

Similar Figures

Warm Up

Solve each proportion.

1. $\frac{3}{9} = \frac{b}{30}$ $b = 10$

2. $\frac{y}{5} = \frac{56}{35}$ $y = 8$

3. $\frac{p}{9} = \frac{4}{12}$ $p = 3$

4. $\frac{28}{26} = \frac{56}{m}$ $m = 52$

Similar Figures

Essential Question:

How do I determine if two figures are similar?

Standard:

MCC8.G.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-D figures, describe a sequence that exhibits the similarity between them.

Vocabulary

similar

corresponding sides

corresponding angles

Similar Figures

Similar figures have the same shape, but not necessarily the same size.

Corresponding sides of two figures are in the same relative position, and **corresponding angles** are in the same relative position. Two figures are similar if the lengths of corresponding sides are proportional and the corresponding angles have equal measures.

different sizes. Two triangles are similar if their corresponding angles are congruent and the lengths of their corresponding sides are proportional.

MCC8.G.5

1 EXPLORE Discovering Angle-Angle Similarity

- A** Use your protractor and a straightedge to draw a triangle. Make one angle measure 45° and another angle measure 60° .
- B** Compare your triangle to those drawn by your classmates. How are the triangles the same?

How are they different?

- C** Use the Triangle Sum Theorem to find the measure of the third angle of your triangle.

REFLECT

- 1a.** If two angles in one triangle are congruent to two angles in another triangle, what do you know about the third pair of angles?

- 1b.** Are two pairs of congruent angles enough information to conclude that two triangles are similar? Explain.

Similar Figures

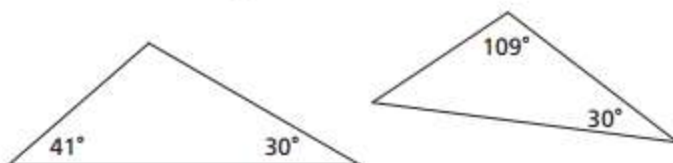
SIMILAR POLYGONS		
Words	Diagram	Corresponding Parts
For two polygons to be similar, corresponding angles must have equal measures, and the ratios of the lengths of the corresponding sides must be proportional.	<p>$\triangle ABC \sim \triangle EFG$</p>	$m\angle A = m\angle E$ $m\angle B = m\angle F$ $m\angle C = m\angle G$ $\frac{AB}{EF} = \frac{BC}{FG} = \frac{AC}{EG} = \frac{2}{1}$

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2

EXAMPLE**Using the AA Similarity Postulate**

Explain whether the triangles are similar.



The figure shows only one pair of congruent angles. Find the measure of the third angle in each triangle. Label the angle measures in the figure.

$$41^\circ + 30^\circ + m\angle 3 = 180^\circ$$

$$71^\circ + m\angle 3 = 180^\circ$$

$$71^\circ + m\angle 3 - \square = 180^\circ - \square$$

$$m\angle 3 = \square$$

$$\square + \square + m\angle 3 = 180^\circ$$

$$\square + m\angle 3 = 180^\circ$$

$$\square + m\angle 3 - \square = 180^\circ - \square$$

$$m\angle 3 = \square$$

Because _____ in one triangle are congruent to _____ in the other triangle, the triangles are _____.

You can also determine whether two triangles are similar by deciding whether the lengths of the corresponding sides are proportional.

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3

EXAMPLE**Using Proportional Side Lengths**

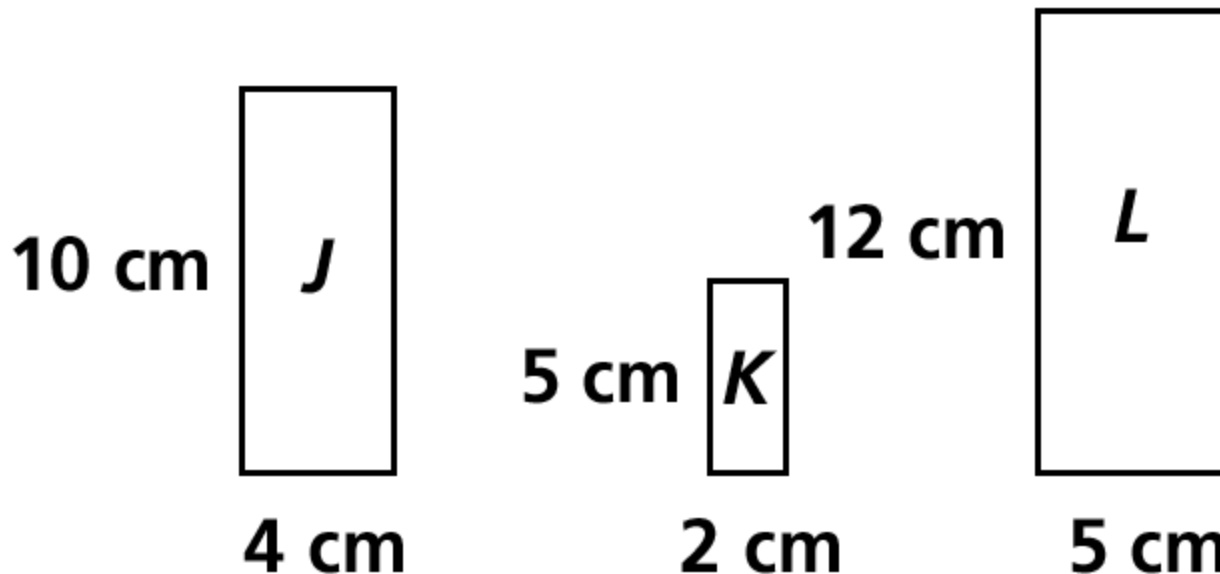
Explain whether $\triangle ABC$ and $\triangle DEF$ are similar.



