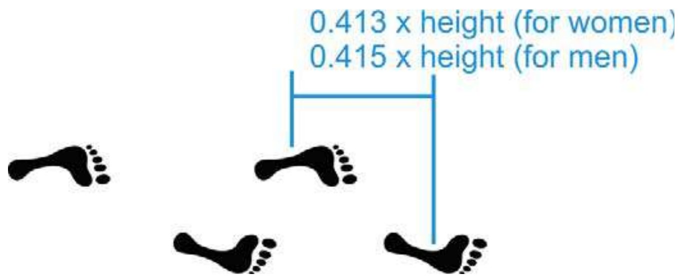
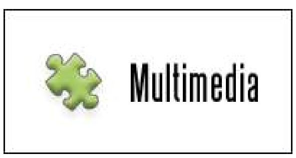
1.8 Composition of Transformations

Here you'll learn how to perform a composition of transformations. You'll also learn some common composition of transformations.

What if you were told that your own footprint is an example of a glide reflection? The equations to find your average footprint are in the diagram below. Determine your average footprint and write the rule for one stride. You may assume your stride starts at (0, 0). After completing this Concept, you'll be able to answer this question.



Watch This



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CK-12 Foundation: Chapter12CompositionofTranformationsA

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Brightstorm:Compositions ofTransformations

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Brightstorm:Glide Reflections

Guidance

Transformations Summary

A **transformation** *is an operation that moves, flips, or otherwise changes a figure to create a new figure. A* **rigid transformation** *(also known as an* **isometry** *or* **congruence transformation***) is a transformation that does not change the size or shape of a figure. The new figure created by a transformation is called the* **image***. The original figure is called the* **preimage***.*

There are three rigid transformations: translations, reflections, and rotations. A **translation** *is a transformation that moves every point in a figure the same distance in the same direction. A* **rotation** *is a transformation where a figure is turned around a fixed point to create an image. A* **reflection** *is a transformation that turns a figure into its mirror image by flipping it over a line.*

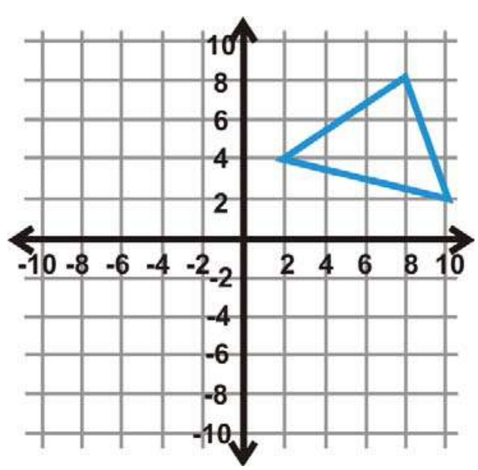
Composition of Transformations

A **composition of transformations** is to perform more than one rigid transformation on a figure. One of the interesting things about compositions is that they can always be written as one rule. What this means is you don't necessarily have to perform one transformation followed by the next. You can write a rule and perform them at the same time. You can compose any transformations, but here are some of the most common compositions.

