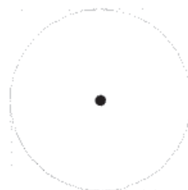
1994 AB 5-BC 2



A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$)

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

1994 AB 3

Consider the curve defined by $x^2 + xy + y^2 = 27$.

(a) Write an expression for the slope of the curve at any point (x, y).

(b) Determine whether the lines tangent to the curve at the $\boldsymbol{x}$-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

(c) Find the points on the curve where the lines tangent to the curve are vertical.

1989 BC3

Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

(a) Find the absolute maximum and minimum values of $f(x)$.

(b) Find the intervals on which f is increasing.

(c) Find the x-coordinate of each point of inflection of the graph of f.

1997 AP Calculus AB: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 (4x^3 - 6x)\,dx =$

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

---

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

---

3. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

---

4. If $f(x) = -x^3 + x + \dfrac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) -1 (D) -3 (E) -5

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

---

6. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

---

7. $\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

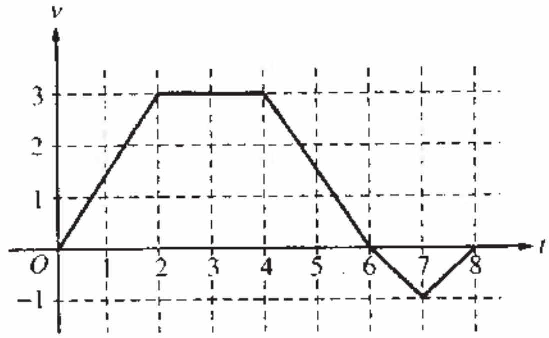
(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

---

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

---

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \dfrac{\pi}{4}$ is

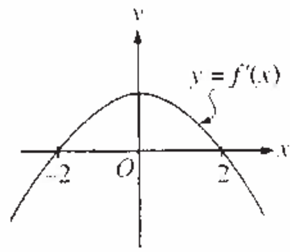
(A) $y - 1 = -\left(x - \dfrac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \dfrac{\pi}{4}\right)$

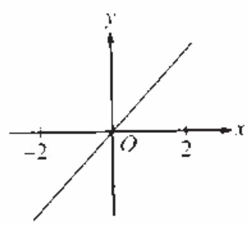
(C) $y = 2\left(x - \dfrac{\pi}{4}\right)$

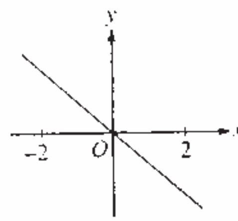
(D) $y = -\left(x - \dfrac{\pi}{4}\right)$

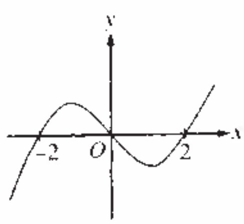
(E) $y = -2\left(x - \dfrac{\pi}{4}\right)$

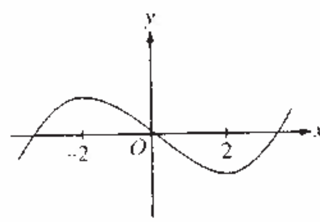


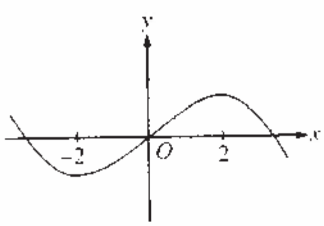
11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

(A)


(B)


(C)


(D)


(E)


12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

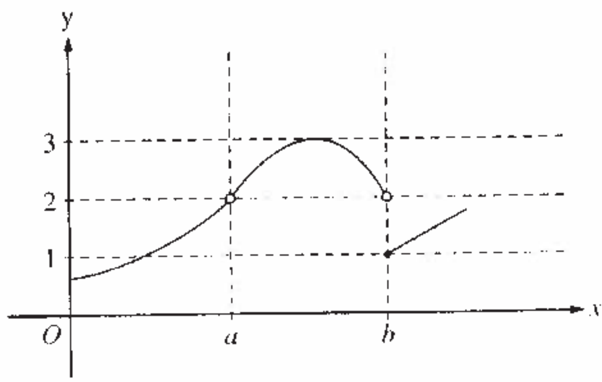
13. Let f be a function defined for all real numbers x. If $f'(x) = \dfrac{|4-x^2|}{x-2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

---

14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$

(B) $\lim_{x\to a} f(x) = 2$

(C) $\lim_{x\to b} f(x) = 2$

(D) $\lim_{x\to b} f(x) = 1$

(E) $\lim_{x\to a} f(x)$ does not exist.

