Warm Up

Determine the coordinates of the image of $P(4, -7)$ under each transformation.

1. a translation 3 units left and 1 unit up

2. a rotation of 90° about the origin

3. a reflection across the *y*-axis

Introduction

First we learned that transformations can be functions in the coordinate plane. Then we learned the definitions and properties of three isometric transformations: rotations, reflections, and translations. Now we are able to apply what we have learned to graph geometric figures and images created through transformations.

Key Concepts

- Transformations can be precisely and accurately graphed using the definitions learned.

- Given a set of points and a target, we can determine the transformation(s) necessary to move the given set of points to the target.

- Observing the orientations of the preimage and image is the first tool in determining the transformations required.

Key Concepts, *continued*

- Graphs can be interpreted differently, allowing for many transformation solution sets. While there are many different solution sets of transformations that will satisfy a particular graph, we will look for the more concise possibilities.

- Formulas can be used to determine translations, reflections, and rotations.

- Translation: $T_{h,\,k}(x, y) = (x + h, y + k)$

Key Concepts, *continued*

- Reflection:
  - through the x-axis: $r_{x\text{-axis}}(x, y) = (x, -y)$
  - through the y-axis: $r_{y\text{-axis}}(x, y) = (-x, y)$
  - through the line $y = x$: $r_{y = x}(x, y) = (y, x)$

- Rotation:
  - 90° rotation about the origin: $R_{90}(x, y) = (-y, x)$
  - 180° rotation about the origin: $R_{180}(x, y) = (-x, -y)$
  - 270° rotation about the origin: $R_{270}(x, y) = (y, -x)$

Common Errors/Misconceptions

- using the incorrect transformation formula for reflections

- using the incorrect transformation formula for rotations

- translating in the positive directions along the axes when the translations are intended to be in the negative directions

- applying the transformations in the wrong order

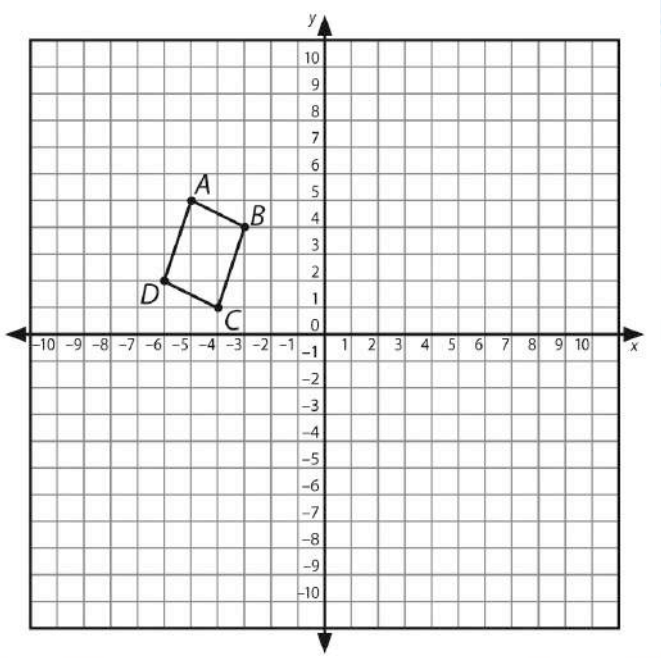
Guided Practice

Example 2

Use the definitions you have learned to graph the reflection of parallelogram *ABCD*, or $\square ABCD$, through the *y*-axis given $\square ABCD$ with the points *A* (–5, 5), *B* (–3, 4), *C* (–4, 1), and *D* (–6, 2).

Guided Practice: Example 2, *continued*

1. Using graph paper, draw the *x*- and *y*-axes and graph $\square ABCD$ with A (–5, 5), B (–3, 4), C (–4, 1), and D (–6, 2).



