1.3 Real World and Mathematical Problems with Rational Numbers - 7.NS.3

Students will change between equivalent forms of rational numbers to perform addition, subtraction, multiplication, and/or division with precision. Mathematical practice will lead to application in real world situations.

Did you know that there is only one thing that is universal? It is mathematics. Every country can talk about mathematics in their own language, but pure mathematics, symbols and numbers, are used the same all over the world.

Basic Words and Phrases into Symbols

The challenge is the changing of words into symbols and numbers. The chart below has shows common English words and phrases with their math translation.

Symbol	Word/Phrase	Example	
		Verbal	Symbolic
+	the sum of more than increased by total	the sum of 4 and 7 8 more than 10 3 increased by 9 the total of 14 and 2	4 + 7 10 + 8 3 + 9 14 + 2
-	the difference less than decreased by subtract from	the difference of 10 and 15 3 less than 7 9 decreased by 11 subtract 7 from 12	10 - 15 7 - 3 9 - 11 12 - 7
X	product times of	the product of 6 and 8 3 times 17 $^1/_4$ of 24	6 x 8 3 x 17 $^1/_4$ x 24
÷	the quotient of divided by split evenly half of	the quotient of 12 and 5 14 divided by 7 split 24 evenly between 2 people half of 30	12 ÷ 5 or $\frac{12}{5}$ 14 ÷ 7 or $\frac{14}{7}$ 24 ÷ 2 or $\frac{24}{2}$ 30 ÷ 2 or $\frac{30}{2}$

You may find more words and phrases to add to this list as more real world situations are modeled. Keeping a list of your own is a great study aid.

Example 1

Jenna has 20 indian head nickels. She gives $\frac{1}{4}$ of them to her sister . How many nickels does she give to her sisiter?

The first sentence gives us our starting number. The second sentence gives us our *math equation* .

$\frac{1}{4}$ of them to her sister

$\frac{1}{4} \times 20 =$ number to her sister

$\frac{1}{4} \times \frac{20}{1}$ make both into fraction form

$\frac{1}{\cancel{4}} \cdot \frac{5 \cdot \cancel{4}}{1}$ factor and cancel

$\frac{5}{1}$ multiply numerators and denominators

5nickels simplify

Example 2

The temperature at 2:30am was -10^0. At noon, the temperature had risen 16^0 . At 5:00pm the temperature had dropped 8^0 . What was the temperature at 5:00pm?

The first sentence gives us our starting temperature. The next two sentences tell the *amount of change and if it is a positive*

$-10+16-8 =$ *the temperature at 5:00pm* $-10+16+-8$ *additive inverse* $-10+-8+16$ *commutative property* $-18+16$

$-10+16-8 =$ the temperature at 5:00pm

$-10+16+-8$ additive inverse

$-10+-8+16$ commutative property

$-18+16$ add like signs

-2 subtract unlike signs

The temperature at 5:00pm was -2^0.

Example 3

The difference in the cost of two skateboards is $13.50. If the first skateboard is priced at $98.45, what are the two possible costs of the second skateboard? Explain your reasoning for each price.

The problem does not tell whether the first skateboard is the one that costs more or the one that costs less. This is where two possible prices can happen.

The price of $98.45 could be the higher price. Difference means to subtract. The first equation would be: $98.45 - 13.50 =$ *price of the second skateboard*

If the $98.45 skateboard is the lower price, we would want the second price to be more. This would change the equation to addition. The second equation would be: $98.45 + 13.50 =$ *price of second skateboard*

Now that we have explained the reason for each equation, the last step is to solve both equations.

$98.45 - 13.50 =$ *price of second skateboard* $98.45 + 13.50 =$ *price of second skateboard*$84.95 =$ *price of second skateboard*

$$98.45 - 13.50 = \text{price of second skateboard} \qquad 98.45 + 13.50 = \text{price of second skateboard}$$
$$84.95 = \text{price of second skateboard} \qquad 111.95 = \text{price of second skateboard}$$

The price of the second skateboard can be $84.95 or $111.95.

http://learnzillion.com/lessons/1150-use-addition-and-subtraction-to-solve-realworld-problems-involving-decimals is a mini lesson on using addition and subtraction in real world problems.

http://www.youtube.com/watch?v=TxxKYne1C-M shows how to use decimals to find the perimeter of a unique polygon.

Modeling Equations to Describe Situations

Math is not always purely equations. Often math comes from problems in real world situations. A problem happens and math is the answer!

