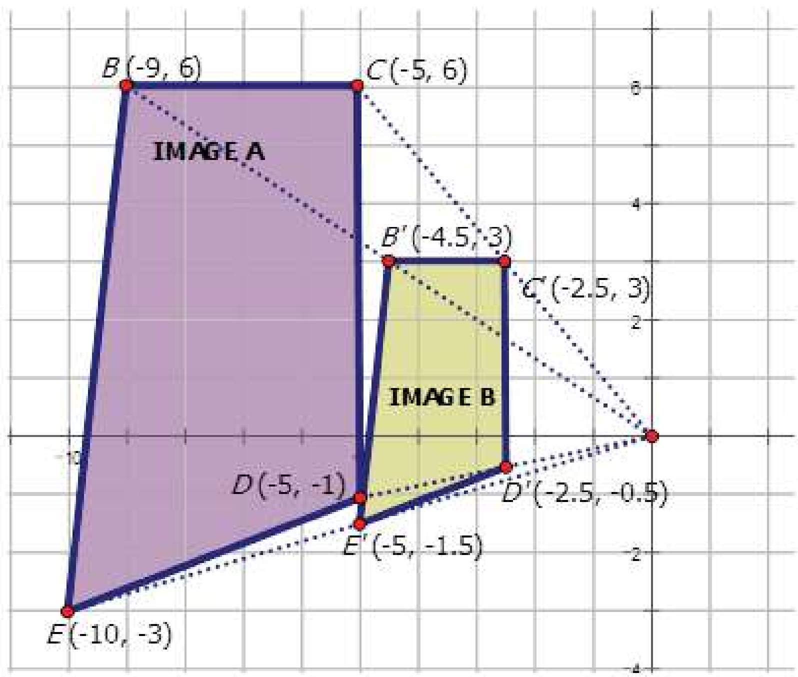
1.6 Rules for Dilations

Here you will learn the notation for describing a dilation.

The figure below shows a dilation of two trapezoids. Write the mapping rule for the dilation of Image A to Image B.



Watch This

First watch this video to learn about writing rules for dilations.

MEDIA
Click image to the left for more content.

CK-12 FoundationChapter10RulesforDilationsA

Then watch this video to see some examples.

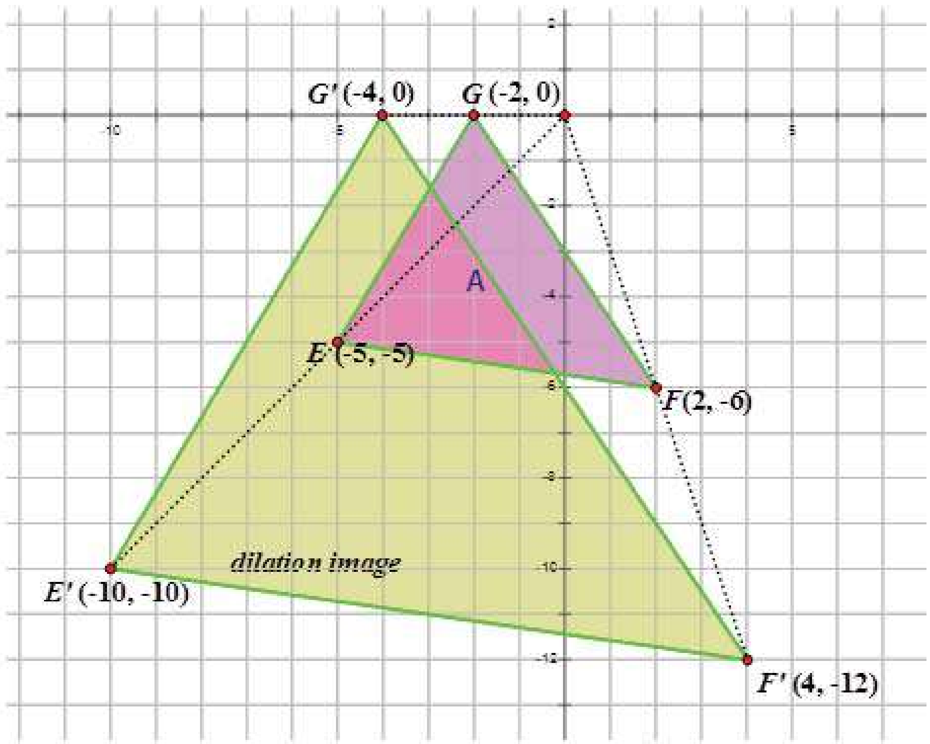
MEDIA
Click image to the left for more content.

