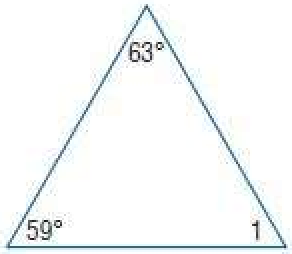
6-1 Angles of Triangles

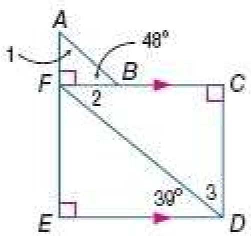
Find the measures of each numbered angle.

1. 

SOLUTION:

The sum of the measures of the angles of a triangle is 180. Let *x* be the measure of unknown angle in the figure.

$$x + 63 + 59 = 180 \quad \text{Triangle Angle-Sum Thm.}$$
$$x + 122 = 180 \quad \text{Add.}$$
$$x + 122 - 122 = 180 - 122 \quad -122 \text{ from each side.}$$
$$x = 58 \quad \text{Simplify.}$$

2. 

SOLUTION:

The sum of the measures of the angles of a triangle is 180. So, $m\angle 1 + 90 + 48 = 180$.

$$m\angle 1 + 90 + 48 = 180 \quad \text{Triangle Angle-Sum Thm.}$$
$$m\angle 1 + 138 = 180 \quad \text{Simplify.}$$
$$m\angle 1 + 138 - 138 = 180 - 138 \quad -138 \text{ from each side.}$$
$$m\angle 1 = 42 \quad \text{Simplify.}$$

In the figure, $m\angle 3 + 39 = 90.$

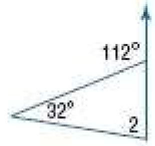
$$m\angle 3 + 39 - 39 = 90 - 39$$
$$m\angle 3 = 51$$

In the figure, $\angle 2$ and the angle measuring 39° are congruent.

So, $m\angle 2 = 39°.$

Find each measure.

3. $m\angle 2$

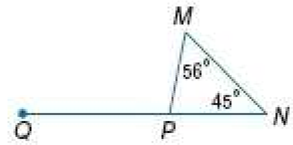


SOLUTION:

By the Exterior Angle Theorem, $m\angle 2 + 32 = 112$.

$$m\angle 2 + 32 = 112 \quad \text{Exterior Angle Thm.}$$
$$m\angle 2 + 32 - 32 = 112 - 32 \quad -32 \text{ from each side.}$$
$$m\angle 2 = 80 \quad \text{Simplify.}$$

4. $m\angle MPQ$



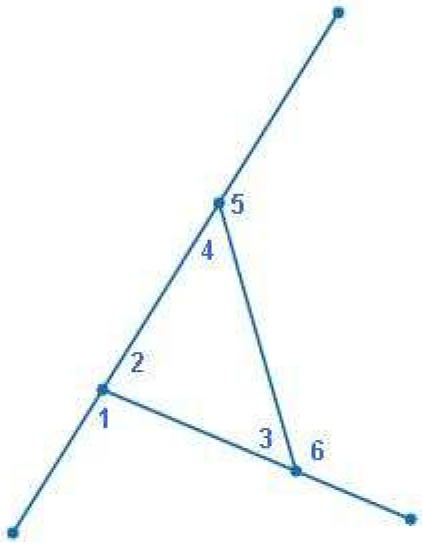
SOLUTION:

By the Exterior Angle Theorem,

$m\angle MPQ = 56 + 45.$

$m\angle MPQ = 101$

DECK CHAIRS The brace of this deck chair forms a triangle with the rest of the chair's frame as shown. If $m\angle 1 = 102$ and $m\angle 3 = 53$, find each measure.



5. $m\angle 4$

SOLUTION:

By the Exterior Angle Theorem, $m\angle 1 = m\angle 3 + m\angle 4$.

Substitute.

$102 = 53 + m\angle 4$	Exterior Angle Thm.
$102 - 53 = 53 + m\angle 4 - 53$	−53 from each side.
$m\angle 4 = 49$	Simplify.

6. $m\angle 6$

SOLUTION:

In the figure, $\angle 3$ and $\angle 6$ form a linear pair. So, $m\angle 3 + m\angle 6 = 180$.

$m\angle 3 + m\angle 6 = 180$	Def. of Linear Pair
$53 + m\angle 6 = 180$	Substitution.
$53 + m\angle 6 - 53 = 180 - 53$	−53 from each side.
$m\angle 6 = 127$	Simplify.

7. $m\angle 2$

SOLUTION:

By the Exterior Angle Theorem, $m\angle 1 = m\angle 3 + m\angle 4$.

Substitute.

$m\angle 1 = m\angle 3 + m\angle 4$	Exterior Angle Thm.
$102 = 53 + m\angle 4$	Substitution.
$102 - 53 = 53 + m\angle 4 - 53$	−53 from each side.
$m\angle 4 = 49$	Simplify.

The sum of the measures of the angles of a triangle is 180.

