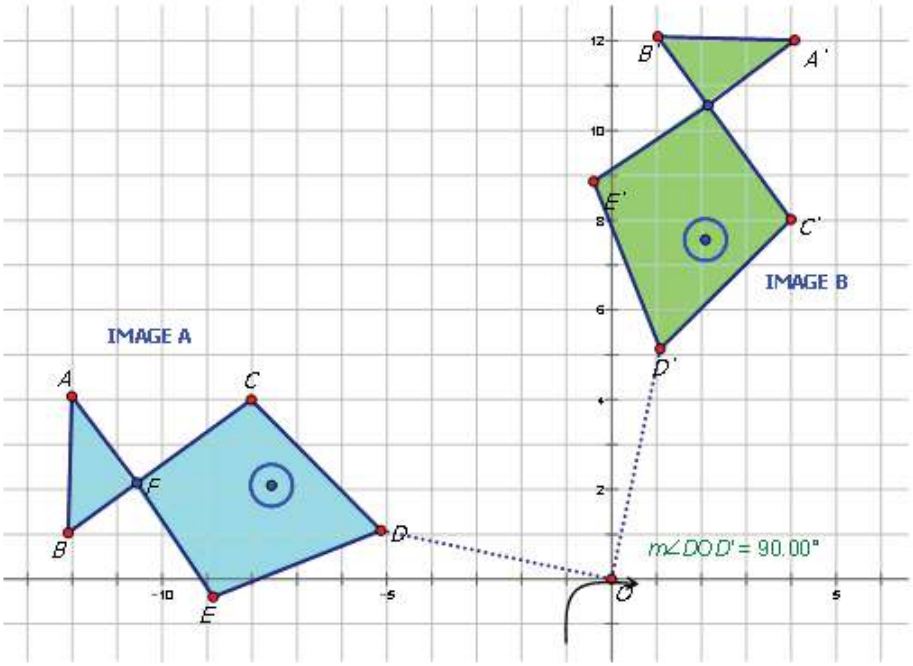
# 1.7 Rules for Rotations

Here you will learn the notation used for rotations.

The figure below shows a pattern of two fish. Write the mapping rule for the rotation of Image A to Image B.




## Watch This

First watch this video to learn about writing rules for rotations.

Multimedia

MEDIA

Click image to the left for more content.

CK-12 FoundationChapter10RulesforRotationsA

Then watch this video to see some examples.

Multimedia

MEDIA

Click image to the left for more content.

CK-12 FoundationChapter10RulesforRotationsB

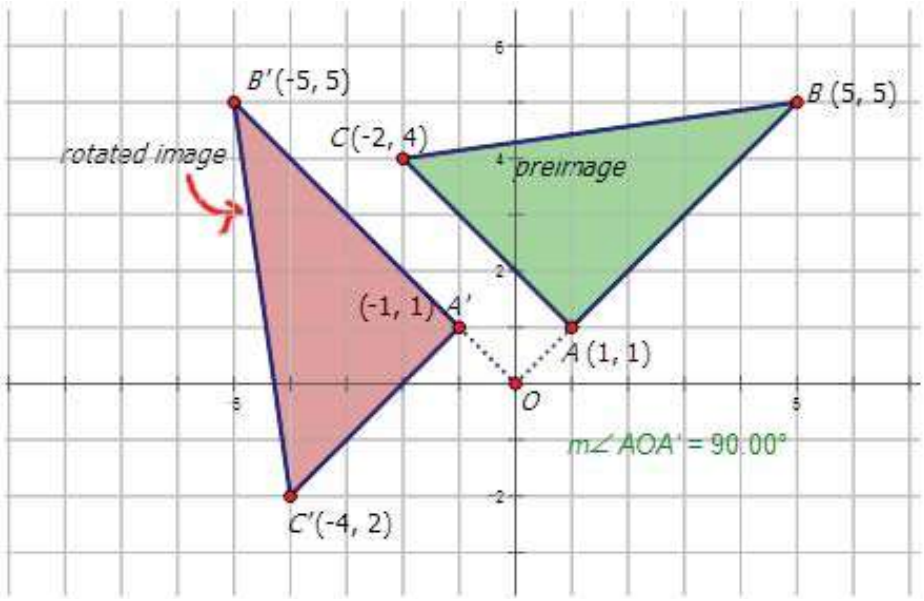
## Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A rotation is an example of a transformation where a figure is rotated about a specific point (called the center of rotation), a certain number of degrees. Common rotations about the origin are shown below:

**TABLE 1.4:**

| Center of Rotation | Angle of Rotation | Preimage (Point $P$) | Rotated Image (Point $P'$) | Notation (Point $P'$) |
|---|---|---|---|---|
| (0, 0) | 90°(or −270°) | $(x, y)$ | $(-y, x)$ | $(x, y) \to (-y, x)$ |
| (0, 0) | 180°(or −180°) | $(x, y)$ | $(-x, -y)$ | $(x, y) \to (-x, -y)$ |
| (0, 0) | 270°(or −90°) | $(x, y)$ | $(y, -x)$ | $(x, y) \to (y, -x)$ |

You can describe rotations in words, or with notation. Consider the image below:




Notice that the preimage is rotated about the origin 90°CCW. If you were to describe the rotated image using notation, you would write the following:

$$R_{90^\circ}(x, y) = (-y, x)$$

### Example A

Find an image of the point (3, 2) that has undergone a clockwise rotation:

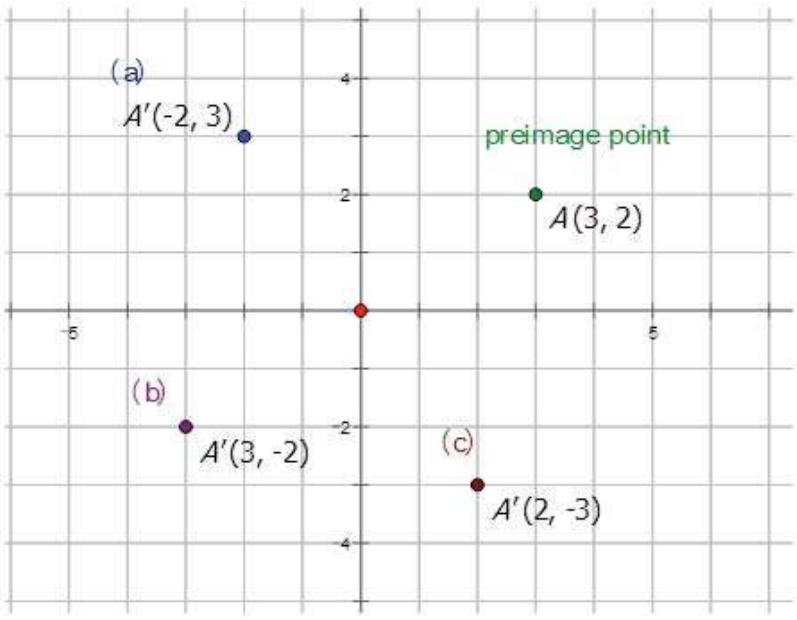
a) about the origin at 90°,

b) about the origin at 180°, and

c) about the origin at 270°.

Write the notation to describe the rotation.

**Solution:**




a) Rotation about the origin at 90° : $R_{90°}(x,y) = (-y,x)$

b) Rotation about the origin at 180° : $R_{180°}(x,y) = (-x,-y)$

c) Rotation about the origin at 270° : $R_{270°}(x,y) = (y,-x)$

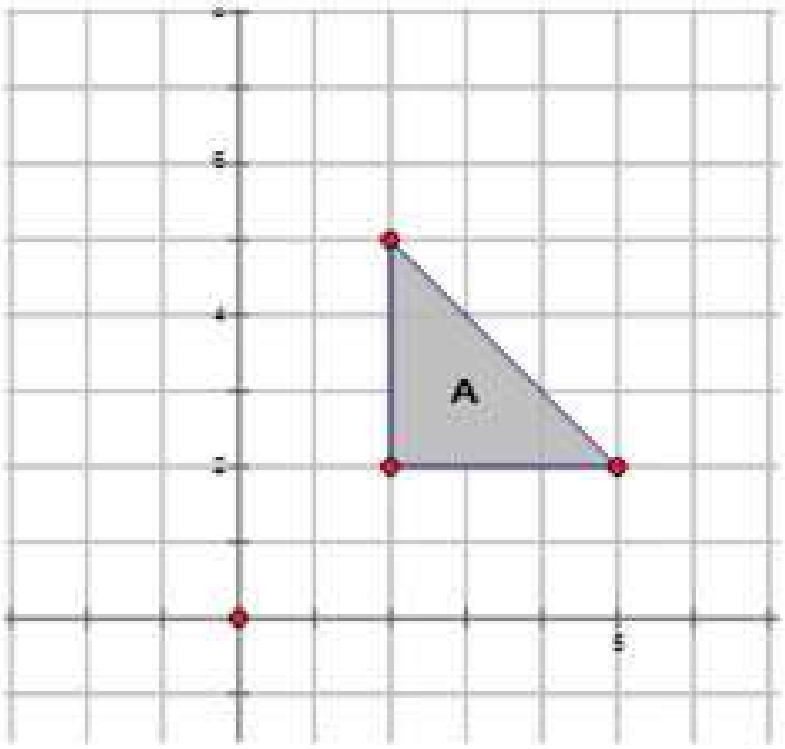
**Example B**

Rotate Image A in the diagram below:

a) about the origin at 90°, and label it $B$.

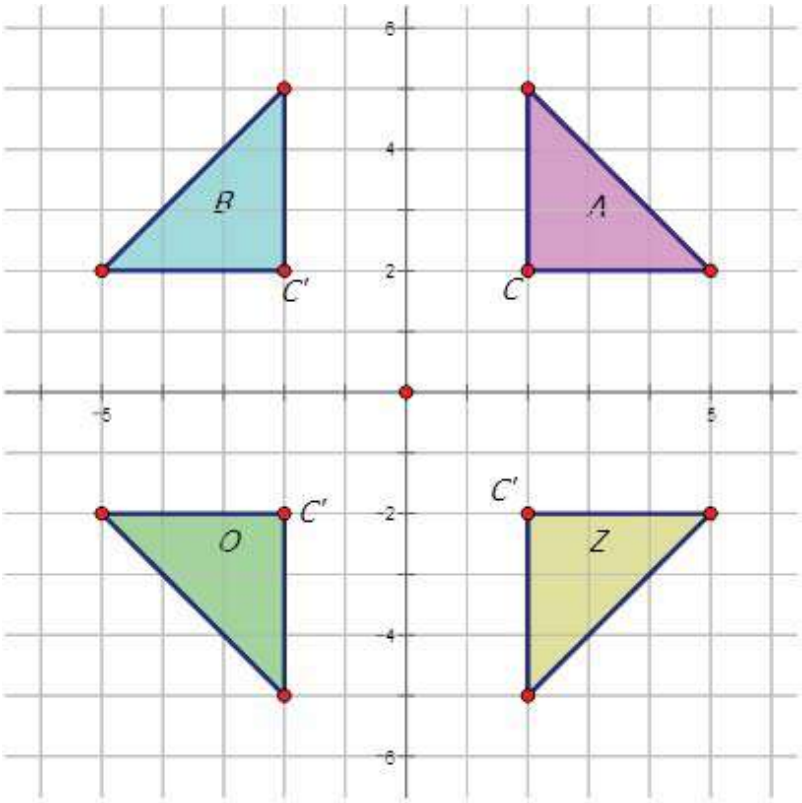
b) about the origin at 180°, and label it $O$.