CK-12 FoundationChapter10RulesforDilationsB

Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A dilation is a type of transformation that enlarges or reduces a figure (called the preimage) to create a new figure (called the image). The scale factor, r, determines how much bigger or smaller the dilation image will be compared to the preimage.

Look at the diagram below:



The Image A has undergone a dilation about the origin with a scale factor of 2. Notice that the points in the dilation image are all double the coordinate points in the preimage. A dilation with a scale factor k about the origin can be described using the following notation:

$$D_k(x,y) = (kx,ky)$$

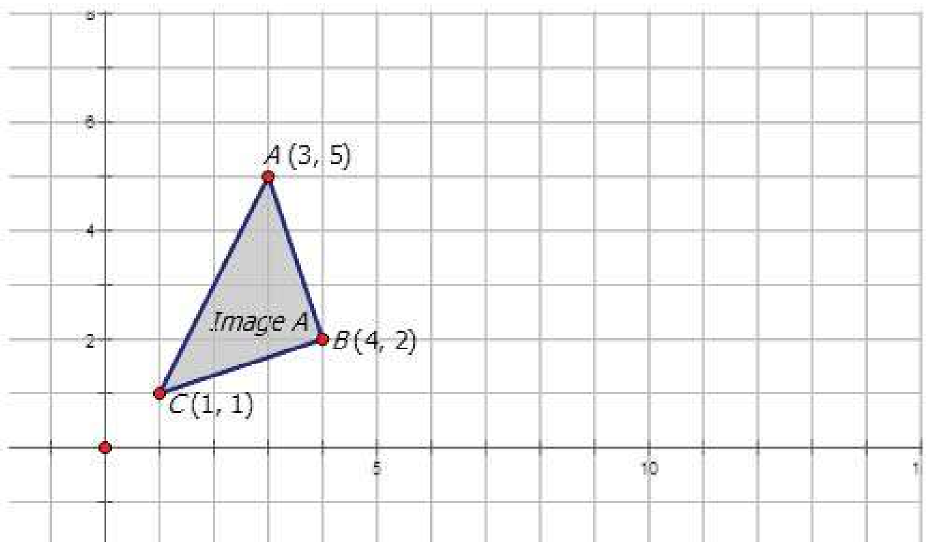
k will always be a value that is greater than 0.

TABLE 1.2:

Scale Factor, k	**Size change for preimage**
$k > 1$	Dilation image is larger than preimage
$0 < k < 1$	Dilation image is smaller than preimage
$k = 1$	Dilation image is the same size as the preimage

Example A

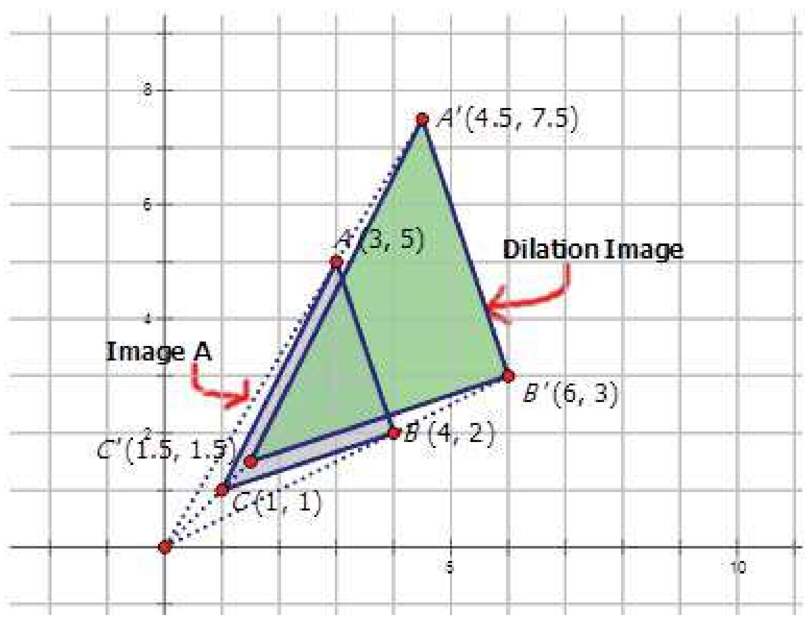
The mapping rule for the dilation applied to the triangle below is $(x,y) \rightarrow (1.5x, 1.5y)$. Draw the dilation image.