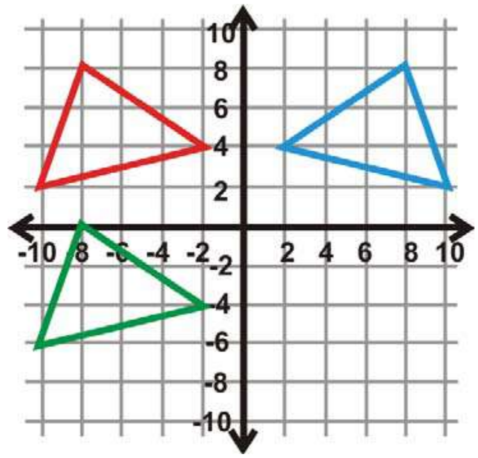
1. **Glide Reflection:** a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.
2. **Reflections over Parallel Lines Theorem:** If you compose two reflections over parallel lines that are h units apart, it is the same as a single translation of $2h$ units. *Be careful with this theorem. Notice, it does not say which direction the translation is in. So, to apply this theorem, you would still need to visualize, or even do, the reflections to see in which direction the translation would be.*
3. **Reflection over the Axes Theorem:** If you compose two reflections over each axis, then the final image is a rotation of 180° of the original. *With this particular composition, order does not matter. Let's look at the angle of intersection for these lines. We know that the axes are perpendicular, which means they intersect at a* 90° *angle. The final answer was a rotation of* 180°*, which is double* 90°*. Therefore, we could say that the composition of the reflections over each axis is a rotation of double their angle of intersection.*
4. **Reflection over Intersecting Lines Theorem:** If you compose two reflections over lines that intersect at x°, then the resulting image is a rotation of $2x^\circ$, where the center of rotation is the point of intersection.

Example A

Reflect $\triangle ABC$ over the $y-$axis and then translate the image 8 units down.



The green image below is the final answer.



$$A(8,8) \rightarrow A''(-8,0)$$
$$B(2,4) \rightarrow B''(-2,-4)$$
$$C(10,2) \rightarrow C''(-10,-6)$$

Example B

Write a single rule for $\triangle ABC$ to $\triangle A''B''C''$ from Example A.

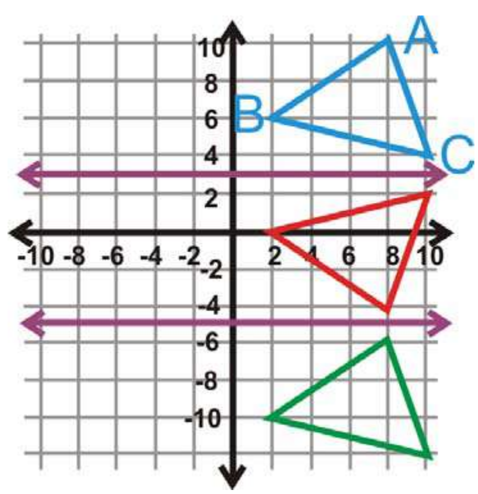
Looking at the coordinates of A to A'', the $x-$value is the opposite sign and the $y-$value is $y-8$. Therefore the rule would be $(x,y) \rightarrow (-x, y-8)$.

Notice that this follows the rules we have learned in previous sections about a reflection over the $y-$axis and translations.

Example C

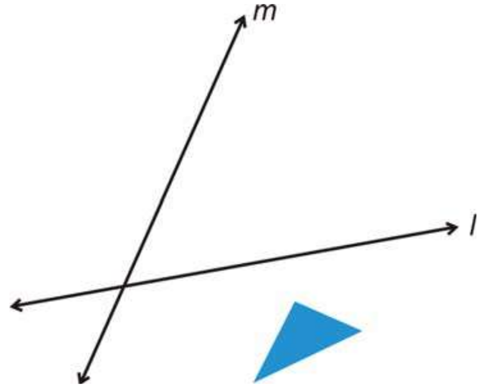
Reflect $\triangle ABC$ over $y = 3$ and $y = -5$.

Unlike a glide reflection, order matters. Therefore, you would reflect over $y = 3$ first, followed by a reflection of this image (red triangle) over $y = -5$. Your answer would be the green triangle in the graph below.

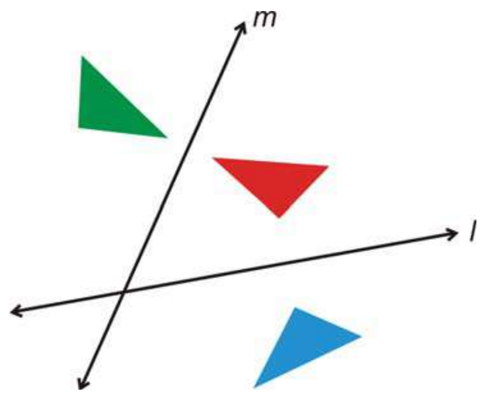


Example D

Copy the figure below and reflect it over l, followed by m.