c) about the origin at 270°, and label it $Z$.




Write notation for each to indicate the type of rotation.

**Solution:**



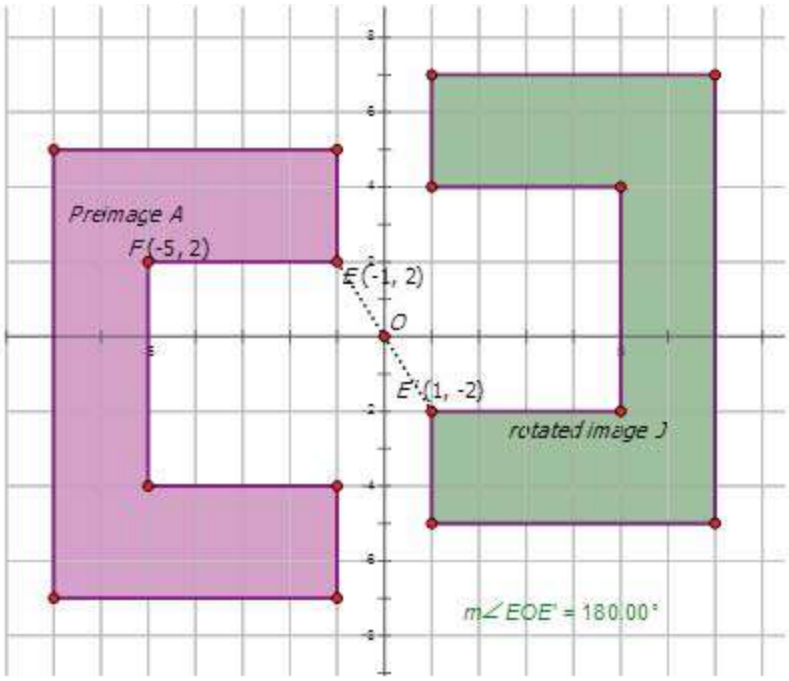

a) Rotation about the origin at 90°: $R_{90^\circ} A \rightarrow B = R_{90^\circ}(x,y) \rightarrow (-y,x)$

b) Rotation about the origin at 180°: $R_{180^\circ} A \rightarrow O = R_{180^\circ}(x,y) \rightarrow (-x,-y)$

c) Rotation about the origin at 270°: $R_{270°}A \to Z = R_{270°}(x,y) \to (y,-x)$

### Example C

Write the notation that represents the rotation of the preimage A to the rotated image J in the diagram below.



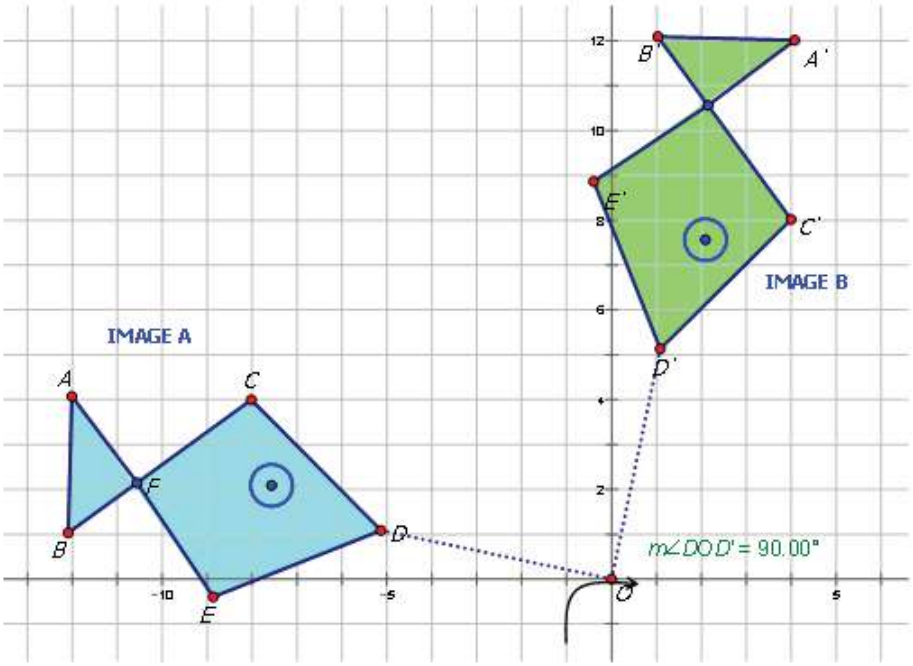

First, pick a point in the diagram to use to see how it is rotated.

$$E:(-1,2) \quad E':(1,-2)$$

Notice how both the $x$- and $y$-coordinates are multiplied by -1. This indicates that the preimage A is reflected about the origin by 180°CCW to form the rotated image J. Therefore the notation is $R_{180°}A \to J = R_{180°}(x,y) \to (-x,-y)$.

