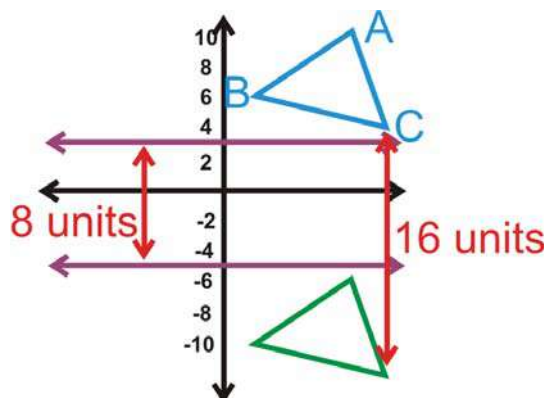
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Composing Transformations CK-12

Guided Practice

1. Write a single rule for $\triangle ABC$ to $\triangle A''B''C''$ from Example C.

2. $\triangle DEF$ has vertices $D(3,-1), E(8,-3)$, and $F(6,4)$. Reflect $\triangle DEF$ over $x = -5$ and then $x = 1$. Determine which one translation this double reflection would be the same as.

3. Reflect $\triangle DEF$ from Question 2 over the $x-$axis, followed by the $y-$axis. Find the coordinates of $\triangle D''E''F''$ and the one transformation this double reflection is the same as.

4. Copy the figure below and reflect the triangle over l, followed by m.
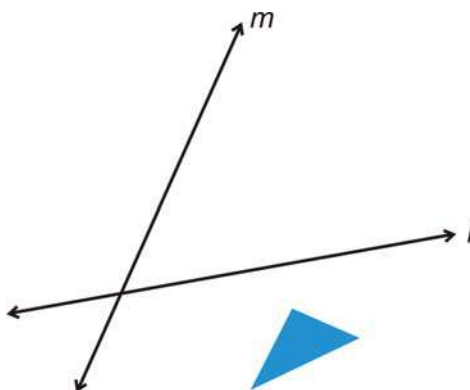
Answers:

1. In the graph, the two lines are 8 units apart $(3-(-5)=8)$. The figures are 16 units apart. The ***double*** reflection is the same as a translation that is ***double*** the distance between the parallel lines. $(x,y) \rightarrow (x, y - 16)$.

2. From the Reflections over Parallel Lines Theorem, we know that this double reflection is going to be the same as a single translation of $2(1(-5))$ or 12 units.
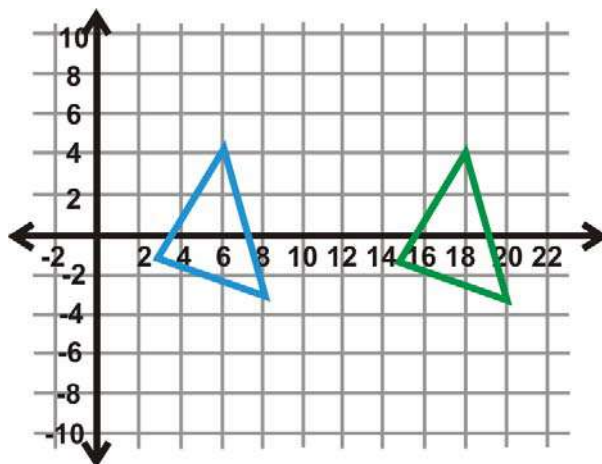
3. $\triangle D''E''F''$ is the green triangle in the graph to the left. If we compare the coordinates of it to $\triangle DEF$, we have:
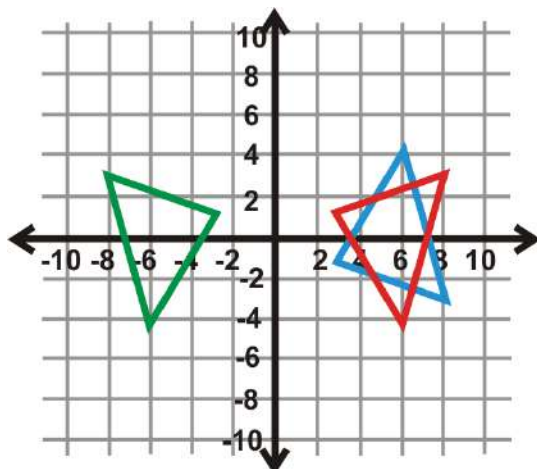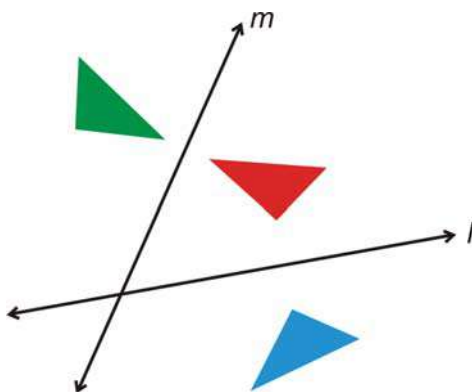