So, $m\angle 2 + m\angle 3 + m\angle 4 = 180$.

Substitute.

$m\angle 2 + m\angle 3 + m\angle 4 = 180$	Triangle Angle-Sum Thm.
$m\angle 2 + 53 + 49 = 180$	Substitution.
$m\angle 2 + 102 = 180$	Add.
$m\angle 2 + 102 - 102 = 180 - 102$	−102 from each side.
$m\angle 2 = 78$	Simplify.

8. $m\angle 5$

SOLUTION:

Angles 4 and 5 form a linear pair. Use the Exterior Angle Theorem to find $m\angle 4$ first and then use the fact that the sum of the measures of the two angles of a linear pair is 180.

By the Exterior Angle Theorem, $m\angle 1 = m\angle 3 + m\angle 4$.

Substitute.

$m\angle 1 = m\angle 3 + m\angle 4$	Exterior Angle Thm.
$102 = 53 + m\angle 4$	Substitution.
$102 - 53 = 53 + m\angle 4 - 53$	− 53 from each side.
$m\angle 4 = 49$	Simplify.

In the figure, $\angle 4$ and $\angle 5$ form a linear pair. So, $m\angle 4 + m\angle 5 = 180$.

$m\angle 4 + m\angle 5 = 180$	Def. of Linear Pair
$49 + m\angle 5 = 180$	Substitution.
$49 + m\angle 5 - 49 = 180 - 49$	−49 from each side.
$m\angle 5 = 131$	Simplify.

CCSS REGULARITY Find each measure.

9. $m\angle 1$

SOLUTION:

The sum of the measures of the angles of a triangle is 180. So, $m\angle 1 + 29 + 90 = 180$.

$$\begin{aligned} m\angle 1 + 29 + 90 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 1 + 119 &= 180 && \text{Simplify.} \\ m\angle 1 + 119 - 119 &= 180 - 119 && -119 \text{ from each side.} \\ m\angle 1 &= 61 && \text{Simplify.} \end{aligned}$$

10. $m\angle 3$

SOLUTION:

In the figure, $\angle 2$ and $29°$ angle form a linear pair. So, $m\angle 2 + 29 = 180$.

$$\begin{aligned} m\angle 2 + 29 &= 180 && \text{Def. of Linear Pair} \\ m\angle 2 + 29 - 29 &= 180 - 29 && -29 \text{ from each side.} \\ m\angle 2 &= 151 && \text{Simplify.} \end{aligned}$$

The sum of the measures of the angles of a triangle is 180. So, $m\angle 3 + m\angle 2 + 17 = 180$. Substitute.

$$\begin{aligned} m\angle 3 + m\angle 2 + 17 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 3 + 151 + 17 &= 180 && \text{Substitution} \\ m\angle 3 + 168 &= 180 && \text{Simplify.} \\ m\angle 3 &= 12 && -168 \text{ from each side.} \end{aligned}$$

11. $m\angle 2$

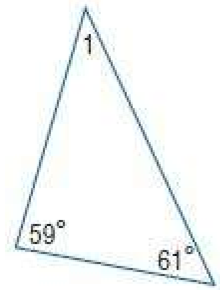
SOLUTION:

In the figure, $\angle 2$ and $29°$ angle form a linear pair. So, $m\angle 2 + 29 = 180$.

$$\begin{aligned} m\angle 2 + 29 &= 180 && \text{Def. of Linear Pair} \\ m\angle 2 + 29 - 29 &= 180 - 29 && -29 \text{ from each side.} \\ m\angle 2 &= 151 && \text{Simplify.} \end{aligned}$$

Find the measure of each numbered angle.

12.



SOLUTION:

The sum of the measures of the angles of a triangle is 180. So, $m\angle 1 + 59 + 61 = 180$.

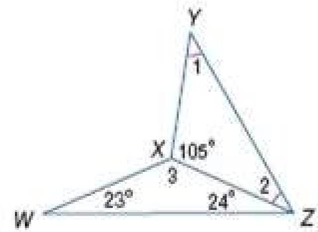
$$\begin{aligned} m\angle 1 + 59 + 61 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 1 + 120 &= 180 && \text{Simplify.} \\ m\angle 1 + 120 - 120 &= 180 - 120 && -120 \text{ from each side.} \\ m\angle 1 &= 60 && \text{Simplify.} \end{aligned}$$

13.

SOLUTION:

The sum of the measures of the angles of a triangle is 180. So, $m\angle 1 + 120 + 30 = 180$.

$$\begin{aligned} m\angle 1 + 120 + 30 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 1 + 150 &= 180 && \text{Simplify.} \\ m\angle 1 + 150 - 150 &= 180 - 150 && -150 \text{ from each side.} \\ m\angle 1 &= 30 && \text{Simplify.} \end{aligned}$$

14. 