### Concept Problem Revisited

The figure below shows a pattern of two fish. Write the mapping rule for the rotation of Image A to Image B.




Notice that the angle measure is 90° and the direction is clockwise. Therefore the Image A has been rotated −90° to form Image B. To write a rule for this rotation you would write: $R_{270°}(x,y) = (-y,x)$.

## Vocabulary

**Notation Rule**

A ***notation rule*** has the following form $R_{180°}A \to O = R_{180°}(x,y) \to (-x,-y)$ and tells you that the image A has been rotated about the origin and both the $x$- and $y$-coordinates are multiplied by -1.

**Center of rotation**

A ***center of rotation*** is the fixed point that a figure rotates about when undergoing a rotation.

**Rotation**

A ***rotation*** is a transformation that rotates (turns) an image a certain amount about a certain point.

**Image**

In a transformation, the final figure is called the ***image***.

**Preimage**

In a transformation, the original figure is called the ***preimage.***

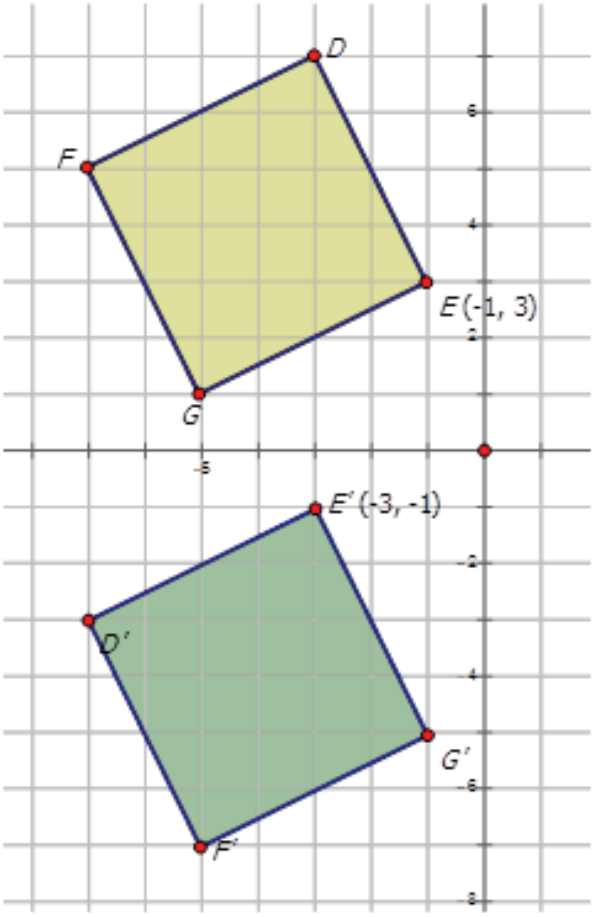
**Transformation**

A ***transformation*** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

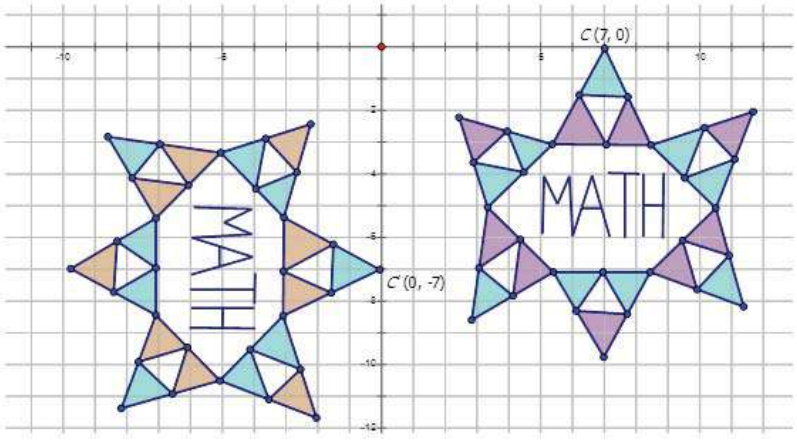
## Guided Practice

1. Thomas describes a rotation as point $J$ moving from $J(-2,6)$ to $J'(6,2)$. Write the notation to describe this rotation for Thomas.

2. Write the notation that represents the rotation of the yellow diamond to the rotated green diamond in the diagram below.

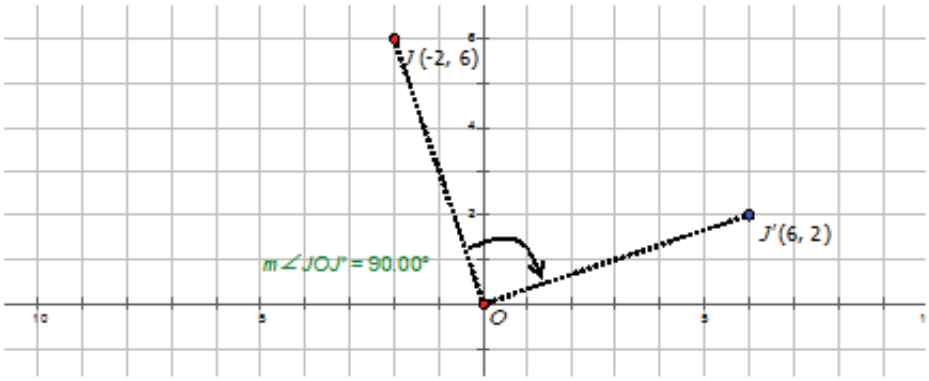



3. Karen was playing around with a drawing program on her computer. She created the following diagrams and then wanted to determine the transformations. Write the notation rule that represents the transformation of the purple and blue diagram to the orange and blue diagram.