$$D(3,-1) \rightarrow D''(-3,1)$$
$$E(8,-3) \rightarrow E''(-8,3)$$
$$F(6,4) \rightarrow F''(-6,-4)$$

4. The easiest way to reflect the triangle is to fold your paper on each line of reflection and draw the image. The final result should look like this (the green triangle is the final answer):


Practice

1. *Explain* why the composition of two or more isometries must also be an isometry.
2. What one transformation is the same as a reflection over two parallel lines?
3. What one transformation is the same as a reflection over two intersecting lines?

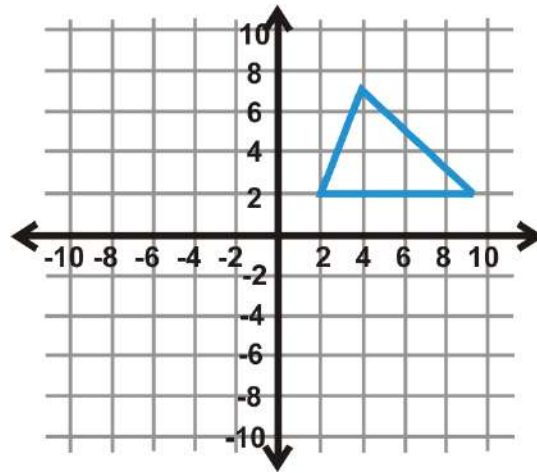Use the graph of the square to the left to answer questions 4-6.
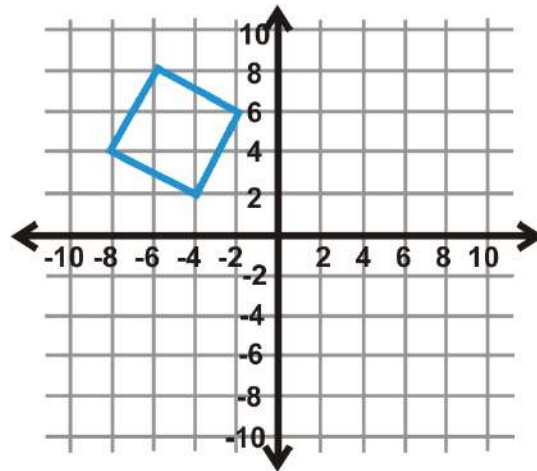
4. Perform a glide reflection over the $x-$axis and to the right 6 units. Write the new coordinates.
5. What is the rule for this glide reflection?
6. What glide reflection would move the image back to the preimage?

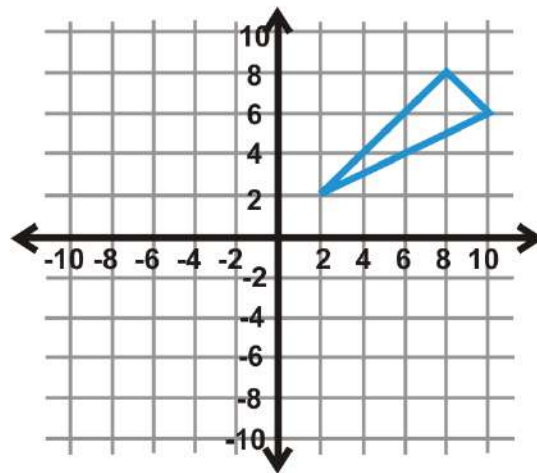Use the graph of the square to the left to answer questions 7-9.
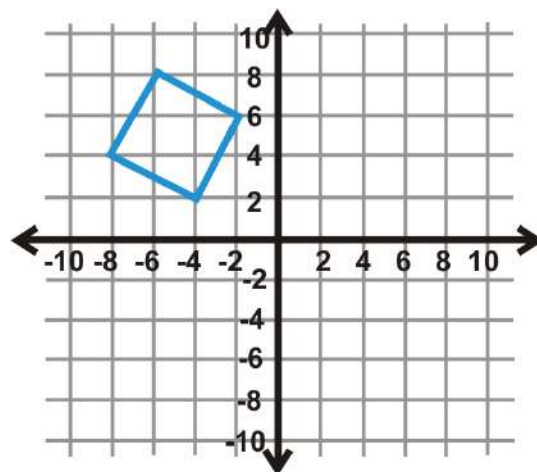
7. Perform a glide reflection to the right 6 units, then over the $x-$axis. Write the new coordinates.
8. What is the rule for this glide reflection?
9. Is the rule in #8 different than the rule in #5? Why or why not?