Solution: With a scale factor of 1.5, each coordinate point will be multiplied by 1.5.

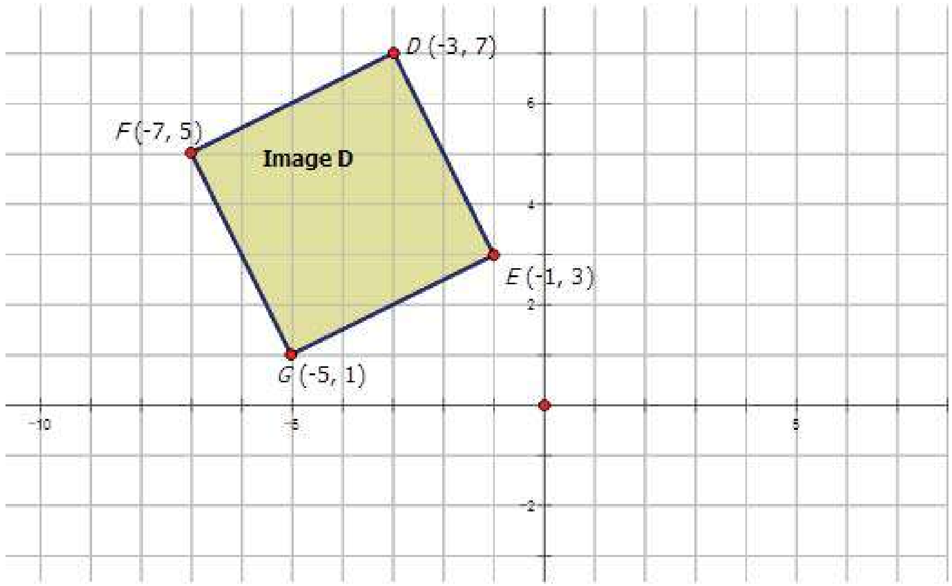
Image A	$A(3,5)$	$B(4,2)$	$C(1,1)$
Dilation Image	$A'(4.5,7.5)$	$B'(6,3)$	$C'(1.5,1.5)$

The dilation image looks like the following:



Example B

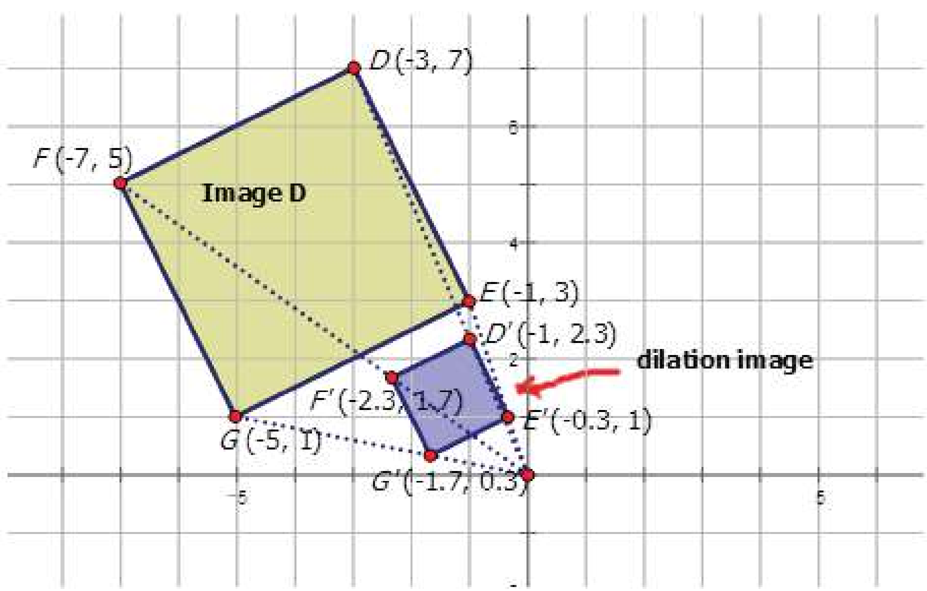
The mapping rule for the dilation applied to the diagram below is $(x,y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)$. Draw the dilation image.