**Answers:**

1. $J:(-2,6)\quad J':(6,2)$

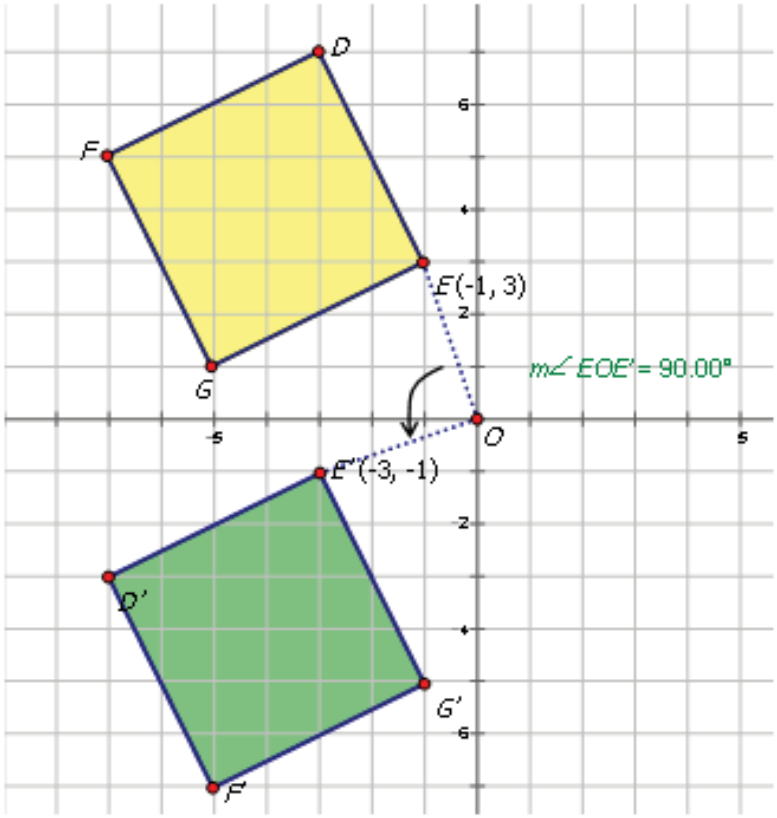



Since the $x$-coordinate is multiplied by -1, the $y$-coordinate remains the same, and finally the $x$- and $y$-coordinates change places, this is a rotation about the origin by 270° or $-90°$. The notation is: $R_{270°}J \to J' = R_{270°}(x,y) \to (y,-x)$

2. In order to write the notation to describe the rotation, choose one point on the preimage (the yellow diamond) and then the rotated point on the green diamond to see how the point has moved. Notice that point $E$ is shown in the diagram:

$$E(-1,3) \to E'(-3,-1)$$

Since both $x$- and $y$-coordinates are reversed places and the $y$-coordinate has been multiplied by -1, the rotation is about the origin 90°. The notation for this rotation would be: $R_{90°}(x,y) \to (-y,x)$.




3. In order to write the notation to describe the transformation, choose one point on the preimage (purple and blue

diagram) and then the transformed point on the orange and blue diagram to see how the point has moved. Notice that point $C$ is shown in the diagram:

$$C(7,0) \rightarrow C'(0,-7)$$

Since the $x$-coordinates only are multiplied by -1, and then $x$- and $y$-coordinates change places, the transformation is a rotation is about the origin by 270°. The notation for this rotation would be: $R_{270°}(x,y) \rightarrow (y,-x)$.

## Practice

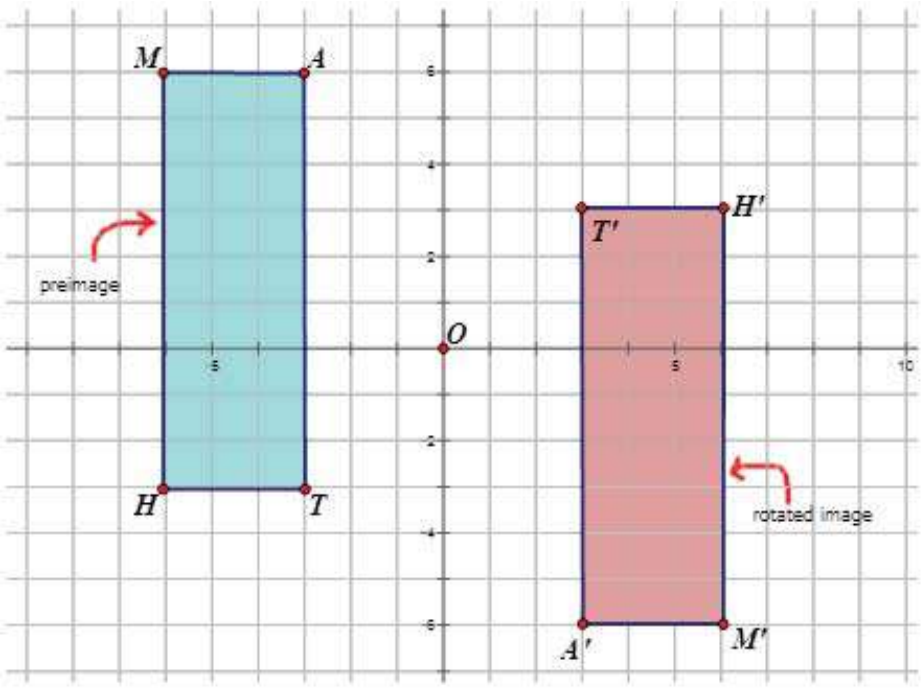
Complete the following table:

**TABLE 1.5:**

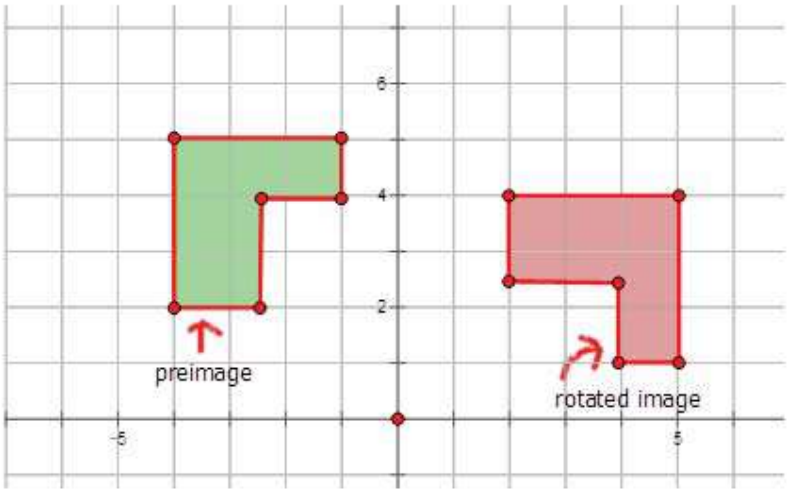
| **Starting Point** | 90° **Rotation** | 180° **Rotation** | 270° **Rotation** | 360° **Rotation** |
|---|---|---|---|---|
| 1. (1, 4) | | | | |
| 2. (4, 2) | | | | |
| 3. (2, 0) | | | | |
| 4. (-1, 2) | | | | |
| 5. (-2, -3) | | | | |

Write the notation that represents the rotation of the preimage to the image for each diagram below.

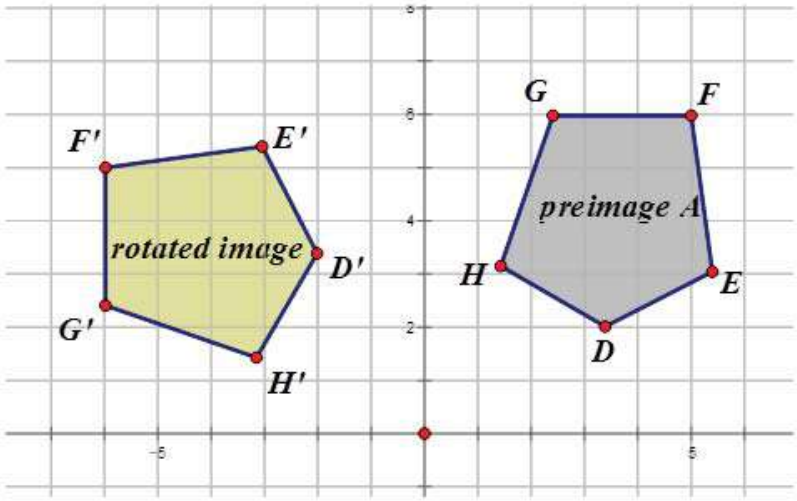
6.



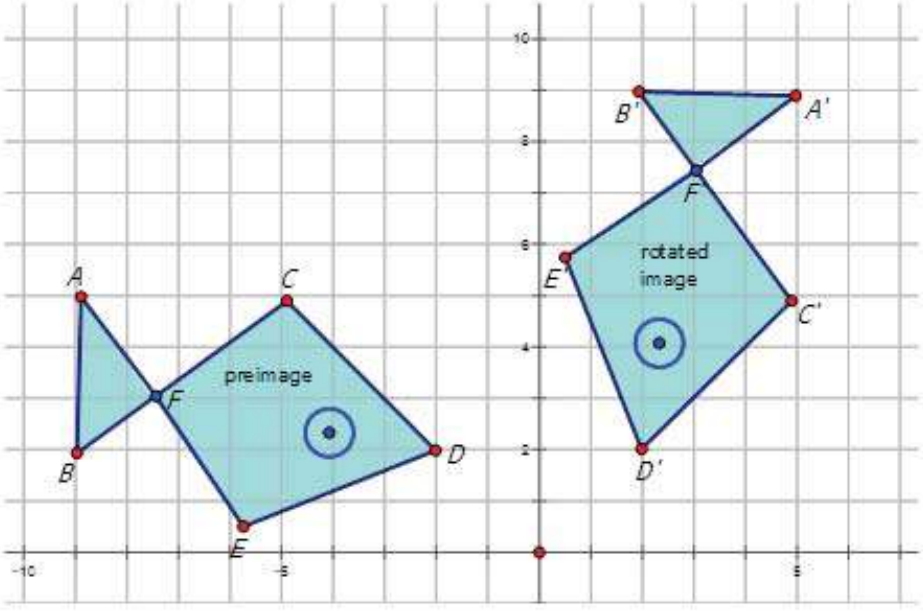
7.