Example 4

A kindergarten class is making clay animals for a play. The teacher has $2\frac{3}{4}$ gallons of clay. Each child needs $^1/_{16}$ of a gallon to make an animal. How many children can make a clay animal?

The problem gives a total amount, $2\frac{3}{4}$ *gallons. The clay needs to be split up so each child gets a certain amount,* $^1/_{16}$ *of a gallon. The units are the same (gallons). To split up means to divide.*

Divide the total amount by how much each child gets.

$$2\frac{3}{4} \div \frac{1}{16}$$
$$\frac{11}{4} \div \frac{1}{16} \qquad \textit{make both fractions}$$
$$\frac{11}{4} \times \frac{16}{1} \qquad \textit{dividing by } \frac{1}{16} \textit{ is the same as multiplying by } \frac{16}{1}$$
$$\frac{11}{\cancel{4}} \times \frac{4 \cdot \cancel{4}}{1} \qquad \textit{cancel out common factors}$$
$$\frac{44}{1} \qquad \textit{multiply across}$$

children 44

Example 5

The stock market measures earnings per share of stock of companies. On Monday, Company ABC opened and closed at $22.31 per share. On Tuesday, it rose $0.27 per share. On Wednesday it fell $1.12 per share. On Thursday it fell another $1.16 per share. On Friday it gained $0.10 per share. What was the price of a share of stock for Compay ABC at the close of the stock market on Friday?

The starting number is the price of the stock at the start of the week.To rise means to add. To fall means to subtract. With this in mind the equation is made by adding and subtracting the numbers from the starting price.

$$\begin{aligned} 22.31+.27-1.12-1.16+.10 &= \textit{ending price of stock} \\ 22.31+.27+-1.12+-1.16+.10 & \qquad \textit{additive inverse} \\ 22.31+.27+.10+-1.12+-1.16 & \qquad \textit{commutative property} \\ 22.68+-2.28 & \qquad \textit{add like signs} \\ 20.40 & \qquad \textit{subtract unlike signs} \end{aligned}$$

The final price of the stock is $20.40.

Example 6

Bob has saved $25.40. Rena saved $\frac{3}{4}$ of the amount that Bob saved. Nick saved twice as much as Rena. How much is the total saved of all three?

To find the answer to this problem, we must first find out how much each person saved.

The first sentence gives the starting number. The second sentence says $\frac{3}{4}$ of. Of means to multiply. To find Rena's amount, multiply $\frac{3}{4}$ by 25.40.

$$\begin{aligned} \frac{3}{4}(25.40) &= \textit{amount Rena saved} \\ (.75)(25.40) & \qquad \textit{make common forms} \\ 19.05 & \qquad \textit{amount Rena saved} \end{aligned}$$

Rena saved $19.05.

The third sentence says twice as much as Rena. Twice means to multiply by 2. To find Nick's amount, multiply 2 by the amount Rena saved.

$$\begin{aligned} 2\,(19.05) &= \textit{amount nick saved} \\ 38.10 & \qquad \textit{amount Nick saved} \end{aligned}$$

Total is to add. We add all three amounts together.

$$\begin{aligned} 24.50+19.05+38.10 &= \textit{total amount saved by all} \\ 81.65 &= \textit{total amount saved by all} \end{aligned}$$

Bob, Rena, and Nick saved $81.65 together.

http://www.mathmaster.org/video/subtracting-mixed-numbers-word-problem/?id=867 is a step by step video that helps understand what to look for in word problems and how to solve them.

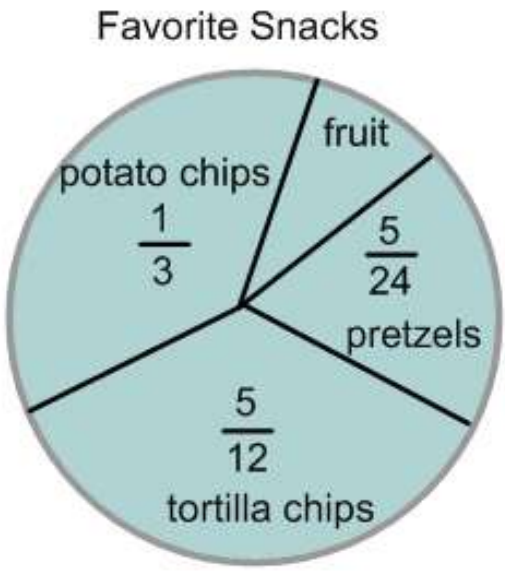
http://www.mathmaster.org/video/dividing-real-numbers-with-different-signs/?id=928 gives a tutorial on mixing up fractions, decimals, and integers in a problem.