Solution: With a scale factor of $\frac{1}{3}$, each coordinate point will be multiplied by $\frac{1}{3}$.

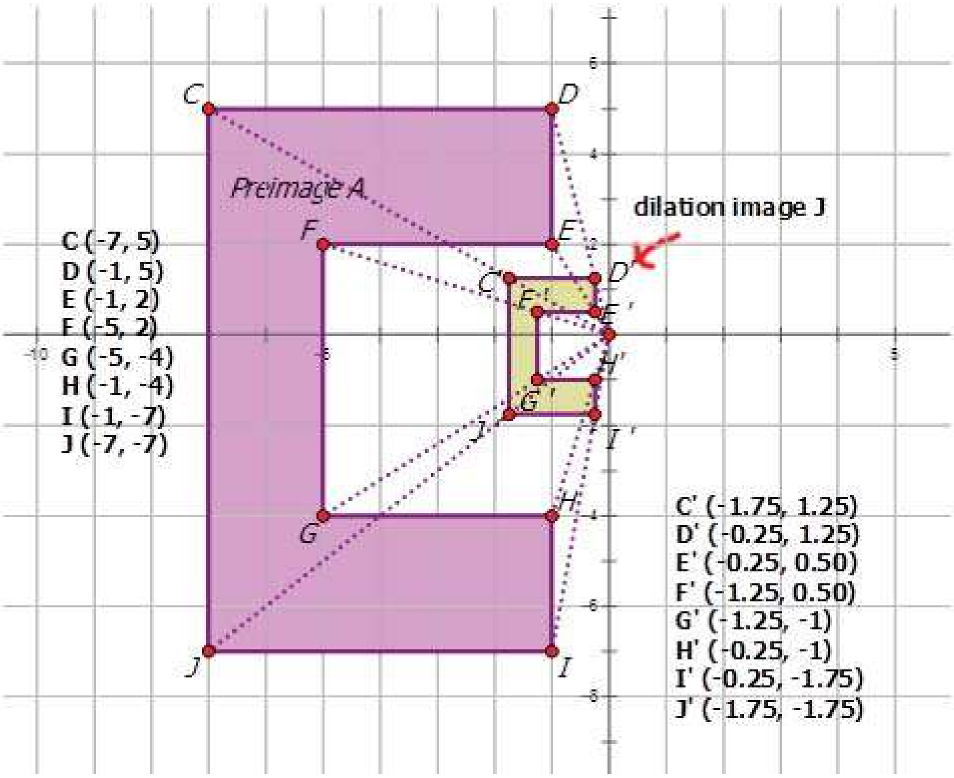
Image *D*	$D(-3,7)$	$E(-1,3)$	$F(-7,5)$	$G(-5,1)$
Dilation Image	$D'(-1,2.3)$	$E'(-0.3,1)$	$F'(-2.3,1.7)$	$G'(-1.7,0.3)$

The dilation image looks like the following:



Example C

Write the notation that represents the dilation of the preimage A to the dilation image J in the diagram below.