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

---

17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

---

18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

---

19. If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\frac{2x}{x^2-1}\right|$

(B) $\frac{2x}{\left|x^2-1\right|}$

(C) $\frac{2|x|}{x^2-1}$

(D) $\frac{2x}{x^2-1}$

(E) $\frac{1}{x^2-1}$

20. The average value of $\cos x$ on the interval $[-3, 5]$ is

(A) $\frac{\sin 5 - \sin 3}{8}$

(B) $\frac{\sin 5 - \sin 3}{2}$

(C) $\frac{\sin 3 - \sin 5}{2}$

(D) $\frac{\sin 3 + \sin 5}{2}$

(E) $\frac{\sin 3 + \sin 5}{8}$

---

21. $\lim_{x \to 1} \frac{x}{\ln x}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

---

22. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x < -1$ and $x > 3$

(C) $-3 < x < 1$

(D) $-1 < x < 3$

(E) All values of x

---

23. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) π

24. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}}+\sqrt{\frac{2}{50}}+\sqrt{\frac{3}{50}}+\cdots+\sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(B) $\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

(E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

---

25. $\int x\sin(2x)\,dx =$

(A) $-\frac{x}{2}\cos(2x)+\frac{1}{4}\sin(2x)+C$

(B) $-\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(C) $\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(D) $-2x\cos(2x)+\sin(2x)+C$

(E) $-2x\cos(2x)-4\sin(2x)+C$

40 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

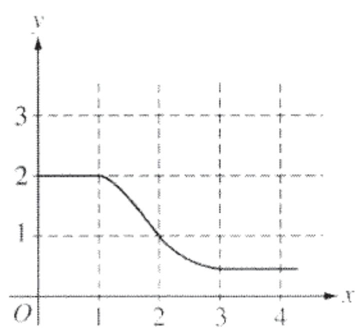
(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

---

77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) −1.58 (B) −1.63 (C) −1.67 (D) −1.89 (E) −2.33



78. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

79. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

---

80. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

---

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

---

82. If $y = 2x - 8$, what is the minimum value of the product xy?

(A) -16 (B) -8 (C) -4 (D) 0 (E) 2

---

83. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

---

84. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

---

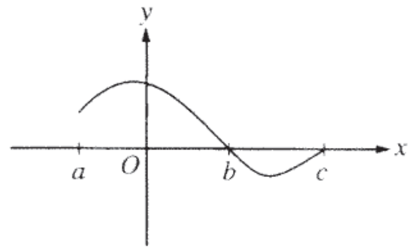
86. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

---

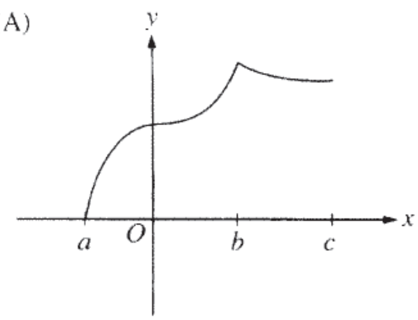
87. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

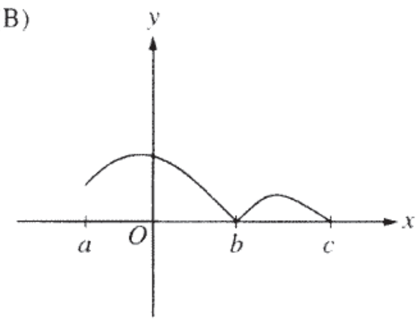


88. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

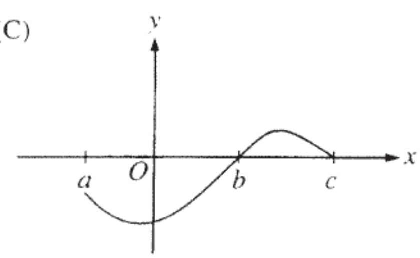
(A)



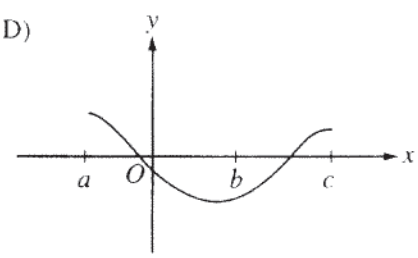
(B)



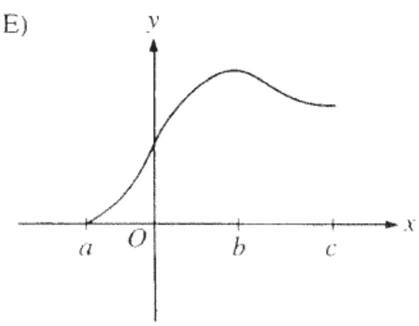
(C)



(D)



(E)



x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0, 2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

---

90. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

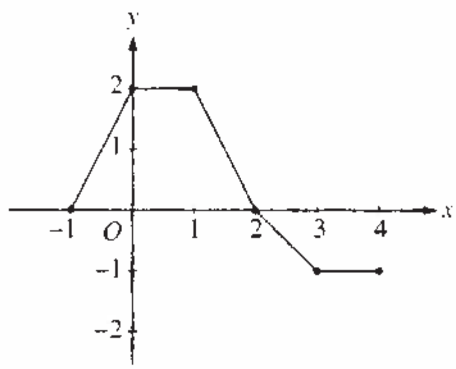
(E) II and III only

55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10



2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

---

3. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

(E) $\int_a^b f(x)dx$ exists.

