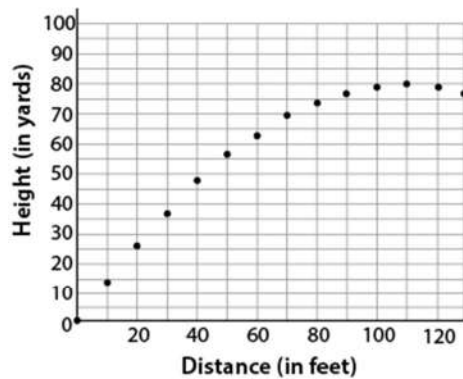


1.



The figure above shows part of the path of a planned roller coaster hill. What is the sum, in feet, of the vertical height and the horizontal distance that the roller coaster will travel while on this particular hill?

- (A) 220
- (B) 300
- (C) 460
- (D) 900

Difficulty: Hard

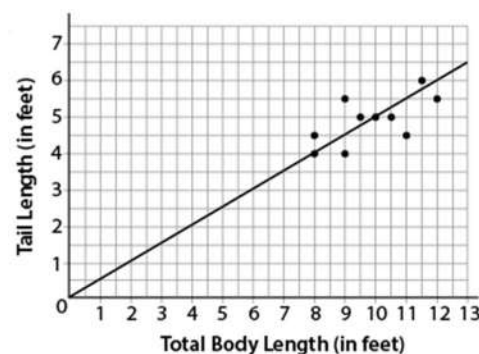
Category: Passport to Advanced Math /

Scatterplots

Strategic Advice: Make sure you read the axis labels and the question carefully. You'll also need to rely on your knowledge of quadratic equations.

Getting to the Answer: The question asks for the sum of the vertical height and the horizontal distance that the roller coaster will travel above ground. The data points follow a parabolic (U-shaped) path, which means you can use properties of quadratic equations to find the solution. The vertical height is fairly straightforward—the vertex of the parabola is located at (110, 80), so the vertical height that the roller coaster reaches is 80 yards (notice the units). To find the horizontal distance, think about symmetry. Because the vertex occurs at a distance of 110 feet, the total horizontal distance that the roller coaster will travel is twice that, or 220 feet. Convert 80 yards to feet and add the result to 220 to arrive at the correct answer, $80 \times 3 = 240$ and $240 + 220 = 460$ feet.

2.



The Florida Department of Wildlife caught and tagged 10 adult female alligators as part of an effort to protect this endangered species. They took various measurements and readings related to body size and health. The total body length is plotted against the tail length in the scatterplot shown above, along with a line of best fit. Which of the following equations best models the data?

- (A) $y = 0.5x$
- (B) $y = 2x$
- (C) $y = 0.4x + 1$
- (D) $y = 0.6x - 1$

Difficulty: Easy

Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: Don't get bogged down in the contextual information in this question. You're looking for the equation that best matches the line drawn through the data points.

Getting to the Answer: The best-fit line begins at the origin, which means the y -intercept is 0 (the b in the equation $y = mx + b$), so you can eliminate C and D. Now, find the slope of the line. Between $(0, 0)$ and $(8, 4)$, the line rises 4 units and runs 8 units, so the slope is $\frac{4}{8} = \frac{1}{2}$ which is equivalent to 0.5. This means (A) is correct.

3. Calculator

The American political system is largely a two-party system. In fact, only six candidates who were not associated with either the Republican or the Democratic Party have been elected governor in any state since 1990. In one such election, the ratio of votes received for the Independent candidate to the Democratic candidate to the Republican candidate was approximately 19:18:13. If 510,000 votes were cast in the election, how many more votes were cast for the Independent candidate than for the Republican candidate?

- (A) 6,000
- (B) 10,200
- (C) 61,200
- (D) 193,800

Difficulty: Easy

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: Don't let the three-way ratio confuse you. You can answer this question just like any other ratio question. Before selecting your answer, make sure you answered the right question (how many *more* votes for the Independent candidate than for the Republican candidate).

Getting to the Answer: Set up an equation using parts: 19 parts of the vote were cast for the Independent candidate, 18 parts for the Democrat, and 13 parts for the Republican. You don't know how big a part is, so call it x . Now, write and solve an equation:

$$19x + 18x + 13x = 510,000$$

$$50x = 510,000$$

$$x = 10,200$$

This means each part is equal to 10,200 votes. Now, you could multiply 19 by this number and 13 by this number, and then subtract. Or, you could recognize that the Independent received $19 - 13 = 6$ more parts of the vote than the Republican, or $6(10,200) = 61,200$ more votes.