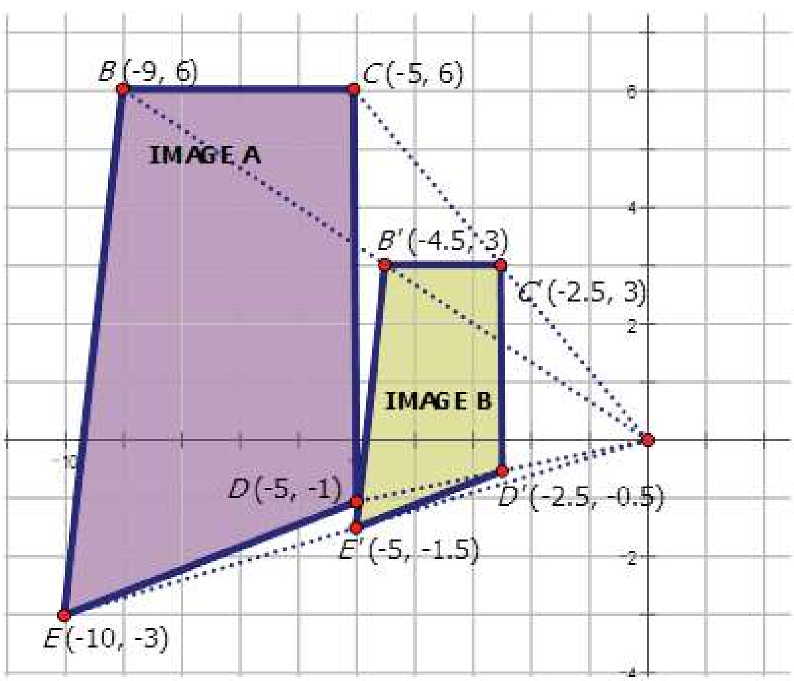


Solution: First, pick a point in the diagram to use to see how it has been affected by the dilation.

$C : (-7, 5) \quad C' : (-1.75, 1.25)$

Notice how both the x- and y-coordinates are multiplied by $\frac{1}{4}$. This indicates that the preimage A undergoes a dilation about the origin by a scale factor of $\frac{1}{4}$ to form the dilation image J. Therefore the mapping notation is $(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$.

Concept Problem Revisited



Look at the points in each image:

Image A	$B(-9,6)$	$C(-5,6)$	$D(-5,-1)$	$E(-10,-3)$
Image B	$B'(-4.5,3)$	$C'(-2.5,3)$	$D'(-2.5,-0.5)$	$E'(-5,-1.5)$

Notice that the coordinate points in Image B (the dilation image) are $\frac{1}{2}$ that found in Image A. Therefore the Image A undergoes a dilation about the origin of scale factor $\frac{1}{2}$. To write a mapping rule for this dilation you would write: $(x,y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$.

Vocabulary

Notation Rule

A ***notation rule*** has the following form $D_k(x,y) = (kx,ky)$ and tells you that the preimage has undergone a dilation about the origin by scale factor k. If k is greater than one, the dilation image will be larger than the preimage. If k is between 0 and 1, the dilation image will be smaller than the preimage. If k is equal to 1, you will have a dilation image that is congruent to the preimage. The mapping rule corresponding to a dilation notation would be: $(x,y) \rightarrow (kx,ky)$

Center Point

The ***center point*** is the center of the dilation. You use the center point to measure the distances to the preimage and the dilation image. It is these distances that determine the scale factor.

Dilation

A ***dilation*** is a transformation that enlarges or reduces the size of a figure.

Scale Factor

The ***scale factor*** determines how much bigger or smaller the dilation image will be compared to the preimage. The scale factor often uses the symbol r.

Image

In a transformation, the final figure is called the ***image***.