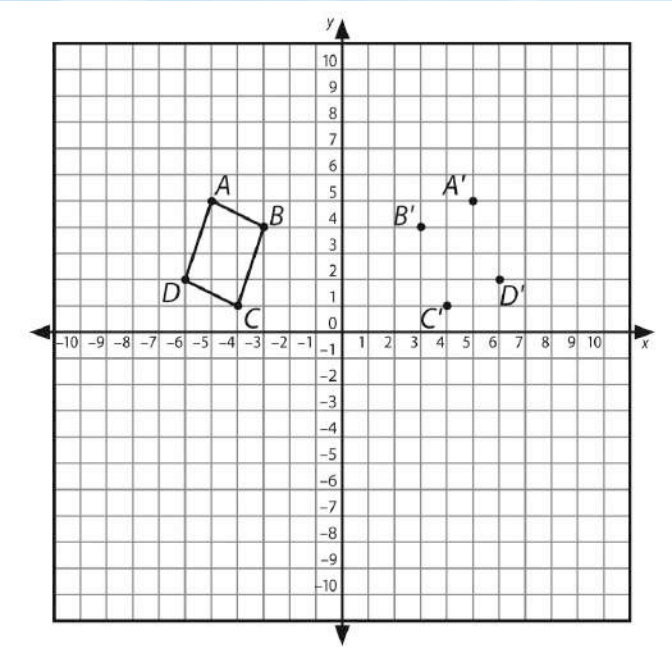
Guided Practice: Example 2, *continued*

2. Write the new points.

$$r_{y\text{-axis}}\left(\square ABCD\right) = \square A'B'C'D' \text{ where}$$

Guided Practice: Example 2, *continued*

3. Plot the new points A', B', C', and D'.



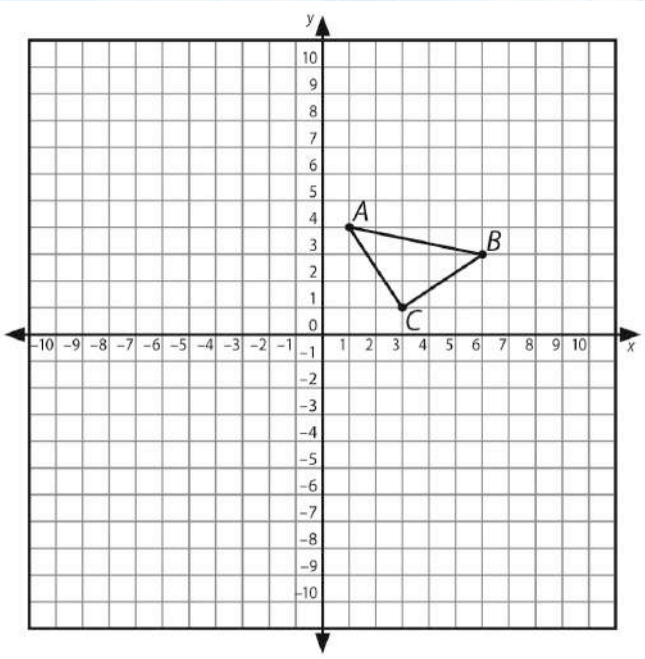
Guided Practice

Example 3

Using the definitions you have learned, graph a 90° rotation of $\triangle ABC$ the points A (1, 4), B (6, 3), and C (3, 1).

Guided Practice: Example 3, *continued*

1. Using graph paper, draw the *x*- and *y*-axes and graph $\triangle ABC$ with the points *A* (1, 4), *B* (6, 3), and *C* (3, 1).



