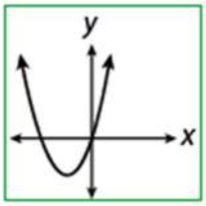
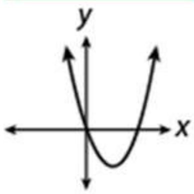
Quadratic Equations, Imaginary Numbers, Lines, Angles, Triangles Notes Solutions

1.

If $a = 0$ and $b < 0$, then which of the following could be the graph of $f(x) = (x - a)(x - b)$?

(A)


(B)


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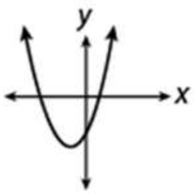

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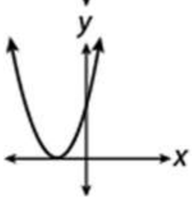


Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: If $a = 0$, then one factor must be $(a - 0)$, which means $x = 0$ is a root. This means the graph must cross through the origin, so you can eliminate C and D right away.

Getting to the Answer: Look at the remaining two choices, (A) and B. The question states that $b < 0$. This means b is negative, which means the other x-intercept must fall to the left of the y-axis, so (A) is correct. Because this question is in the calculator section of the test, you could also use the Picking Numbers strategy. Choose a value for b (that is less than 0), such as −2, and graph the equation $y = (x - 0)(x - (-2))$ to see what the graph looks like.

2.

Which of the following are solutions to the quadratic equation $(x - 2)^2 = \frac{16}{25}$?

(A) $x = \pm\sqrt{\frac{4}{5}}$

(B) $x = -\frac{4}{5}, x = \frac{4}{5}$

(C) $x = \frac{6}{5}, x = \frac{14}{5}$

(D) $x = \frac{14}{5}, x = -\frac{14}{5}$

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: Taking the square root is the inverse operation of squaring, and both sides of the equation are already perfect squares, so take their square roots. Then solve the resulting equations. Remember, there will be two equations to solve.

Getting to the Answer:

$$(x-2)^2 = \frac{16}{25}$$
$$\sqrt{(x-2)^2} = \sqrt{\frac{16}{25}}$$
$$x-2 = \pm\frac{\sqrt{16}}{\sqrt{25}}$$
$$x = 2 \pm \frac{4}{5}$$

Now, simplify each equation: $x = 2 - \frac{4}{5} = \frac{10}{5} - \frac{4}{5} = \frac{6}{5}$ and $x = 2 + \frac{4}{5} = \frac{10}{5} + \frac{4}{5} = \frac{14}{5}$.

3.

$$\frac{4+\sqrt{-16}}{2+\sqrt{-4}}$$

Use the definition $\sqrt{-1} = i$ to simplify the expression above.

Difficulty: Medium

Category: Additional Topics / Imaginary Numbers

Strategic Advice: Because $\sqrt{-1} = i$, rewrite each number under the radical as a product of −1 and itself. Then take the square root of each. If possible, cancel any factors that are common to the numerator and the denominator.

Getting to the Answer:

$$\frac{4+\sqrt{-16}}{2+\sqrt{-4}} = \frac{4+\sqrt{16\times -1}}{2+\sqrt{4\times -1}}$$
$$= \frac{4+4i}{2+2i}$$
$$= \frac{2\cancel{(2+2i)}}{\cancel{2+2i}}$$
$$= 2$$

4.

$$\frac{1}{4}i^{42} + i^{60}$$

What is the value of the complex number given above?

Difficulty: Hard

Category: Additional Topics in Math / Imaginary Numbers

Strategic Advice: To evaluate a high power of i, look for patterns and use the definition $\sqrt{-1} = i$, which can be written in a more useful form as $i^2 = -1$.

Getting to the Answer: Write out enough powers of i that allow you to see the pattern:

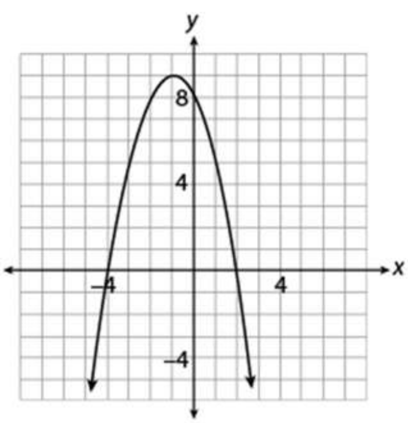
$i^1 = i$
$i^2 = -1$ (by definition)
$i^3 = i \times i^2 = i \times -1 = -i$
$i^4 = i^2 \times i^2 = -1 \times -1 = 1$
$i^5 = i^4 \times i = 1 \times i = i$
$i^6 = i^4 \times i^2 = 1 \times -1 = -1$
$i^7 = i^6 \times i = -1 \times i = -i$
$i^8 = i^4 \times i^4 = 1 \times 1 = 1$

Notice that the pattern (i, −1, $-i$, 1, i, −1, $-i$, 1) repeats on a cycle of 4. To evaluate i^{42}, divide 42 by 4. The result is 10, remainder 2, which means 10 full cycles, and then back to i^2. This means i^{42} is equivalent to i^2, which is −1. Do the same for i^{60}: 60 ÷ 4 = 15, remainder 0, which means stop on the 4th cycle to find that $i^{60} = 1$. Make these substitutions in the original equation:

$$\frac{1}{4}i^{42} + i^{60} = \frac{1}{4}(-1) + 1 = -\frac{1}{4} + 1 = \frac{3}{4}$$

Grid in the answer as 3/4 or .75.