Preimage

In a transformation, the original figure is called the ***preimage.***

Transformation

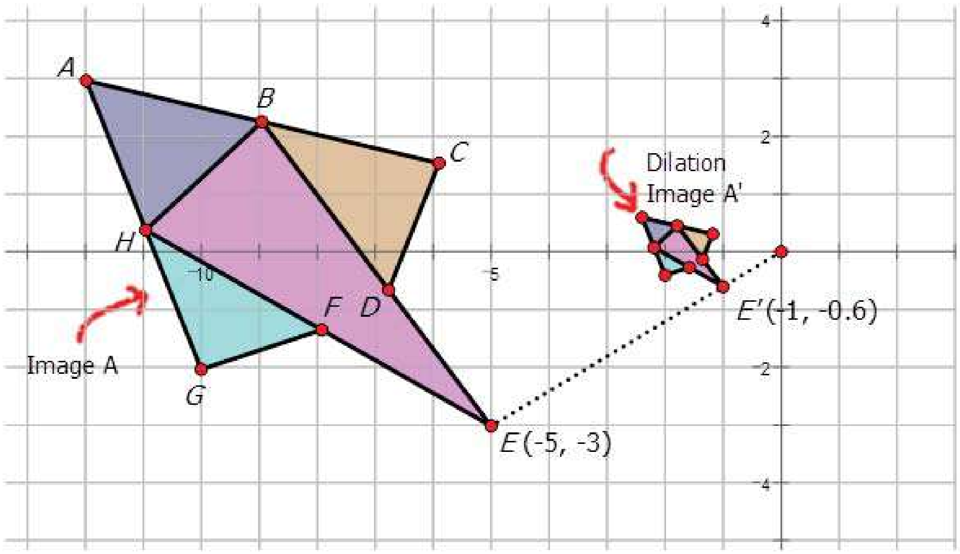
A ***transformation*** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

Guided Practice

1. Thomas describes a dilation of point JT with vertices $J(-2,6)$ to $T(6,2)$ to point $J'T'$ with vertices $J'(-4,12)$ and $T'(12,4)$. Write the notation to describe this dilation for Thomas.

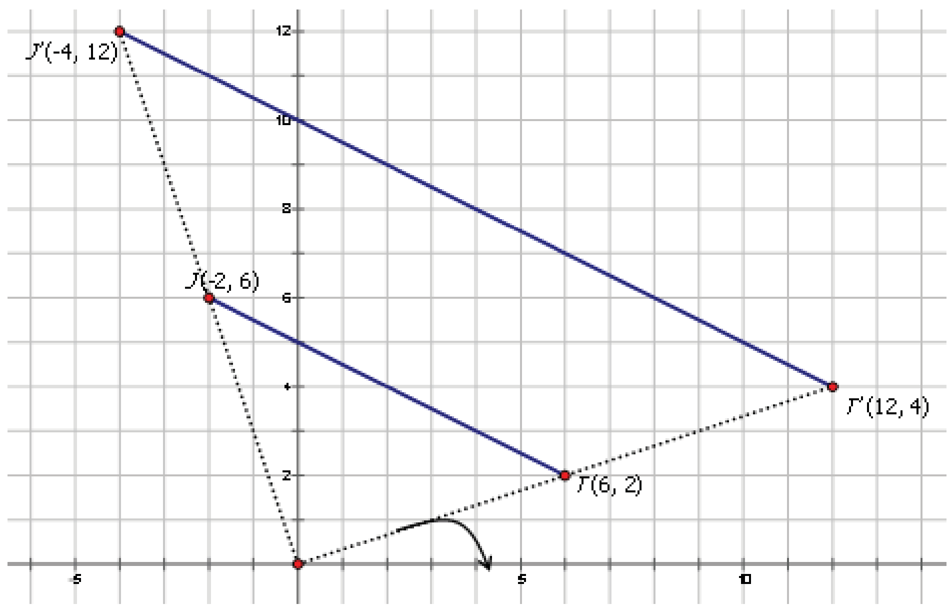
2. Given the points $A(12,8)$ and $B(8,4)$ on a line undergoing a dilation to produce $A'(6,4)$ and $B'(4,2)$, write the notation that represents the dilation.

3. Janet was playing around with a drawing program on her computer. She created the following diagrams and then wanted to determine the transformations. Write the notation rule that represents the transformation of the purple and blue diagram to the orange and blue diagram.



Answers:

1.



Since the x- and y-coordinates are each multiplied by 2, the *scale factor* is 2. The mapping notation is: $(x,y) \rightarrow (2x,2y)$

2. In order to write the notation to describe the dilation, choose one point on the preimage and then the corresponding point on the dilation image to see how the point has moved. Notice that point EA is:

$$A(12,8) \rightarrow A'(6,4)$$

Since both x- and y-coordinates are multiplied by $\frac{1}{2}$, the dilation is about the origin has a scale factor of $\frac{1}{2}$. The notation for this dilation would be: $(x,y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$.