preimage
rotated image

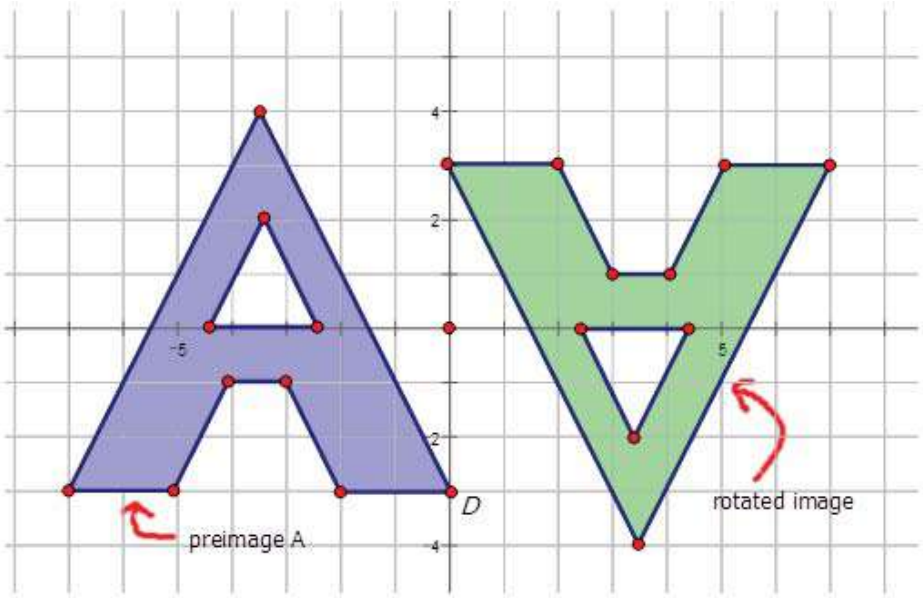
8.


F'
E'
rotated image
D'
G'
H'
G
F
preimage A
H
E
D

9.

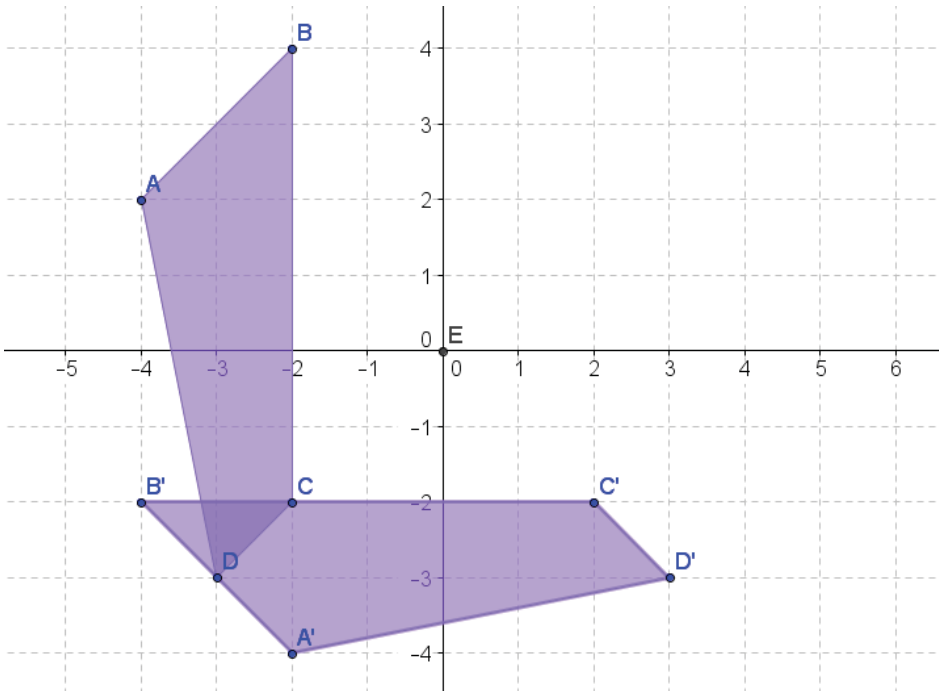

A
C
preimage
F
B
D
E
B'
A'
F
rotated image
E'
C'
D'

10.

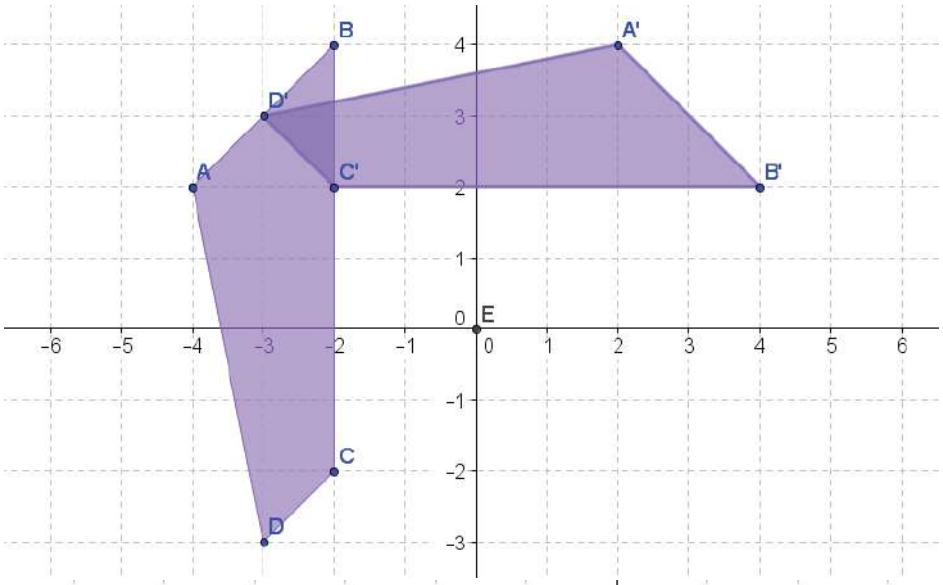



Write the notation that represents the rotation of the preimage to the image for each diagram below.

