Exponents, Radicals, Polynomials, Advanced Equations, Functions Notes Solutions

1. Calculator

If $M = 3x^2 + 9x - 4$ and $N = 5x^2 - 12$, what is $2(M - N)$?

(A) $-2x^2 + 9x + 8$
(B) $-4x^2 + 18x - 32$
(C) $-4x^2 + 18x + 16$
(D) $8x^2 + 9x - 16$

Difficulty: Medium

Category: Passport to Advanced Math / Exponents

Strategic Advice: Adding polynomials is typically safer than subtracting them, because you may forget to distribute the negative sign when subtracting more than one term.

Getting to the Answer: To find $M - N$, multiply each term of N by -1 and then add the two polynomials by combining like terms.

$-1N = -5x^2 + 12$

$M + (-N) = 3x^2 + 9x - 4 - 5x^2 + 12$

$= -2x^2 + 9x + 8$

Don't forget to multiply the resulting expression by 2 to get $2(-2x^2 + 9x + 8) = -4x^2 + 18x + 16$.

2.

If h is a function defined over the set of all real numbers and $h(x - 4) = 6x^2 + 2x + 10$, then which of the following defines $h(x)$?

(A) $h(x) = 6x^2 - 2x + 114$
(B) $h(x) = 6x^2 - 46x + 98$
(C) $h(x) = 6x^2 + 2x + 98$
(D) $h(x) = 6x^2 + 50x + 114$

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Strategic Advice: The key to answering this question is in having a conceptual understanding of function notation. Here, the input $(x - 4)$ has already been substituted and simplified in the given function. Your job is to determine what the function would have looked like had x been the input instead.

Getting to the Answer: To keep things organized, let $u = x - 4$, the old input. This means $x = u + 4$. Substitute this into $h(x - 4)$ and simplify:

$$h(x-4) = 6x^2 + 2x + 10$$
$$h(u) = 6(u+4)^2 + 2(u+4) + 10$$
$$= 6(u^2 + 8u + 16) + 2u + 8 + 10$$
$$= 6u^2 + 48u + 96 + 2u + 8 + 10$$
$$= 6u^2 + 50u + 114$$

This means $h(u) = 6u^2 + 50u + 114$.

When working with function notation, you evaluate the function by substituting a given input value for the variable in the parentheses. Here, if the input value is x, then $h(x) = 6x^2 + 50x + 114$.

3.

Which of the following functions has a domain of $x \geq 2$?

(A) $f(x) = -x^2 + 2$
(B) $g(x) = -\sqrt{x-2}$
(C) $h(x) = -\sqrt{x} + 2$
(D) $k(x) = -|x - 2|$

Difficulty: Medium

Category: Passport to Advanced Math / Functions

Strategic Advice: The domain of a function is the set of x-values (inputs) for which the function is defined. Of all the parent functions, the only ones that have a *restricted* domain (a domain that is not all real numbers) are the square root function (because the square root of a negative number is imaginary) and the rational function (because you cannot divide by 0).

Getting to the Answer: The domain in the question is restricted to numbers greater than or equal to 2, so you can immediately eliminate A and D—the domain of a quadratic function and an absolute function is all real numbers. To choose between (B) and C, you can draw a quick sketch or think about how transformations affect the domain of each function. The domain of the parent function $f(x) = \sqrt{x}$ is $x \geq 0$ (nonnegative numbers). In (B), the parent function is reflected vertically across the

horizontal axis (which doesn't change the domain) and then shifted to the right 2 (making the domain $x \geq 2$), so (B) is correct. Note that in C, the function is reflected across the horizontal axis and then shifted *up* 2 units, which adds 2 to the *range* of the function, not the domain.

4.

$$16^{\frac{3}{2}}$$

Which of the following represents the number shown above as an integer?

Ⓐ 4
Ⓑ 12
Ⓒ 48
Ⓓ 64

Difficulty: Hard

Category: Passport to Advanced Math / Exponents

Strategic Advice: Because this is a non-calculator question, you need to rewrite the exponent in a way that makes it easier to evaluate. Unit fractions, as exponents, are easy to evaluate because they can be rewritten as radicals.