3. In order to write the notation to describe the dilation, choose one point on the preimage A and then the corresponding point on the dilation image A' to see how the point has changed. Notice that point E is shown in the diagram:

$$E(-5,-3) \rightarrow E'(-1,-0.6)$$

Since both x- and y-coordinates are multiplied by $\frac{1}{5}$, the dilation is about the origin has a scale factor of $\frac{1}{5}$. The notation for this dilation would be: $(x,y) \rightarrow \left(\frac{1}{5}x, \frac{1}{5}y\right)$.

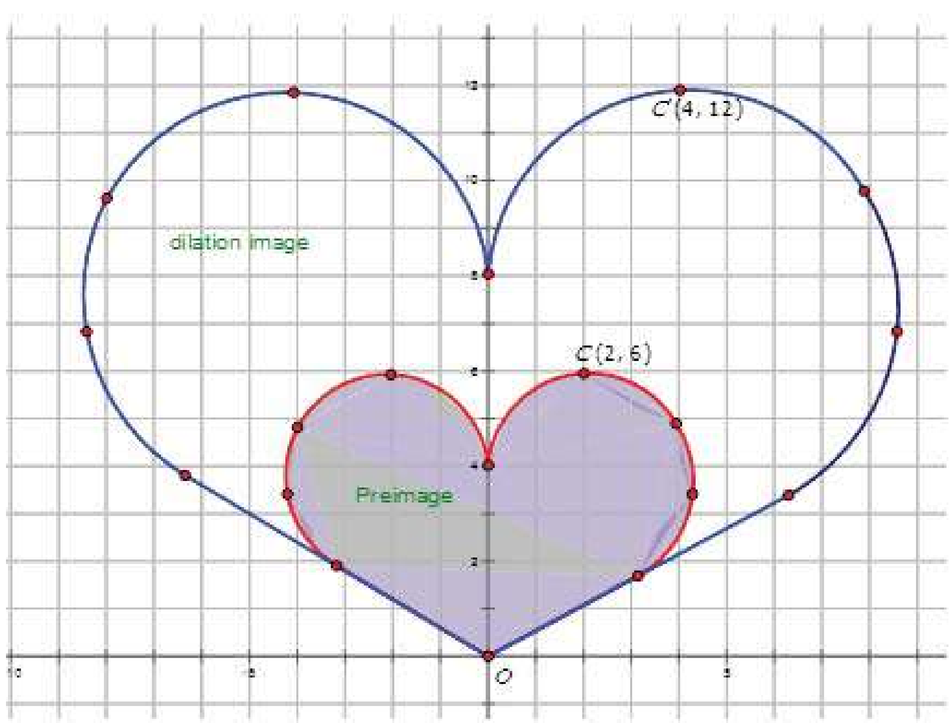
Practice

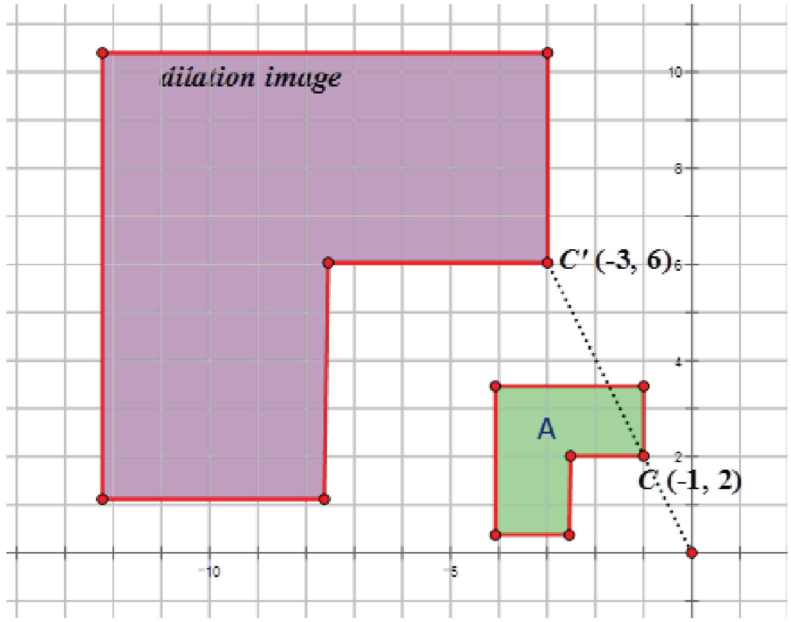
Complete the following table. Assume that the center of dilation is the origin.