Modeling Equations Using Diagrams and Charts

Often diagrams are used to help visualize word problems. Diagrams and charts help us see the relationships between the numbers. Pictures are sometimes easier to understand than words.

Example 7

The following circle graph shows the five most popular snacks for people ages 20-24. There were120 people in this survey.



What fractional part of people ages 20-24 like fruit?

How many people prefer chips based on the survey?

For the first question, the sum of the parts equals the whole. In other words, all the sections must add up to 1. To find the missing part (fruit) we subtract what we know (potato chips, tortilla chips, and pretzels) from the whole (1).

$$1-\left(\frac{1}{3}+\frac{5}{24}+\frac{5}{12}\right)$$

We group the parts we know and subtract it from the total.

First we combine the three fractions. Find a common denominator for all three fractions. Make equivalent fractions to add.

$$1-\left(\frac{1\cdot 8}{3\cdot 8}+\frac{5}{24}+\frac{5\cdot 2}{12\cdot 2}\right) \qquad \text{common denominator of 24}$$
$$1-\left(\frac{8}{24}+\frac{5}{24}+\frac{10}{24}\right)$$
$$1-\left(\frac{23}{24}\right) \qquad \text{add numerators}$$

Finally we subtract the two numbers. One can be expressed as any fraction when both the numerator and the denominator are the same.

$$1-\left(\frac{23}{24}\right)$$
$$\frac{24}{24}-\left(\frac{23}{24}\right) \qquad \textit{change to common denominator}$$
$$\frac{24}{24}-\frac{23}{24}=\frac{1}{24}$$

So never guess just by looking at a diagram what the answer might be. Do the math. Only $\frac{1}{24}$ of the people surveyed said fruit was their favorite snack.

The second question asks to find the actual number of people that like chips. Since it does not say what kind of chips, we combine both chips together. Combine is to add.

$$\frac{1}{3}+\frac{5}{12} \qquad \textit{add both kinds of chips}$$
$$\frac{1\cdot 4}{3\cdot 4}+\frac{5}{12} \qquad \textit{get a common denominator}$$
$$\frac{4}{12}+\frac{5}{12}=\frac{9}{12} \qquad \textit{add numerators}$$

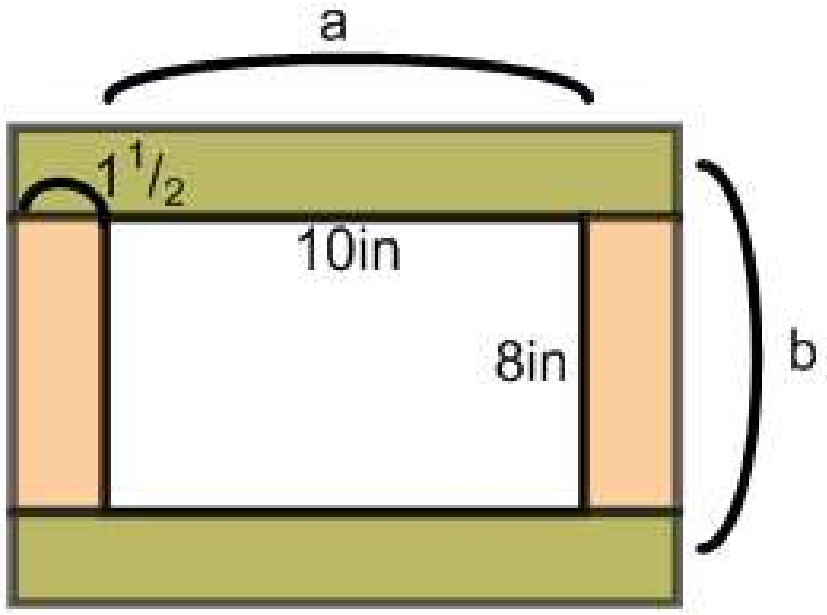
Now to make fraction form into an actual number of people, we need to know what $\frac{9}{12}$ of 120 is.