The easiest way to reflect the triangle is to fold your paper on each line of reflection and draw the image. It should look like this:



The green triangle would be the final answer.

Watch this video for help with the Examples above.

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CK-12 Foundation: Chapter12CompositionofTransformationsB

Concept Problem Revisited

The average 6 foot tall man has a $0.415 \times 6 = 2.5$ foot stride. Therefore, the transformation rule for this person would be $(x, y) \rightarrow (-x, y + 2.5)$.

Vocabulary

A ***transformation*** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A ***rigid transformation*** (also known as an ***isometry*** or ***congruence transformation***) is a transformation that does not change the size or shape of a figure. The new figure created by a transformation is called the ***image***. The original figure is called the ***preimage***.

There are three rigid transformations: translations, reflections, and rotations. A ***translation*** is a transformation that moves every point in a figure the same distance in the same direction. A ***rotation*** is a transformation where a figure is turned around a fixed point to create an image. A ***reflection*** is a transformation that turns a figure into its mirror image by flipping it over a line.

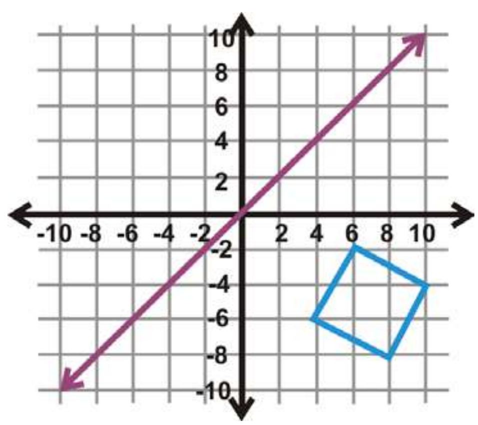
A ***composition (of transformations)*** is when more than one transformation is performed on a figure. A ***glide reflection*** is a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.

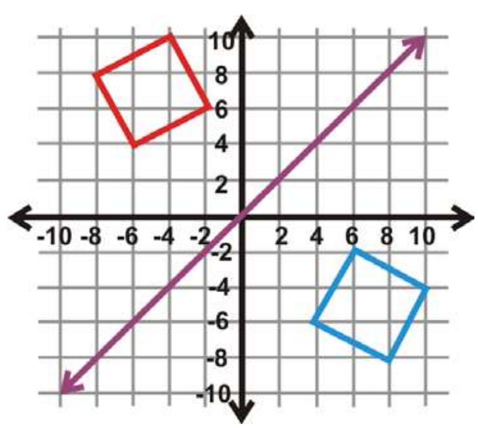
Guided Practice

1. $\triangle DEF$ has vertices $D(3,-1), E(8,-3)$, and $F(6,4)$. Reflect $\triangle DEF$ over $x=-5$ and $x=1$. This double reflection would be the same as which one translation?

2. Reflect $\triangle DEF$ from #1 over the $x-$axis, followed by the $y-$axis. Determine the coordinates of $\triangle D''E''F''$ and what one transformation this double reflection would be the same as.

3. Reflect the square over $y=x$, followed by a reflection over the $x-$axis.





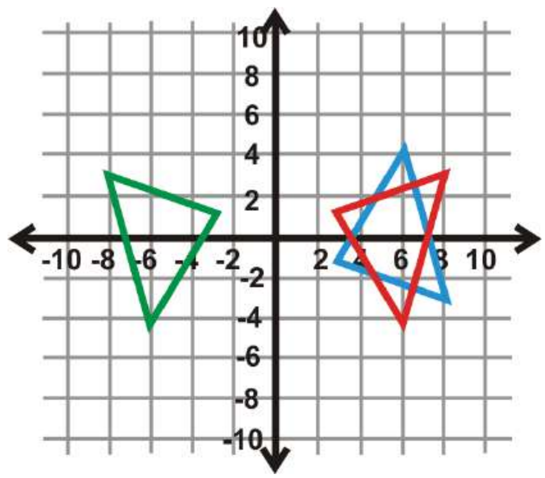
4. Determine the one rotation that is the same as the double reflection from #3.