4. Calculator

Selection Method	Number of States
Election	22
Gubernatorial Appointment	11
Legislative Appointment	2
Missouri Plan	15

There are four ways in which state judges are selected for their positions. One is by election, another is appointment by the governor (usually with the confirmation by the state legislature), and a third is appointment by the state legislature. The final way is a hybrid of the last two, called the Missouri Plan, in which a nonpartisan legislative committee recommends a list of candidates and the governor chooses from this list. The table above shows the number of states that engage in each process for the highest court of the state, usually called the state Supreme Court. What percent of states select judges using the Missouri Plan?

- (A) 17%
- (B) 30%
- (C) 33%
- (D) 43%

Difficulty: Easy

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: Don't spend too much time reviewing the context of the question. Focus on the last couple of sentences, which tell you what you're looking for.

Getting to the Answer: You need to find the percent of states that use the Missouri Plan. There are 15 states that use the Missouri Plan and 50 states total, so use the formula $\text{Whole} \times \text{percent} = \text{part}$. The whole is 50, the percent is unknown so call it x , and the part is 15, resulting in the equation $50x = 15$. Use division to find that x is $15 \div 50 = 0.3 = 30\%$.

5. Calculator

A dendrologist (a botanist who studies trees exclusively) is examining the way in which a certain tree sheds its leaves. He tracks the number of leaves shed each day over the period of a month, starting when the first leaf is shed. He organizes the data in a scatterplot and sees that the data can be modeled using an exponential function. He determines the exponential model to be $f(x) = 6(1.92)^x$, where x is the number of days after the tree began to shed its leaves. What does the value 1.92 in the function tell the dendrologist?

- (A) The number of leaves shed almost doubles each day.
- (B) The number of leaves shed almost doubles every six days.
- (C) The number of leaves left on the tree is reduced by about 92% each day.
- (D) The number of leaves left on the tree is reduced by about 92% every six days.

Difficulty: Medium

Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: The dendrologist uses an exponential function to model the data. When an exponential equation is written in the form of $f(x) = ab^x$, a is the starting amount and b is the rate of growth or decay.

Getting to the Answer: Read the question carefully. The dendrologist is studying the number of leaves shed, not the number of leaves left on the tree, so you can eliminate C and D. Remember, a is the starting amount, not the unit of time, so it can't represent the number of days, which means you can also eliminate B. Choice (A) is correct because 1.92 is b in the equation, which represents the growth rate, so it tells the dendrologist that the number of leaves shed almost doubles (192% is very close to 200%) each day.

6 and 7. Calculator

Plants are capable of cross-pollinating with related but different plants. This creates a hybrid. Sometimes, a hybrid plant is superior to the two different plants from which it was derived. This is "hybrid vigor." Scientists can examine the DNA of a plant to see if it is a hybrid. This can be information because if the plant appears superior, it would be beneficial to develop more of these. An agricultural scientist examines an orchard that has several types of apple trees and orange trees which ones are hybrids. Some of the information she collected is shown in the table below.

	Apple Trees	Orange Trees	Total
Hybrid			402
Non-hybrid		118	
Totals			628

6. Based on the data, if 45% of the apple trees are not hybrids, how many apple trees are hybrids?

- (A) 50
- (B) 132
- (C) 226
- (D) 240

Difficulty: Medium

Category: Problem Solving and Data Analysis / Statistics and Probability

Strategic Advice: Start by filling in any cells in the table that you can, using the information provided in the table itself (kind of like a sudoku puzzle).

Getting to the Answer: Because there are 628 trees total and 402 are hybrids, you know that $628 - 402 = 226$ are not hybrids. Then, because 118 orange trees are not hybrids, you know that $226 - 118 = 108$ apple trees are not hybrids. Now, you've reached the point at which the table can't help you anymore. So, look at the question. It says that 45% of the apple trees are not hybrids.

Use the formula $\text{Percent} \times \text{whole} = \text{part}$ to arrive at the equation $0.45w = 108$. Then, solve for w by dividing: $108 \div 0.45 = 240$, which tells you there are 240 apple trees in total. This means there are $240 - 108 = 132$ apple trees that are hybrids.

7.

The scientist wants to study the orange trees to check for hybrid vigor. If she chooses one orange tree at random, what is the probability that it will be a hybrid?

- (A) $\frac{59}{194}$
(B) $\frac{97}{157}$
(C) $\frac{135}{314}$
(D) $\frac{135}{194}$

Difficulty: Medium

Category: Problem Solving and Data Analysis / Statistics and Probability

Strategic Advice: Start by completing the rest of the table. Use the information you found in the previous question.

Getting to the Answer: Because there are 402 hybrids in total, there are $402 - 132 = 270$ orange trees that are hybrids, which means there are $270 + 118 = 388$ orange trees in total. Now, find the probability that if the scientist selects one orange tree, it will be a hybrid. There are 388 orange trees total, and of those, 270 are hybrids, so the probability of picking a hybrid is $\frac{270}{388} = \frac{135}{194}$.