5.



The graph of the function $f(x) = -x^2 - 2x + 8$ is shown in the figure above. For what values of x does $f(x) = 5$?

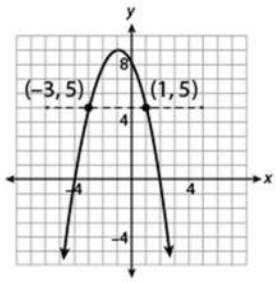
Ⓐ −4 and 2
Ⓑ −3 and 1
Ⓒ −1 and 9
Ⓓ 5 and 8

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: This question is very straightforward if you understand the language of functions. Although you could set the second equation equal to 0 and solve for x, the solution can be found simply by looking at the graph.

Getting to the Answer: The statement $f(x) = 5$ means to find the x-values on the graph when y is 5. To do this, draw a horizontal line across the graph at $y = 5$ and read the x-coordinates of the points where the line intersects the parabola.



The function $f(x) = -x^2 - 2x + 8$ has x-values of -3 and 1 when $y = 5$.

6.

If the equation $\frac{2}{9}x^2 + \frac{8}{3}x - 7 = 3$ has solutions x_1 and x_2, what is the product of x_1 and x_2?

Ⓐ −45
Ⓑ −15
Ⓒ −5
Ⓓ 3

Difficulty: Hard

Category: Passport to Advanced Math / Quadratics

Strategic Advice: This is a quadratic equation, so you need one side to equal 0 and then, best-case scenario, you'll be able to factor. If not, you can rely on the quadratic formula.

Getting to the Answer: First, subtract 3 from both sides of the equation. Then multiply everything by 9 to clear the fractions.

$$\frac{2}{9}x^2 + \frac{8}{3}x - 7 = 3$$
$$\frac{2}{9}x^2 + \frac{8}{3}x - 10 = 0$$
$$9\left(\frac{2}{9}x^2 + \frac{8}{3}x - 10 = 0\right)$$
$$2x^2 + 24x - 90 = 0$$

Each number in the equation is divisible by 2, so factor out a 2 and go from there.

$$2x^2 + 24x - 90 = 0$$
$$2(x^2 + 12x - 45) = 0$$
$$2(x + 15)(x - 3) = 0$$

The solutions are −15 and 3, but be careful! The question asks for the product of the solutions, so the correct answer is (−15)(3) = −45.

7. Calculator

$$\begin{cases} y = 3x \\ -3x^2 + 2y^2 = 180 \end{cases}$$

If (x, y) is a solution to the system of equations above, what is the value of x^2?

Ⓐ 12
Ⓑ 20
Ⓒ 60
Ⓓ 144

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

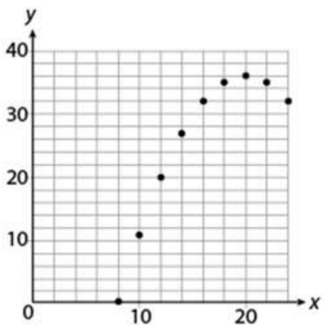
Strategic Advice: Even though one of the equations in this system isn't linear, you can still solve the system using substitution.

Getting to the Answer: You already know that *y* is equal to 3*x*, so substitute 3*x* for *y* in the second equation. Don't forget that when you square 3*x*, you must square both the coefficient and the variable.

$$-3x^2+2y^2=180$$
$$-3x^2+2(3x)^2=180$$
$$-3x^2+2(9x^2)=180$$
$$-3x^2+18x^2=180$$
$$15x^2=180$$
$$x^2=12$$

The question asks for the value of x^2, not *x*, so there is no need to take the square root of 12 to find the value of *x*. The answer is (A).

8. Calculator



If a quadratic equation is used to model the data shown in the scatterplot above, and the model fits the data exactly, which of the following is a solution to the quadratic equation?

Ⓐ 28
Ⓑ 32
Ⓒ 34
Ⓓ 36

Difficulty: Hard

Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: This question requires a conceptual understanding of modeling data and properties of quadratic equations. You also need to recall that a *solution* to an equation is the same as the *x*-intercept of the equation's graph.

Getting to the Answer: The graph of a quadratic equation is symmetric with respect to its axis of symmetry. The axis of symmetry occurs at the *x*-value of the vertex, which according to the graph is 20. You can also see from the graph that one of the *x*-intercepts is $x = 8$. This means that 8 is a solution to the quadratic equation. Unfortunately, 8 isn't one of the answer choices. However, because the graph of a quadratic equation is symmetric, the other solution (*x*-intercept) must be the same distance from the vertex as 8 is, which is $20 - 8 = 12$ units. Therefore, the other solution to the equation is $x = 20 + 12 = 32$.