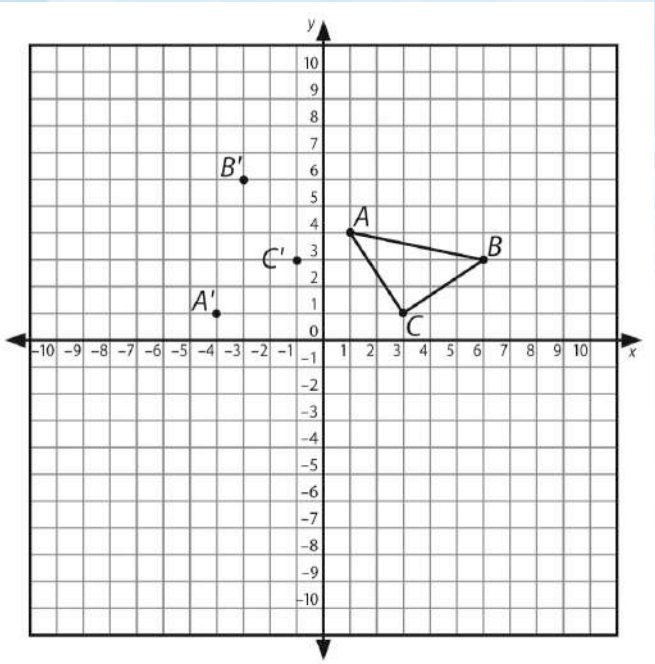
Guided Practice: Example 3, *continued*

2. **Write the new points.**

$R_{90}(\triangle ABC) = \triangle A'B'C'$ where

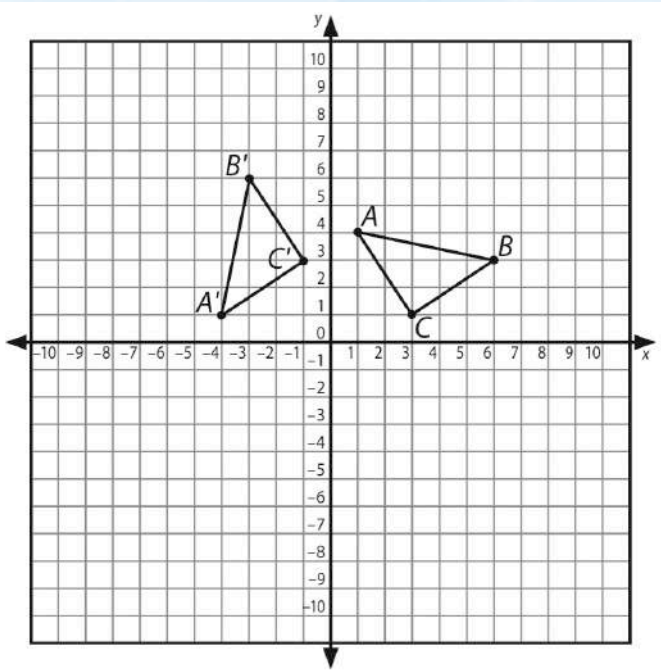
Guided Practice: Example 3, *continued*

3. Plot the new points A', B', and C'.



Guided Practice: Example 3, *continued*

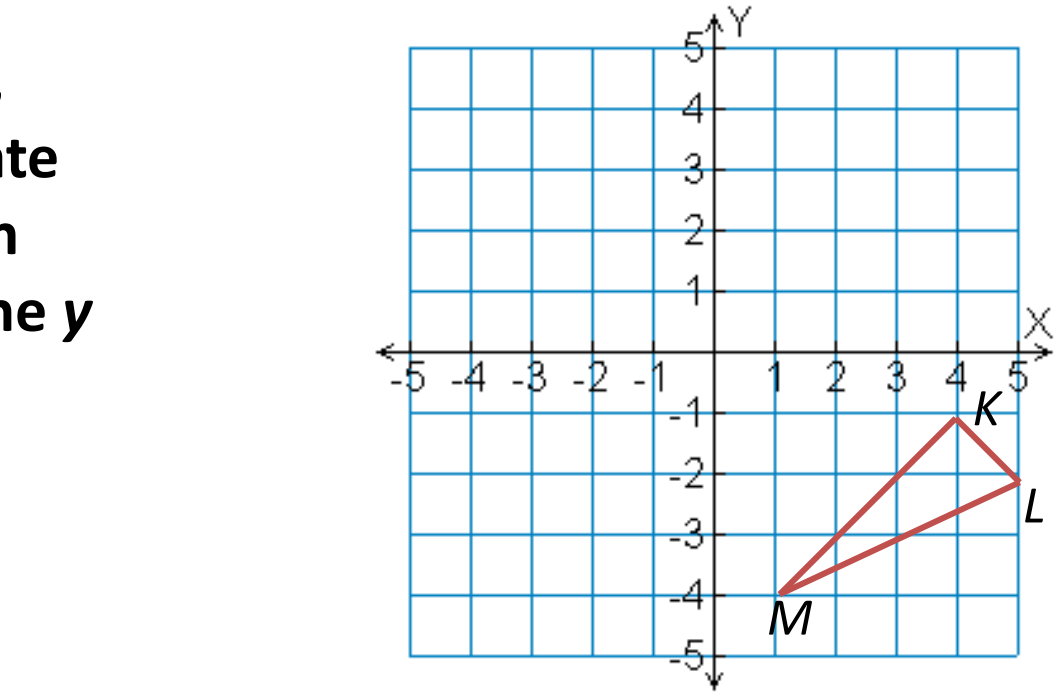
4. Connect the vertices to graph a 90° rotation of $\triangle ABC$.



Example 1B: Drawing Compositions of Isometries