$\frac{9}{12}$ *of* $120 =$ *people who like chips*

$$\frac{9}{12}\times 120 \qquad \textit{of means times}$$
$$\frac{9}{12}\times\frac{120}{1} \qquad \textit{make equivalent forms}$$
$$\frac{9}{\cancel{12}}\times\frac{\cancel{2}\cdot 10}{1} \qquad \textit{factor to cancel}$$
$$\frac{90}{1} \qquad \textit{multiply numerators and denominators}$$
$$\frac{90}{1}\Rightarrow 90 \textit{ people}$$

Example 8

George is making a picture frame. He wants the frame to be $1\frac{1}{2}$ inches wide. The picture is an 8in x 10in photo. How long should he cut the length (side a) of the frame? Use the diagram as a model.



We will find each side separately. For the length (side a), the inside is 10 inches. There is also an extra $1\frac{1}{2}$ inches added on to each side for the frame.

$1\frac{1}{2}+10+1\frac{1}{2}=$ length of side a

At this point, we can add as fractions or change to decimals and add. That is the great thing about rational numbers. We can use any form we want. We will show both ways here. You decide which you would prefer.

using fractions

$$1\frac{1}{2}+10+1\frac{1}{2}$$

$$1\frac{1}{2}+1\frac{1}{2}+10$$

$$\frac{3}{2}+\frac{3}{2}+\frac{20}{2}$$

$$\frac{26}{2}$$

$$13$$

using decimals

$$1\frac{1}{2}+10+1\frac{1}{2}$$

$$1\frac{1}{2}+1\frac{1}{2}+10$$

use commutative property to put fractions together

$$1.5+1.5+10$$

put numbers in common format to add

$$13$$

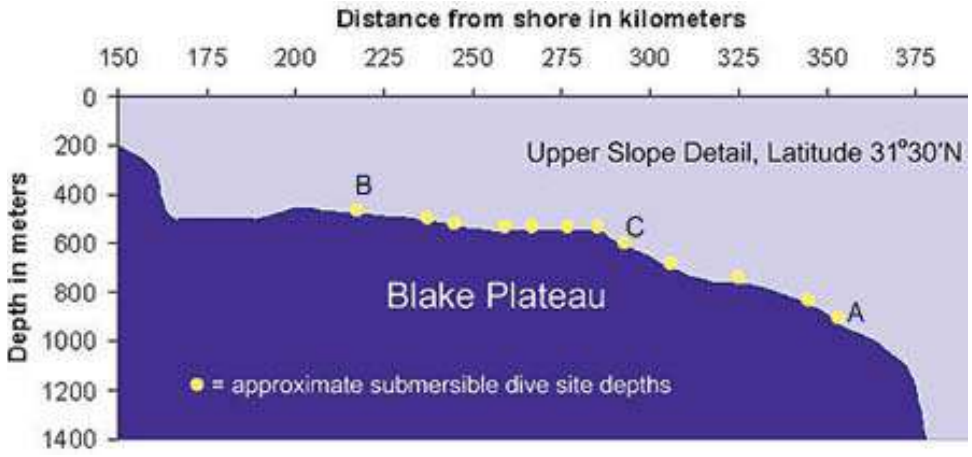
add and reduce

$$13$$

The length is 13 inches.

Example 9

A scuba diver starts at 150km from the shore and descends to point B. Then he travels to point A. Realizing his depth could not be held for a long time, he ascended to point C. Using the chart below, model an equation to find his depth at point C.



Looking at the vertical axis, we see that 0 is our maximum number. This means that all numbers below it are negative. The diver enters the water at the 0m,150km mark. Descend means to go down. So the distance from 0 to point B is -450m.

Is point A going down or up from point B? Up means add, down is subtract. Writing this part of the equation is: $-450-450$.

Now the diver goes up to point C. This is adding. The next part of the equation is now: $-450-450+300$

We know the diver is now at point C. Point C is the answer to complete our equation. Solve to find C. Use the rules for adding and subtracting integers.

$$\begin{aligned} -450-450+300 & \\ -450+-450+300 & \quad \textit{additive inverse} \\ -900+300 & \quad \textit{add negatives} \\ -600 & \quad \textit{unlike signs subtract} \\ & \quad \textit{still underwater so sign is negative} \end{aligned}$$

Now we write our model equation. $-450-450+300=-600$

We can even check to see if our equation makes sense. Go back to the graph and see where point C is located. It is about -600meters. We did it right!

Veiw a video lesson on mixing up operations with rational numbers in real world situations at: http://learnzillion.com/lessons/1152-use-addition-and-multiplication-to-solve-realworld-problems-with-rational-numbers

integer, rational number, distributive property, commutative property, multiplicative inverse, additive inverse, divisor, dividend, quotient, factor, product, sum, difference, subtract from, order of operations,