Use the graph of the triangle to the left to answer questions 10-12.

10. Perform a glide reflection over the $y-$axis and down 5 units. Write the new coordinates.
11. What is the rule for this glide reflection?
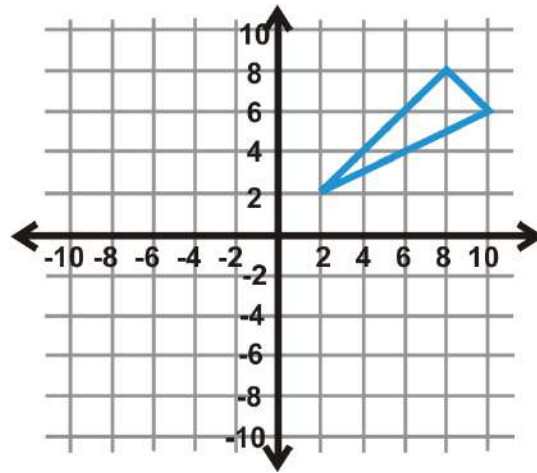12. What glide reflection would move the image back to the preimage?

Use the graph of the triangle to the left to answer questions 13-15.

13. Reflect the preimage over $y = -1$ followed by $y = -7$. Draw the new triangle.
14. What one transformation is this double reflection the same as?
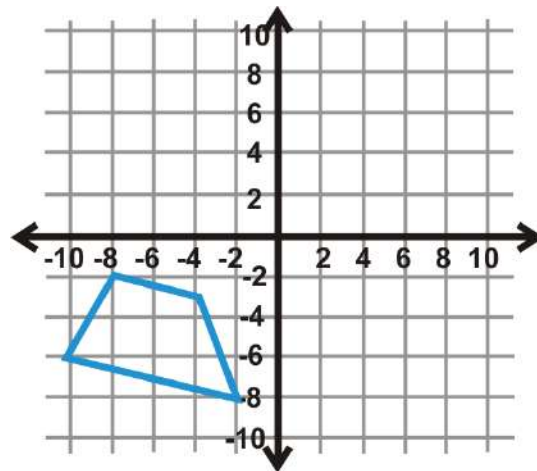15. Write the rule.

Use the graph of the triangle to the left to answer questions 16-18.

16. Reflect the preimage over $y = -7$ *followed* by $y = -1$. Draw the new triangle.
17. What one transformation is this double reflection the same as?
18. Write the rule.
19. How do the final triangles in #13 and #16 differ?

Use the trapezoid in the graph to the left to answer questions 20-22.

20. Reflect the preimage over the $x-$axis then the $y-$axis. Draw the new trapezoid.
21. Now, start over. Reflect the trapezoid over the $y-$axis then the $x-$axis. Draw this trapezoid.
22. Are the final trapezoids from #20 and #21 different? Why do you think that is?

Answer the questions below. Be as specific as you can.

23. Two parallel lines are 7 units apart. If you reflect a figure over both how far apart with the preimage and final image be?
24. After a double reflection over parallel lines, a preimage and its image are 28 units apart. How far apart are the parallel lines?
25. Two lines intersect at a 165° angle. If a figure is reflected over both lines, how far apart will the preimage and image be?
26. What is the center of rotation for #25?
27. Two lines intersect at an 83° angle. If a figure is reflected over both lines, how far apart will the preimage and image be?

28. A preimage and its image are 244° apart. If the preimage was reflected over two intersecting lines, at what angle did they intersect?
29. A preimage and its image are 98° apart. If the preimage was reflected over two intersecting lines, at what angle did they intersect?
30. After a double reflection over parallel lines, a preimage and its image are 62 units apart. How far apart are the parallel lines?