Similar Figures

Additional Example 1: Identifying Similar Figures

Which rectangles are similar?



You can check to see if the corresponding sides are congruent by setting up a proportion.

Since the three figures are all rectangles, all the angles are right angles. So the corresponding angles are congruent.

Similar Figures

Additional Example 1 Continued

Compare the ratios of corresponding sides to see if they are equal.

$$\frac{\text{length of rectangle } J}{\text{length of rectangle } K} \rightarrow \frac{10}{5} \stackrel{?}{=} \frac{4}{2} \leftarrow \frac{\text{width of rectangle } J}{\text{width of rectangle } K}$$
$$20 = 20$$

The ratios are equal. Rectangle J is similar to rectangle K . The notation $J \sim K$ shows similarity.

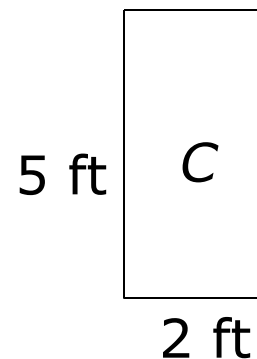
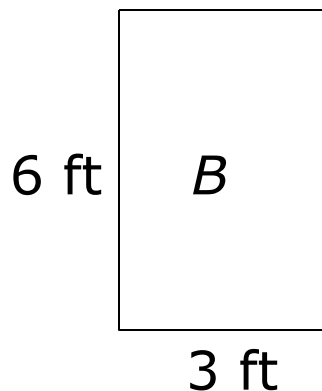
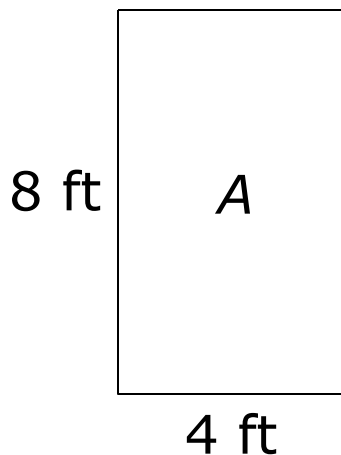
$$\frac{\text{length of rectangle } J}{\text{length of rectangle } L} \rightarrow \frac{10}{12} \stackrel{?}{=} \frac{4}{5} \leftarrow \frac{\text{width of rectangle } J}{\text{width of rectangle } L}$$
$$50 \neq 48$$

The ratios are not equal. Rectangle J is not similar to rectangle L . Therefore, rectangle K is not similar to rectangle L .

Similar Figures

Check It Out: Example 1

Which rectangles are similar?



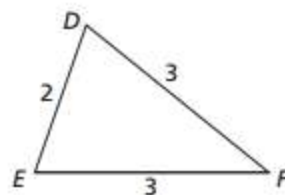
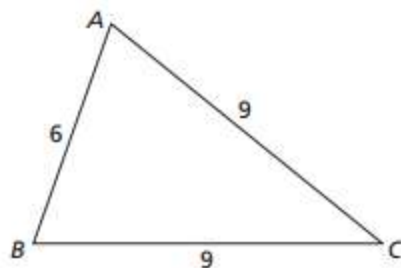
Since the three figures are all rectangles, all the angles are right angles. So the corresponding angles are congruent.

You can also determine whether two triangles are similar by deciding whether the lengths of the corresponding sides are proportional.

MCC8.G.5

3 EXAMPLE Using Proportional Side Lengths

Explain whether $\triangle ABC$ and $\triangle DEF$ are similar.



Corresponding parts of triangles are listed in the same order, so \overline{AB} corresponds to \overline{DE} , \overline{BC} corresponds to _____, and \overline{AC} corresponds to _____.

Determine whether the lengths of corresponding sides are proportional.

$$\frac{AB}{DE} = \frac{BC}{EF} \rightarrow \frac{6}{2} = \frac{\quad}{\quad} \quad \text{Substitute the lengths from the figure.}$$

$$\frac{\quad}{\quad} = \frac{\quad}{\quad} \quad \text{Simplify the ratios.}$$

AC is congruent to BC and DF is congruent to EF , so you do not need to set up a second proportion. Because the lengths of corresponding sides are _____, the triangles are _____.