Draw the result of the composition of isometries.

Δ*KLM* has vertices *K*(4, –1), *L*(5, –2), and *M*(1, –4). Rotate Δ*KLM* 180° about the origin and then reflect it across the *y* -axis.



Example 1B Continued

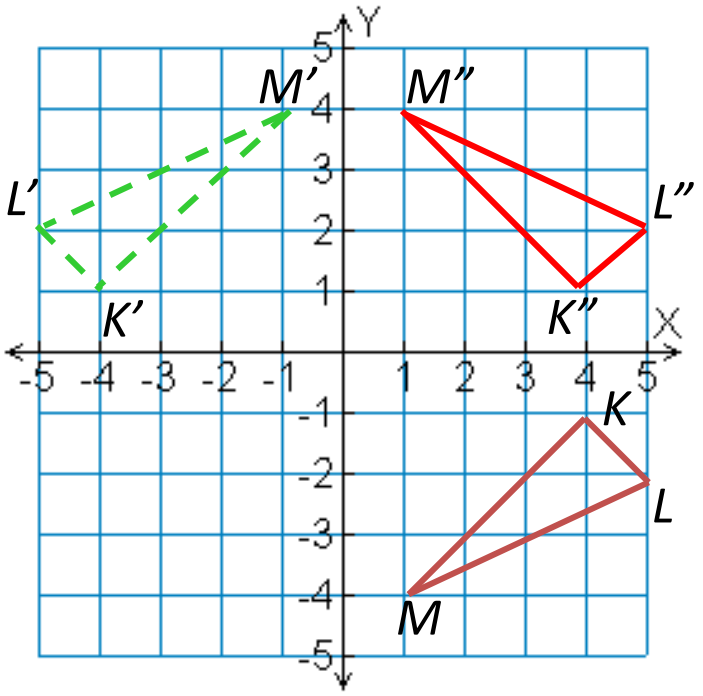
Step 1 The rotational image of (x, y) is $(-x, -y)$.

$K(4, -1) \rightarrow K'(-4, 1)$,
$L(5, -2) \rightarrow L'(-5, 2)$, and
$M(1, -4) \rightarrow M'(-1, 4)$.

Step 2 The reflection image of (x, y) is $(-x, y)$.

