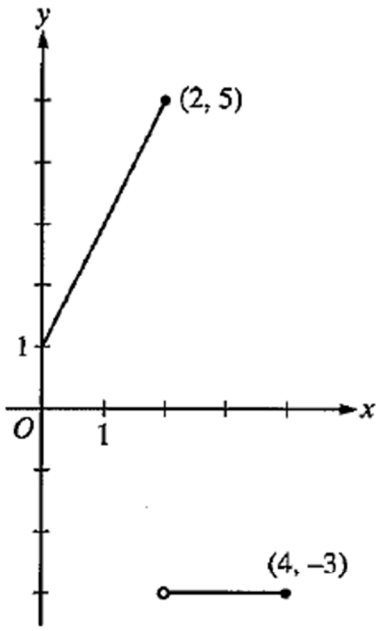
1. If $y = \sin^3 x$, then $\frac{dy}{dx} =$

(A) $\cos^3 x$ (B) $3\cos^2 x$ (C) $3\sin^2 x$ (D) $-3\sin^2 x \cos x$ (E) $3\sin^2 x \cos x$

2. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest?

(A) $(-4, 12)$ (B) $(-3, 6)$ (C) $(-2, 9)$ (D) $(0, 0)$ (E) $(3, 4)$



Graph of f

3. The graph of f is shown above for $0 \le x \le 4$. What is the value of $\int_0^4 f(x)\,dx$?

(A) -1 (B) 0 (C) 2 (D) 6 (E) 12

4. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}}\,dx$

(B) $\int_1^2 \left(1 + \frac{1}{x^2}\right) dx$

(C) $\int_1^2 \sqrt{1 + e^{2x}}\,dx$

(D) $\int_1^2 \sqrt{1 + (\ln x)^2}\,dx$

(E) $\int_1^2 \left(1 + (\ln x)^2\right) dx$

5. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

(A) -3 (B) $-\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{13}{16}$ (E) 4

6. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x\left(x^2 - 3\right)^5 \, dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5 \, du$

(B) $\int_{-2}^{13} u^5 \, du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5 \, du$

(D) $\int_{-1}^{4} u^5 \, du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5 \, du$

7. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

(A) $\frac{y}{\sqrt{1-x^2}}$

(B) $\frac{xy}{\sqrt{1-x^2}}$

(C) $\frac{y}{1+x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1+x^2}$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

10. $\int_1^4 t^{-3/2}\, dt =$

(A) -1 (B) $-\frac{7}{8}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 1

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

12. The points $(-1, -1)$ and $(1, -5)$ are on the graph of a function $y = f(x)$ that satisfies the differential equation $\frac{dy}{dx} = x^2 + y$. Which of the following must be true?

(A) $(1, -5)$ is a local maximum of f.

(B) $(1, -5)$ is a point of inflection of the graph of f.

(C) $(-1, -1)$ is a local maximum of f.

(D) $(-1, -1)$ is a local minimum of f.

(E) $(-1, -1)$ is a point of inflection of the graph of f.

13. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

(A) $2\sqrt{3}$ (B) 3 (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) 0

14. Let k be a positive constant. Which of the following is a logistic differential equation?

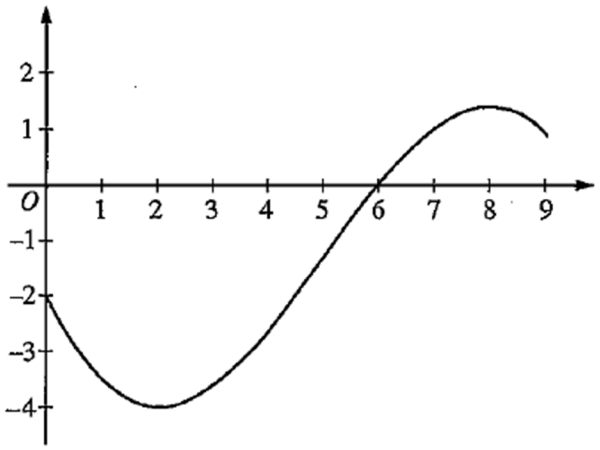
(A) $\frac{dy}{dt} = kt$

(B) $\frac{dy}{dt} = ky$

(C) $\frac{dy}{dt} = kt(1-t)$

(D) $\frac{dy}{dt} = ky(1-t)$

(E) $\frac{dy}{dt} = ky(1-y)$



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

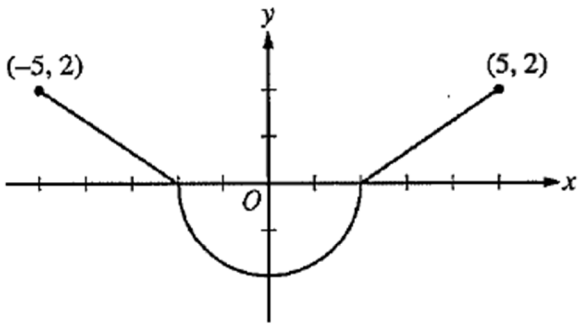
(E) $h''(6) < h'(6) < h(6)$

16. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

(A) $-\frac{5}{4}$ (B) 1 (C) $\frac{7}{4}$ (D) 2 (E) $\frac{21}{4}$

17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \cdots$ converges to which of the following?

(A) $\cos x$ (B) $\sin x$ (C) $\frac{\sin x}{x}$ (D) $e^x - e^{x^2}$ (E) $1 + e^x - e^{x^2}$



Graph of f'

18. The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2},\frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4,\frac{2}{3}\right)$

(E) There are no such points.

20. $\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is

(A) $\ln(8)$ (B) $\ln\left(\frac{27}{2}\right)$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \frac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \frac{1}{x-5}$ (D) $y = \frac{5x}{1-x}$ (E) $y = \frac{20x^2 - x}{1+4x^2}$

22. The power series $\sum_{n=0}^{\infty} a_n (x-3)^n$ converges at $x = 5$. Which of the following must be true?

(A) The series diverges at $x = 0$.

(B) The series diverges at $x = 1$.

(C) The series converges at $x = 1$.

(D) The series converges at $x = 2$.

(E) The series converges at $x = 6$.

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

24. Let f be a differentiable function such that $\int f(x)\sin x\,dx = -f(x)\cos x + \int 4x^3 \cos x\,dx$. Which of the following could be $f(x)$?

(A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ (E) x^4

25. $\int_1^{\infty} xe^{-x^2}dx$ is

(A) $-\frac{1}{e}$ (B) $\frac{1}{2e}$ (C) $\frac{1}{e}$ (D) $\frac{2}{e}$ (E) divergent

26. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

27. For what values of p will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converge?

(A) $-2 < p < 2$ only

(B) $-\frac{1}{2} < p < \frac{1}{2}$ only

(C) $\frac{1}{2} < p < 2$ only

(D) $p < \frac{1}{2}$ and $p > 2$

(E) There are no such values of p.

28. Let g be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \to 1} \frac{\int_1^x g(t)\,dt}{g(x) - 6}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) The limit does not exist.