Getting to the Answer: Use the rules of exponents to rewrite $\frac{3}{2}$ as a unit fraction raised to a power. Then write the expression in radical form and simplify.

$$\begin{aligned} 16^{\frac{3}{2}} &= (16^{\frac{1}{2}})^3 \\ &= (\sqrt{16})^3 \\ &= 4^3 \\ &= 4 \times 4 \times 4 \\ &= 64 \end{aligned}$$

5.

$$g(x) = \begin{cases} x^2 - 1, \text{ if } x \leq 0 \\ \dfrac{x^2}{3} + 1, \text{ if } 0 < x \leq 3 \\ 5x + 3, \text{ if } x > 3 \end{cases}$$

For the piecewise defined function $g(x)$ shown above, what is the value of $g(2)$?

Difficulty: Medium

Category: Passport to Advanced Math / Functions

Strategic Advice: Piecewise defined functions look intimidating, but they are usually very simple functions —they're just written in pieces. Your job is to figure out which piece of the function you need to use to answer the question.

Getting to the Answer: The right-hand side of each piece of the function tells you what part of the domain (which x-values) goes with that particular function. In this function, only values of x that are less than zero go with the top function, values of x between 0 and 3 go with the middle function, and values of x that are greater than 3 go with the bottom function. Because 2 is between 0 and 3, plug it into the middle function and simplify:

$$\begin{aligned} g(2) &= \frac{(2)^2}{3}+1 \\ &= \frac{4}{3}+1 \\ &= \frac{4}{3}+\frac{3}{3}=\frac{7}{3} \end{aligned}$$

6.

$$\frac{x}{x-1}-\frac{2}{x}=\frac{1}{x-1}$$

What is one possible solution to the rational equation shown above?

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Strategic Advice: Solving a rational equation takes patience and a good deal of algebraic manipulation. You'll need to find a common denominator and multiply both sides of the equation by that denominator. The next steps will depend on what kind of equation results from the previous steps.

Getting to the Answer: Start by multiplying both sides of the equation (all three terms) by the common denominator, which is $x(x + 1)$. Try to write neatly, especially when canceling terms, so you don't lose track of anything.

$$\begin{aligned} x(x-1)\left(\frac{x}{x-1}\right)-x(x-1)\left(\frac{2}{x}\right) &= x(x-1)\left(\frac{1}{x-1}\right) \\ x(x)-2(x-1) &= x(1) \\ x^2-2x+2 &= x \end{aligned}$$

The resulting equation is quadratic, so set it equal to zero and either try to factor it or use the quadratic formula to solve it.

$$\begin{aligned} x^2-2x+2 &= x \\ x^2-3x+2 &= 0 \\ (x-1)(x-2) &= 0 \end{aligned}$$

$$\begin{aligned} x-1 &= 0 \\ x &= 1 \\ &\text{or} \\ x-2 &= 0 \\ x &= 2 \end{aligned}$$

Be careful here—whenever there is a variable in the denominator of an equation, you must check to make sure that the solutions do not result in division by zero. The solution $x = 1$ *does* result in division by 0, so it is invalid. That means the only correct solution is $x = 2$.

7.

$$\left(36x^4y^7\right)^{\frac{1}{2}}$$

Which of the following is equivalent to the expression given above?

Ⓐ $\frac{36x^4y^7}{2}$

Ⓑ $6xy^2\sqrt{y}$

Ⓒ $6x^2y^3\sqrt{y}$

Ⓓ $\left(36x^4y^7\right)^{-2}$

Difficulty: Medium

Category: Passport to Advanced Math / Exponents

Strategic Advice: To make the expression look more familiar, rewrite the fraction exponent as a radical. Then, find the largest perfect squares of each factor and take their square roots (which allows you to bring them outside the radical).

Getting to the Answer:

$\left(36x^4y^7\right)^{\frac{1}{2}}$
$= \sqrt{36x^4y^7}$
$= \sqrt{(6^2)(x^2)^2(y^3)^2y}$
$= 6x^2y^3\sqrt{y}$

You could also use prime factorization and look for pairs of factors that are the same in order to bring them outside the radical.