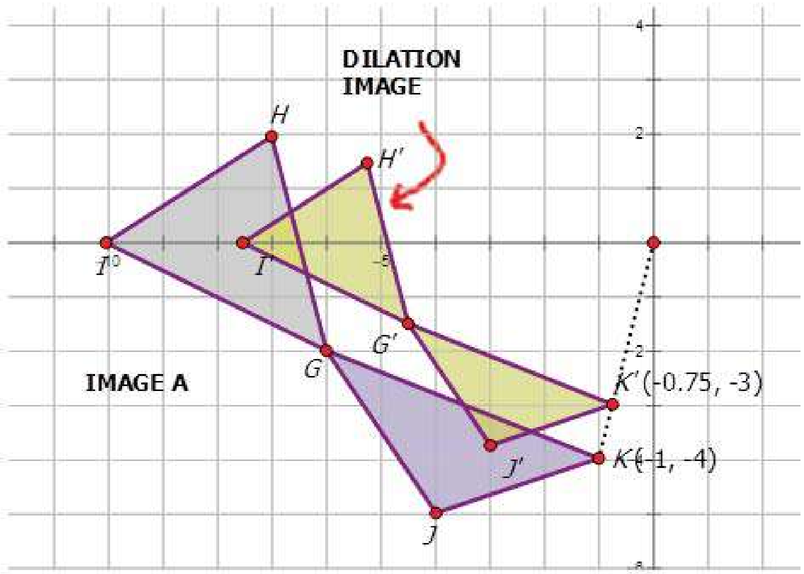
TABLE 1.3:

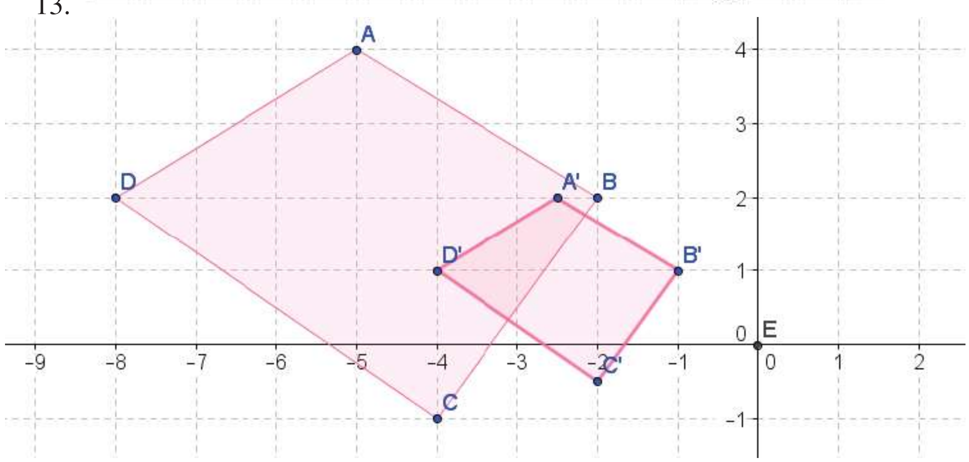
Starting Point	D_2	D_5	$D_{\frac{1}{2}}$	$D_{\frac{3}{4}}$
1. (1, 4)				
2. (4, 2)				
3. (2, 0)				
4. (-1, 2)				
5. (-2, -3)				
6. (9, 4)				
7. (-1, 3)				
8. (-5, 2)				
9. (2, 6)				
10. (-5, 7)				

Write the notation that represents the dilation of the preimage to the image for each diagram below.

11. 

12. 
dilation image
C′ (-3, 6)
A
C (-1, 2)

13. 
DILATION
IMAGE
H
H′
I
I′
G′
G
IMAGE A
K′ (-0.75, -3)
J′
K (-1, -4)
J

14. 
A
D
A′
B
D′
B′
E
C′
C

15.