Answers:

1. From the Reflections over Parallel Lines Theorem, we know that this double reflection is going to be the same as a single translation of $2(1-(-5))$ or 12 units. Now, we need to determine if it is to the right or to the left. Because we first reflect over a line that is further away from $\triangle DEF$, to the ***left***, $\triangle D''E''F''$ will be on the ***right*** of $\triangle DEF$. So,

it would be the same as a translation of 12 units to the right. If the lines of reflection were switched and we reflected the triangle over $x = 1$ followed by $x = -5$, then it would have been the same as a translation of 12 units to the ***left.***'

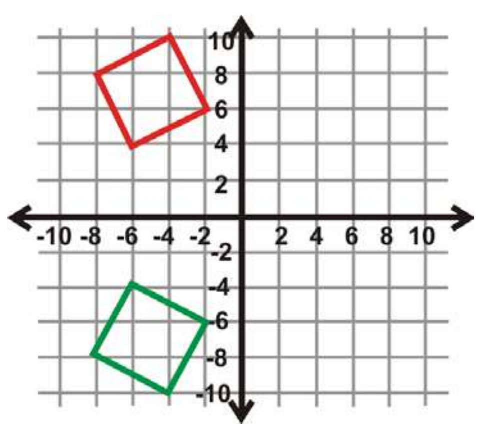
2. $\triangle D''E''F''$ is the green triangle in the graph below. If we compare the coordinates of it to $\triangle DEF$, we have:



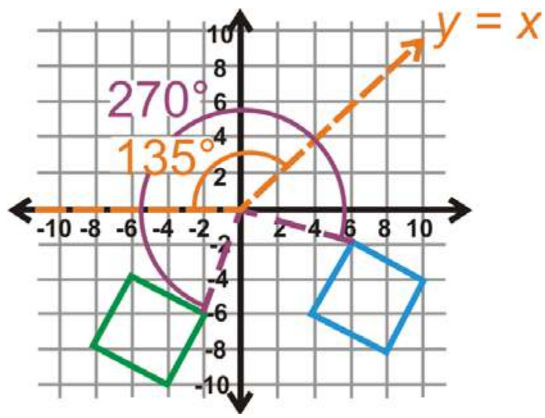
$$\begin{aligned} D(3,-1) &\rightarrow D'(-3,1) \\ E(8,-3) &\rightarrow E'(-8,3) \\ F(6,4) &\rightarrow F'(-6,-4) \end{aligned}$$

If you recall the rules of rotations from the previous section, this is the same as a rotation of 180°.

3. First, reflect the square over $y = x$. The answer is the red square in the graph above. Second, reflect the red square over the $x-$axis. The answer is the green square below.



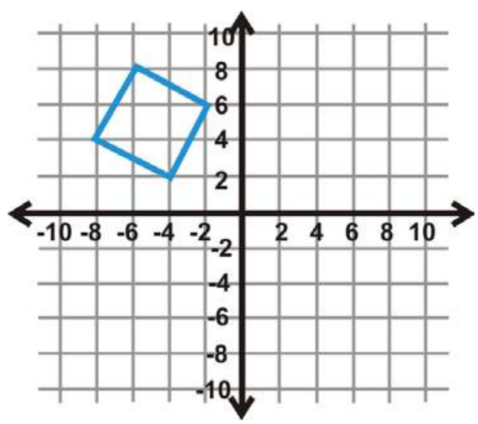
4. Let's use the theorem above. First, we need to figure out what the angle of intersection is for $y = x$ and the $x-$axis. $y = x$ is halfway between the two axes, which are perpendicular, so is 45° from the $x-$axis. Therefore, the angle of rotation is 90° clockwise or 270° counterclockwise. The correct answer is 270° counterclockwise because we always measure angle of rotation in the coordinate plane in a counterclockwise direction. From the diagram, we could have also said the two lines are 135° apart, which is supplementary to 45°.