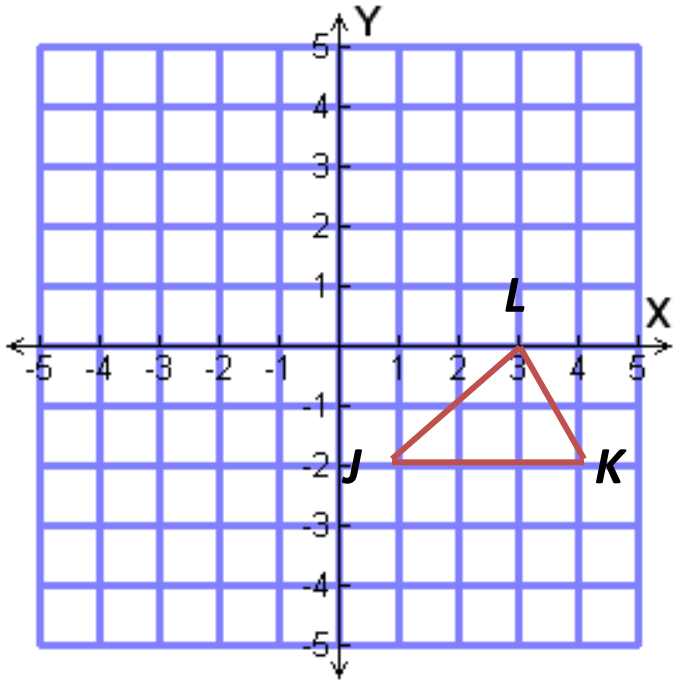
$K'(-4, 1) \rightarrow K''(4, 1)$,
$L'(-5, 2) \rightarrow L''(5, 2)$, and $M'(-1, 4) \rightarrow M''(1, 4)$.

Step 3 Graph the image and preimages.



Check It Out! Example 1

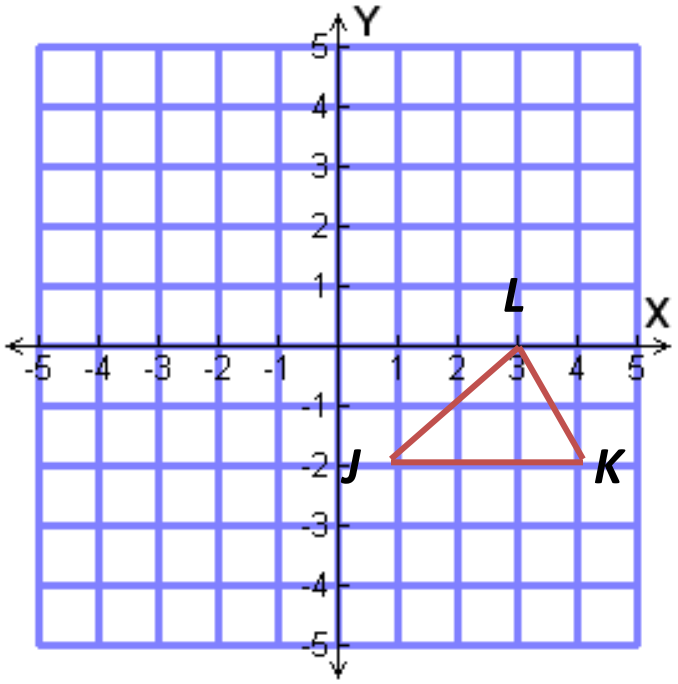
Δ*JKL* has vertices *J*(1,–2), *K*(4, –2), and *L*(3, 0). Reflect Δ*JKL* across the *x*-axis and then rotate it 180° about the origin.



Check It Out! Example 1 Continued

Step 1 The reflection image of (x, y) is $(-x, y)$.

Step 2 The rotational image of (x, y) is $(-x, -y)$.



Step 3 Graph the image and preimages.

Lesson Quiz: Part I

***PQR* has vertices *P*(5, –2), *Q*(1, –4), and *P*(–3, 3).**

1. Translate Δ*PQR* along the vector <–2, 1> and then reflect it across the *x*-axis.

2. Reflect Δ*PQR* across the line $y = x$ and then rotate it 90° about the origin.