8. Calculator

Mount Fuji in Japan was first climbed by a monk in 663 ad and subsequently became a Japanese religious site for hundreds of years. It is now a popular tourist site. When ascending the mountain, tourists drive part of the distance and climb the rest of the way. Suppose a tourist drove to an elevation of 2,390 meters and from that point climbed to the top of the mountain, and then descended back to the car taking the same route. If it took her a total of 7 hours to climb up and back down, and she climbed at an average rate of 264 vertical meters per hour going up and twice that going down, approximately how tall is Mount Fuji?

- (A) 1,386 meters
(B) 2,772 meters
(C) 3,776 meters
(D) 5,172 meters

Difficulty: Medium

Category: Problem Solving and Data Analysis / Rates, Ratios, Proportions, and Percentages

Strategic Advice: There are a few ways to answer this question, but the quickest way is to average the tourist's climb rate over the 7 hours.

Getting to the Answer: The distance going up and back down is the same (because she uses the same route), so find the average of the tourist's speed over both the ascent and the descent. She climbed 264 meters per hour going up and twice that, 528 meters per hour, going back down, so her average climb rate was $264 + 528 = 792 \div 2 = 396$ meters per hour. It took her 7 hours. Use the formula

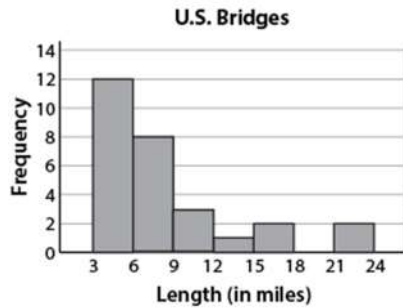
Distance = rate \times time to find the distance:

$$\text{Distance} = 396 \times 7$$

$$\text{Distance} = 2,772$$

But remember, this amount represents both up and down the mountain, so divide by 2 to find that the vertical distance between the point where she started climbing and the top of the mountain is 1,386 meters. Be careful—this is not the answer! The question asks how tall Mount Fuji is, so don't forget to add the vertical distance she drove, 2,390 meters, to get $2,390 + 1,386 = 3,776$ meters.

9. Calculator



The Lake Pontchartrain Causeway Bridge in Louisiana is the longest bridge in the United States, at 23.83 miles long. The histogram above shows the distribution of the lengths, in miles, of 28 of the longest bridges in the United States, including Lake Pontchartrain Causeway Bridge. Which of the following could be the median length of the 28 bridges represented in the histogram?

- (A) 5.9
- (B) 7.9
- (C) 9.2
- (D) 9.9

Difficulty: Medium

Category: Problem Solving and Data Analysis / Statistics and Probability

Strategic Advice: The *median* of a data set is the middle value when the data points are arranged from least to greatest (or greatest to least). When there is an even number of data points, the median is the average of the two middle values.

Getting to the Answer: The histogram represents the lengths of 28 bridges, so the median length is the average of the 14th and 15th longest bridges. Because the number of bridges that are less than 6 miles long is 12, and the number of bridges that are less than 9 miles long is $12 + 8 = 20$, the median length of the 28 bridges must be between 6 and 9 miles (because 14 and 15 lie between 12 and 20). Of the choices given, only (B) matches this criterion.

10. Calculator

In the United States, the original full retirement age was 65. The retirement age has since been pushed to 66 and will soon move to 67, as life expectancies go up. The Social Security Administration periodically conducts studies regarding retirement age. One such study focused on whether or not retiring early lowers a person's life expectancy. The study found a weak positive correlation between retirement age and life expectancy. If data from the study were graphed in a scatterplot, which of the following statements would be true?

- (A) The slope of the line of best fit would be a large positive number.
- (B) The slope of the line of best fit would be a negative number close to 0.
- (C) The data points would follow, but not closely, an increasing line of best fit.
- (D) The data points would be closely gathered around an increasing line of best fit.

Difficulty: Medium

Category: Problem Solving and Data Analysis / Scatterplots

Strategic Advice: It's a good idea to get comfortable with the vocabulary used in statistics questions. *Correlation* simply means relationship. The word *weak* refers to the strength of the relationship, which has no effect on slope, but rather on how closely the data points follow the line of best fit.

Getting to the Answer: Be careful not to confuse slope and strength. Simply because a data set shows a weak correlation does not mean the slope will be close to zero. The data can still be gathered around a steep line of best fit. So, you can eliminate A and B. Also, keep in mind that the terms *weak* and *positive* are not related but rather are two independent descriptors of the correlation. So, the fact that the rate of change is positive has nothing to do with the strength of the correlation. In a weak correlation, the data points will follow the line of best fit, but not as closely as in a strong correlation, which means (C) is correct.