Similar Figures

Additional Example 2: Finding Missing Measures in Similar Figures

A picture 10 in. tall and 14 in. wide is to be scaled to 1.5 in. tall to be displayed on a Web page. How wide should the picture be on the Web page for the two pictures to be similar?

Set up a proportion. Let w be the width of the picture on the Web page.

$$\begin{array}{l} \text{width of a picture} \longrightarrow \frac{14}{w} = \frac{10}{1.5} \longleftarrow \text{height of picture} \\ \text{width on Web page} \longrightarrow \end{array}$$

$$14 \cdot 1.5 = w \cdot 10 \quad \text{Find the cross products.}$$

$$21 = 10w$$

$$\frac{21}{10} = \frac{10w}{10} \quad \text{Divide both sides by 10.}$$

$$2.1 = w$$

The picture on the Web page should be 2.1 in. wide.

Similar Figures

Check It Out: Example 3

A flag in the shape of an isosceles triangle with side lengths 18 ft, 18 ft, and 24 ft is hanging on a pole outside a campground. A camp t-shirt shows a smaller version of the triangle with two sides that are each 4 in. long. What is the length of the third side of the triangle on the t-shirt?

$$\frac{18 \text{ ft}}{4 \text{ in.}} = \frac{24 \text{ ft}}{x \text{ in.}}$$

Set up a proportion.

$$18 \cdot x = 24 \cdot 4$$

Find the cross products.

Similar Figures

Check It Out: Example 3 Continued

$$18x = 96$$

Multiply.

$$x = \frac{96}{18} \approx 5.3$$

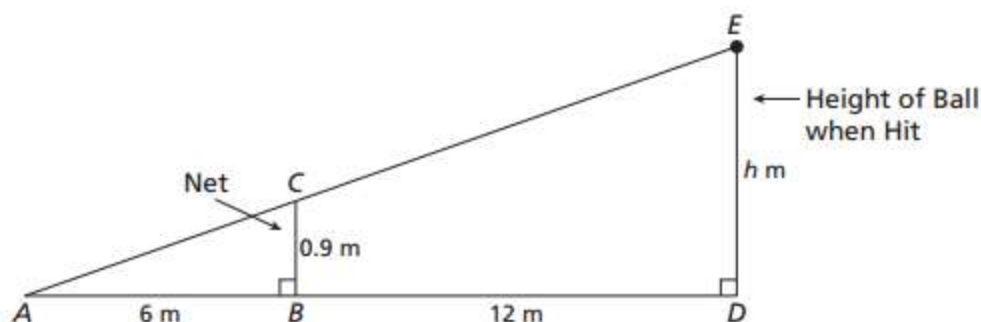
Solve for x.

The third side of the triangle is about 5.3 in. long.

MCC8.G.5

4 EXAMPLE Finding Missing Measures in Similar Triangles

While playing tennis, Matt is 12 meters from the net that is 0.9 meter high. He needs to hit the ball so that it just clears the net and lands 6 meters beyond the base of the net. At what height in meters should Matt hit the tennis ball?



- A** Both triangles contain $\angle A$; $\angle A$ in $\triangle ABC$ is congruent to _____ in $\triangle ADE$.

The net (\overline{BC}) is perpendicular to the ground (\overline{AB}), so $\triangle ABC$ is a right angle. The line representing the height of the ball (\overline{DE}) is perpendicular to the ground (\overline{AD}), so $\angle ADE$ is a _____.

There are two pairs of _____ angles, so $\triangle ABC$ and $\triangle ADE$ are _____.

- B** In similar triangles, corresponding side lengths are proportional.

$$\frac{AD}{AB} = \frac{DE}{BC} \rightarrow \frac{6 + 12}{6} = \frac{h}{0.9} \quad \text{Substitute the lengths from the figure.}$$

$$\square \times \frac{18}{6} = \frac{h}{0.9} \times \square \quad \text{Use properties of equality to get } h \text{ by itself.}$$

$$0.9 \times \square = h \quad \text{Simplify.}$$

$$\square \times \frac{18}{\square} = \frac{h}{\square} \times \square \quad \text{Use properties of equality to get } h \text{ by itself.}$$

$$0.9 \times \square = h \quad \text{Simplify.}$$

$$\square = h \quad \text{Multiply.}$$

Matt should hit the ball at a height of _____ meter(s).

TRY THIS!

4. What if you set up a proportion so that each ratio compares parts of one triangle?

$$\begin{array}{l} \text{height of } \triangle ABC \rightarrow \frac{BC}{AB} = \frac{DE}{AD} \leftarrow \text{height of } \triangle ADE \\ \text{base of } \triangle ABC \rightarrow \frac{BC}{AB} = \frac{DE}{AD} \leftarrow \text{base of } \triangle ADE \end{array}$$

Show that this proportion leads to the same value for h as in **B**.

PRACTICE

1. Two transversals intersect two parallel lines as shown. Explain whether $\triangle ABC$ and $\triangle DEC$ are similar.