Practice

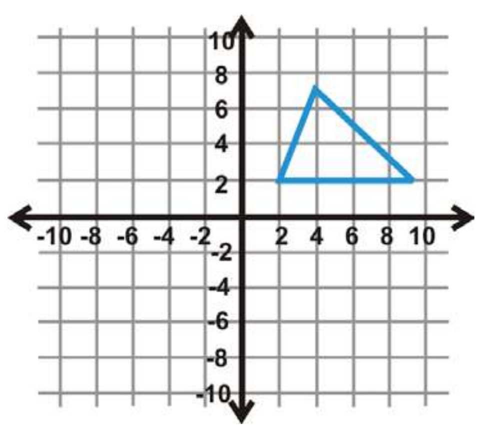
1. What one transformation is equivalent to a reflection over two parallel lines?
2. What one transformation is equivalent to a reflection over two intersecting lines?

Use the graph of the square below to answer questions 3-6.



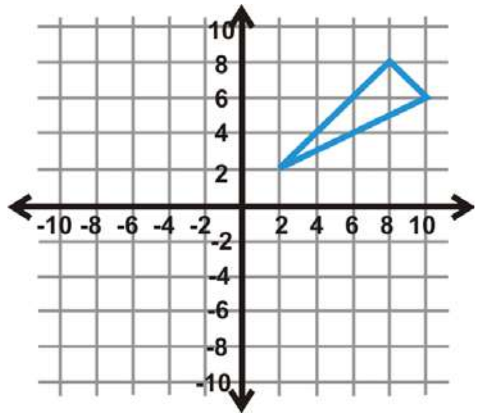
3. Perform a glide reflection over the $x-$axis and to the right 6 units. Write the new coordinates.
4. What is the rule for this glide reflection?
5. What glide reflection would move the image back to the preimage?
6. Start over. Would the coordinates of a glide reflection where you move the square 6 units to the right and then reflect over the $x-$axis be any different than #3? Why or why not?

Use the graph of the triangle below to answer questions 7-9.



7. Perform a glide reflection over the $y-$axis and down 5 units. Write the new coordinates.
8. What is the rule for this glide reflection?
9. What glide reflection would move the image back to the preimage?

Use the graph of the triangle below to answer questions 10-14.



10. Reflect the preimage over $y = -1$ followed by $y = -7$. Write the new coordinates.
11. What one transformation is this double reflection the same as?
12. What one translation would move the image back to the preimage?
13. Start over. Reflect the preimage over $y = -7$, then $y = -1$. How is this different from #10?
14. Write the rules for #10 and #13. How do they differ?

Fill in the blanks or answer the questions below.

15. Two parallel lines are 7 units apart. If you reflect a figure over both how far apart with the preimage and final image be?
16. After a double reflection over parallel lines, a preimage and its image are 28 units apart. How far apart are the parallel lines?
17. A double reflection over the x and y axes is the same as a _________ of ________.
18. What is the center of rotation for #17?
19. Two lines intersect at an 83° angle. If a figure is reflected over both lines, how far apart will the preimage and image be?
20. A preimage and its image are 244° apart. If the preimage was reflected over two intersected lines, at what angle did they intersect?
21. After a double reflection over parallel lines, a preimage and its image are 62 units apart. How far apart are the parallel lines?
22. A figure is to the left of $x = a$. If it is reflected over $x = a$ followed by $x = b$ and $b > a$, then the preimage and image are _________ units apart and the image is to the _________ of the preimage.