---

5. $\int_0^x \sin t \, dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

---

6. If $x^2 + xy = 10$, then when $x = 2$, $\dfrac{dy}{dx} =$

(A) $-\dfrac{7}{2}$ (B) -2 (C) $\dfrac{2}{7}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{7}{2}$

---

7. $\int_1^e \left(\dfrac{x^2 - 1}{x}\right) dx =$

(A) $e - \dfrac{1}{e}$ (B) $e^2 - e$ (C) $\dfrac{e^2}{2} - e + \dfrac{1}{2}$ (D) $e^2 - 2$ (E) $\dfrac{e^2}{2} - \dfrac{3}{2}$

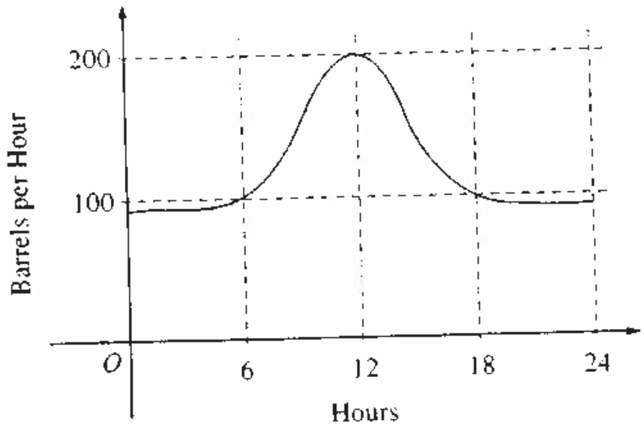
---

8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

---

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{2}$ (D) 2 (E) 6

---

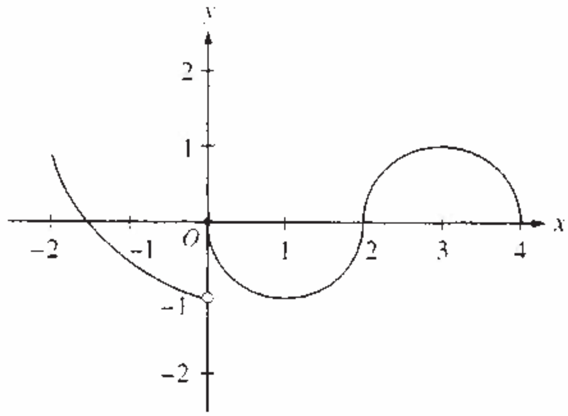
11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\dfrac{ab}{2}$ (D) $b - a$ (E) $\dfrac{b^2 - a^2}{2}$

---

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim\limits_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

---

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

15. If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

---

16. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

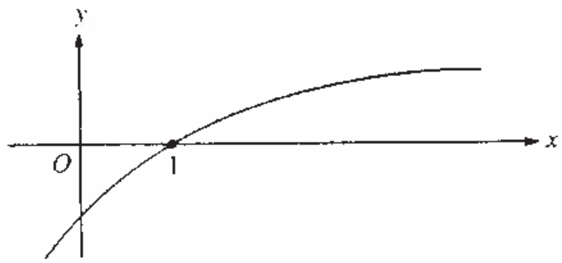
(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

---

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0,1)$ is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

---

19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) −1 only (B) 2 only (C) −1 and 0 only (D) −1 and 2 only (E) −1, 0, and 2 only

---

20. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

(A) −3 (B) 0 (C) 3 (D) −3 and 3 (E) −3, 0, and 3

---

21. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

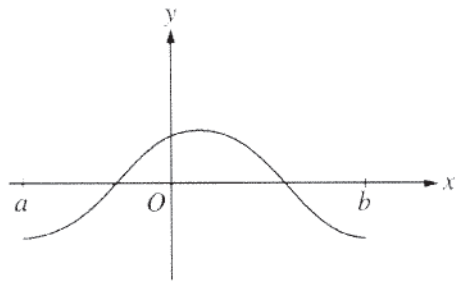
(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

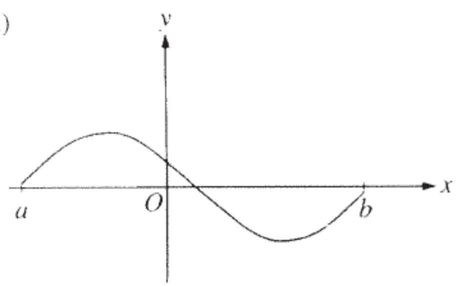
(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