8.

Given the function $f(x) = \frac{1}{4}x - 2$, what domain value corresponds to a range value of $-\frac{5}{3}$?

Ⓐ $-\frac{29}{12}$

Ⓑ $\frac{4}{3}$

Ⓒ $\frac{7}{3}$

Ⓓ $\frac{29}{12}$

Difficulty: Medium

Category: Passport to Advanced Math / Functions

Strategic Advice: Don't answer this question too quickly—you may be tempted to substitute $-\frac{5}{3}$ for *x*, but $-\frac{5}{3}$ is the output (range), not the input (domain).

Getting to the Answer: The given range value is an output value, so substitute $-\frac{5}{3}$ for *f*(*x*) and use inverse operations to solve for *x*, which gives you the corresponding domain value. Start by multiplying the equation by the greatest common multiple of 3 and 4, which is 12, in order to clear the fractions.

$$-\frac{5}{3} = \frac{1}{4}x - 2$$
$$(12)\left(-\frac{5}{3}\right) = (12)\frac{1}{4}x - 2(12)$$
$$-20 = 3x - 24$$
$$4 = 3x$$
$$\frac{4}{3} = x$$

9.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

When a spring is pressed tightly between two objects, it remains still. When one or both of those objects is disturbed, the spring starts to move. The equation above can be used to find the time period *T* in which a mass *m*, attached to a spring, makes a single oscillation (going all the way down and then back up). The variable *k* is a constant. Which of the following equations could be used to find the mass of the object?

Ⓐ $m = \frac{2\pi k}{T^2}$

Ⓑ $m = \frac{kT^2}{4\pi^2}$

Ⓒ $m = \frac{T^2}{4\pi^2 k}$

Ⓓ $m = \sqrt{\frac{T}{2\pi k}}$

Difficulty: Medium

Category: Passport to Advanced Math / Exponents

Strategic Advice: Don't spend too much time reading the scientific explanation of the equation. Focus on the question at the very end—it's just asking you to solve the equation for *m*.

Getting to the Answer: First, square both sides of the equation to get *m* out from under the radical. Then, divide both sides by $4\pi^2$. Finally, multiply both sides by *k*.

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T^2 = (2\pi)^2\left(\frac{m}{k}\right)$$
$$T^2 = 4\pi^2\left(\frac{m}{k}\right)$$
$$\frac{T^2}{4\pi^2} = \frac{m}{k}$$
$$\frac{kT^2}{4\pi^2} = m$$

10.

$$\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

In electronic circuits, resistors are often paired to manage the flow of the electrical current. To find the total resistance of a pair of parallel resistors, electricians use the formula shown above, where R_1 is the resistance of the first resistor and R_2 is the resistance of the second resistor. Which of the following is another way to represent this formula?

Ⓐ $\frac{R_1R_2}{R_1+R_2}$

Ⓑ $\frac{R_1+R_2}{R_1R_2}$

Ⓒ $\frac{1}{R_2}-\frac{1}{R_1}$

Ⓓ R_1+R_2

Difficulty: Hard

Category: Passport to Advanced Math / Exponents

Strategic Advice: Simplifying a complex rational expression requires planning and patience. Here, you need to write the denominator of the big expression as a single fraction and then you can simply "flip it" to adjust for the "1 over."

Getting to the Answer: Start by writing $\frac{1}{R_1}+\frac{1}{R_2}$ as a single term. To do this, find the common denominator and write each piece of the sum in terms of that denominator. The common denominator is $R_1 R_2$.

$$\begin{aligned}\frac{1}{R_1}+\frac{1}{R_2}&=\frac{R_2}{R_2}\left(\frac{1}{R_1}\right)+\frac{R_1}{R_1}\left(\frac{1}{R_2}\right)\\&=\frac{R_2}{R_1R_2}+\frac{R_1}{R_1R_2}\\&=\frac{R_1+R_2}{R_1R_2}\end{aligned}$$

But remember, this fraction was the denominator under 1, so you need to write the reciprocal (flip it); the correct expression is $\frac{R_1R_2}{R_1+R_2}$.