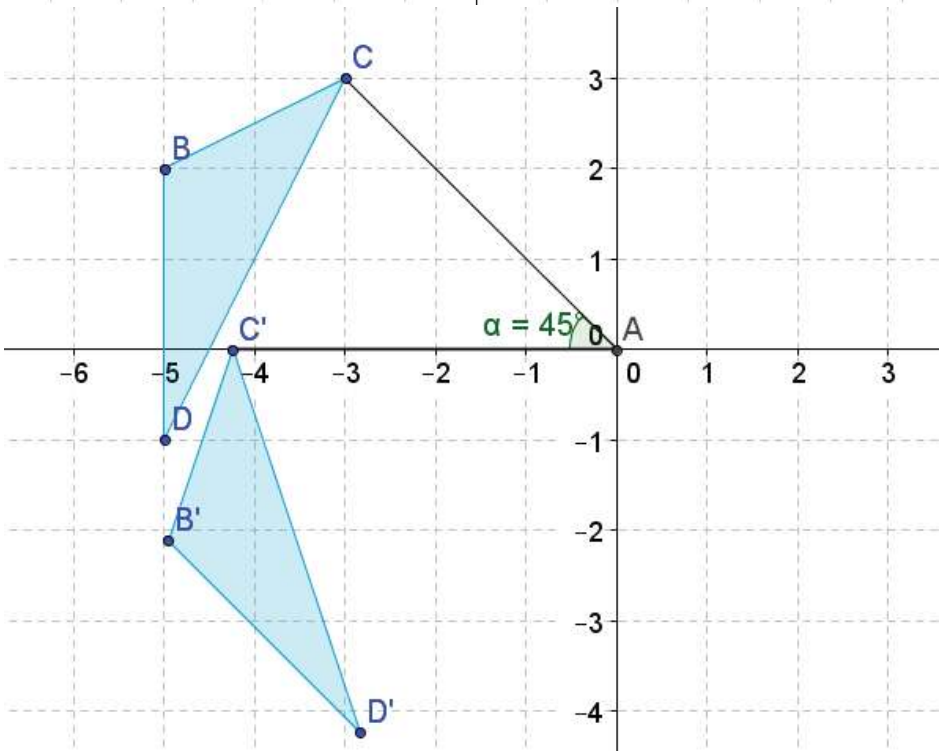
11.



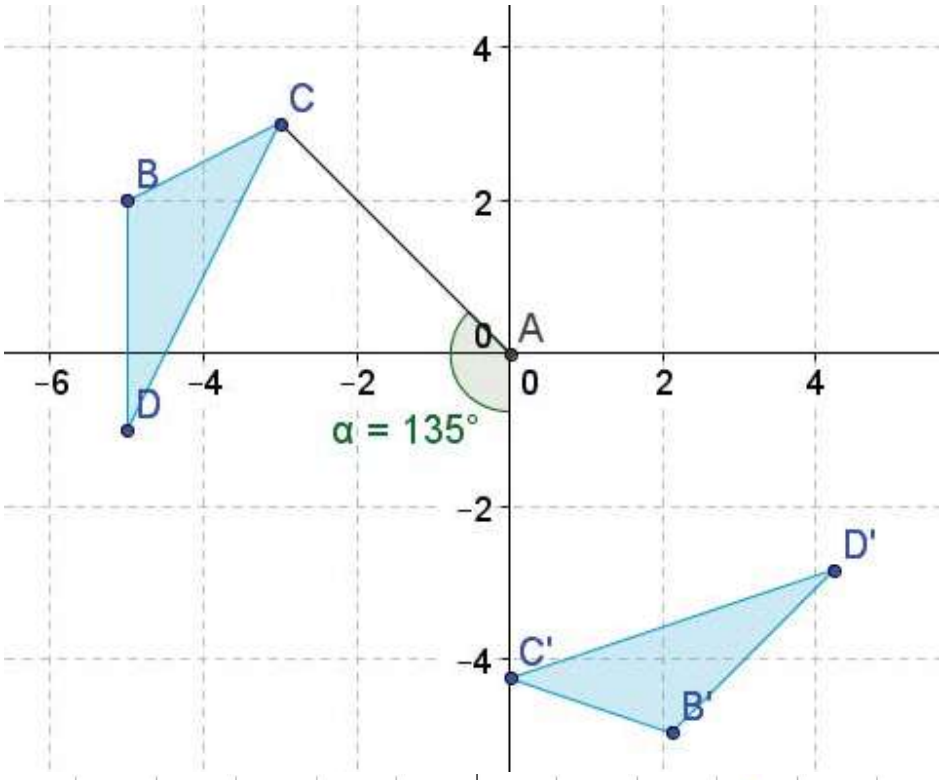
12.




13.



14.




15.