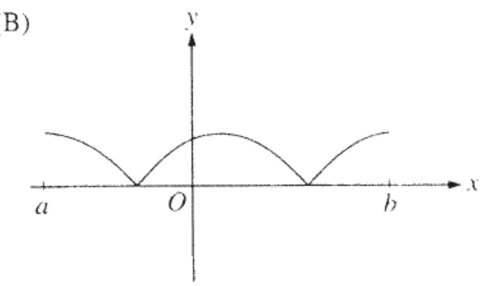


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

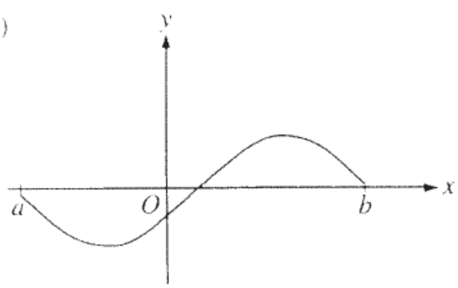
(A)



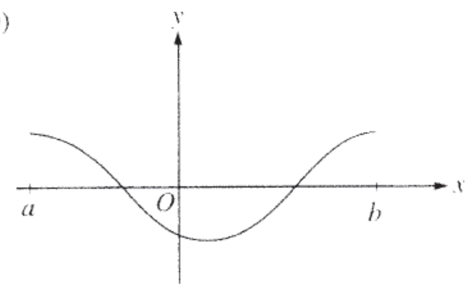
(B)



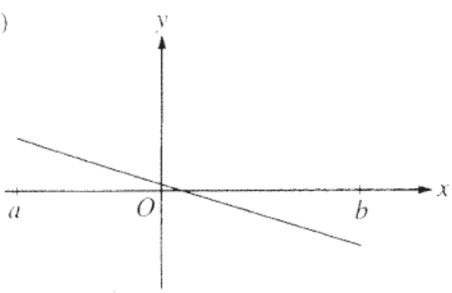
(C)



(D)



(E)



24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

---

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

---

x	0	1	2
$f(x)$	1	k	2

26. The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

---

27. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0,2]$?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

---

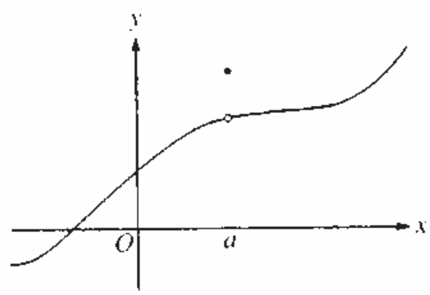
28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

---

77. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

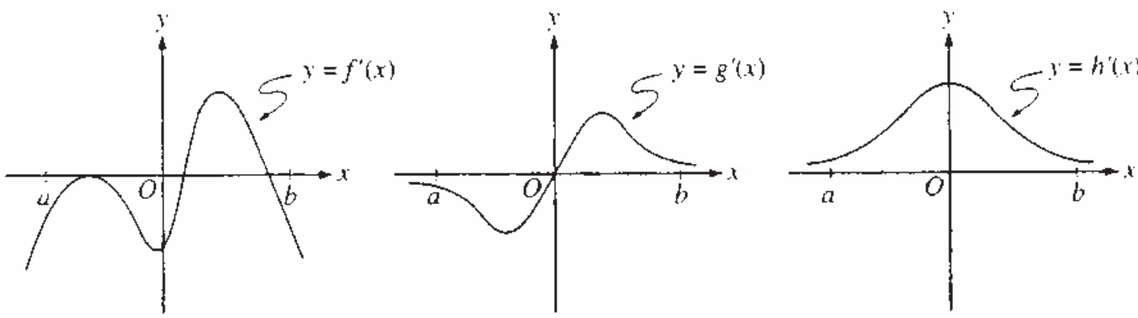
(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\dfrac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

80. The first derivative of the function f is given by $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

---

82. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

---

83. If $a \neq 0$, then $\lim\limits_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

---

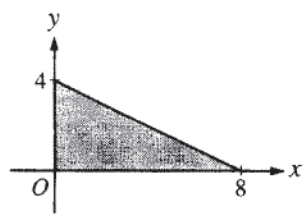
84. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210



86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

---

87. Which of the following is an equation of the line tangent to the graph of $f(x)=x^4+2x^2$ at the point where $f'(x)=1$?

(A) $y=8x-5$
(B) $y=x+7$
(C) $y=x+0.763$
(D) $y=x-0.122$
(E) $y=x-2.146$

---

88. Let $F(x)$ be an antiderivative of $\dfrac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

---

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

---

91. Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

---

92. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436