SOLUTION:

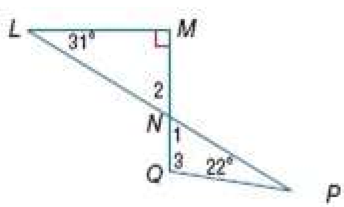
The sum of the measures of the angles of a triangle is 180. In $\triangle XWZ$, $m\angle 3 + 23 + 24 = 180$.

$$\begin{aligned} m\angle 3 + 23 + 24 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 3 + 47 &= 180 && \text{Simplify.} \\ m\angle 3 + 47 - 47 &= 180 - 47 && \text{-47 from each side.} \\ m\angle 3 &= 133 && \text{Simplify.} \end{aligned}$$

Here, $\angle 1$ and $\angle 2$ are congruent angles. By the definition of congruence, $m\angle 1 = m\angle 2$.
In $\triangle XYZ$, $m\angle 1 + m\angle 2 + 105 = 180$.

$$\begin{aligned} m\angle 1 + m\angle 2 + 105 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 1 + m\angle 1 + 105 &= 180 && \text{Substitution.} \\ 2m\angle 1 + 105 &= 180 && \text{Simplify.} \\ 2m\angle 1 + 105 - 105 &= 180 - 105 && \text{-105 from each side.} \\ 2m\angle 1 &= 75 && \text{Simplify.} \\ m\angle 1 &= 37.5 && \text{Divide each side by 2.} \end{aligned}$$

Since $m\angle 1 = 37.5$, $m\angle 2 = 37.5$.

15. 

SOLUTION:

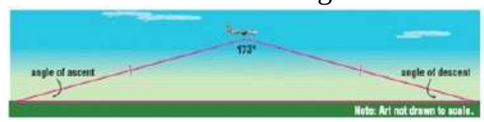
The sum of the measures of the angles of a triangle is 180. In $\triangle LMN$, $m\angle 2 + 90 + 31 = 180$.

$$\begin{aligned} m\angle 2 + 90 + 31 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 2 + 121 &= 180 && \text{Simplify.} \\ m\angle 2 + 121 - 121 &= 180 - 121 && \text{-121 from each side.} \\ m\angle 2 &= 59 && \text{Simplify.} \end{aligned}$$

In the figure, $\angle 2$ and $\angle 1$ are vertical angles. Since vertical angles are congruent, $m\angle 2 = m\angle 1 = 59$.
In $\triangle PQN$, $m\angle 1 + m\angle 3 + 22 = 180$.
Substitute.

$$\begin{aligned} m\angle 1 + m\angle 3 + 22 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 2 + m\angle 3 + 22 &= 180 && \text{Def. of Vertical Angles} \\ 59 + m\angle 3 + 22 &= 180 && \text{Substitution.} \\ m\angle 3 + 81 &= 180 && \text{Simplify.} \\ m\angle 3 + 81 - 81 &= 180 - 81 && \text{-81 from each side.} \\ m\angle 3 &= 99 && \text{Simplify.} \end{aligned}$$

16. **AIRPLANES** The path of an airplane can be modeled using two sides of a triangle as shown. The distance covered during the plane's ascent is equal to the distance covered during its descent.



a. Classify the model using its sides and angles.

b. The angles of ascent and descent are congruent. Find their measures.

SOLUTION:

a. The triangle has two congruent sides. So, it is isosceles. One angle of the triangle measures 173, so it is a obtuse angle. Since the triangle has an obtuse angle, it is an obtuse triangle.

b. Let *x* be the angle measure of ascent and descent. We know that
the sum of the measures of the angles of a triangle is 180. So, $x + x + 173 = 180$.

$$\begin{aligned} x + x + 173 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 2x + 173 &= 180 && \text{Simplify.} \\ 2x + 173 - 173 &= 180 - 173 && \text{-173 from each side.} \\ 2x &= 7 && \text{Simplify.} \\ x &= 3.5 && \text{Divide each side by 2.} \end{aligned}$$

The angle of ascent is 3.5 and the angle of descent is 3.5.

Find each measure.

17. $m\angle 1$

SOLUTION:

By the Exterior Angle Theorem, $m\angle 1 = 27 + 52$.
Find $m\angle 1$.

$$\begin{aligned} m\angle 1 &= 27 + 52 && \text{Exterior Angle Thm.} \\ &= 79 && \text{Add.} \end{aligned}$$

18. $m\angle 3$

SOLUTION:

By the Exterior Angle Theorem, $m\angle 3 = 43 + 22.$

Find $m\angle 3$.

$m\angle 3 = 43 + 22$ Exterior Angle Thm.

$= 65$ Simplify.

19. $m\angle 2$

SOLUTION:

By the Exterior Angle Theorem, $92 = m\angle 2 + 71.$

Solve for $m\angle 2$.

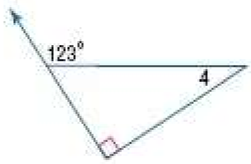
$92 = m\angle 2 + 71$ Exterior Angle Thm.

$92 - 71 = m\angle 2 + 71 - 71$ –71 from each side.

$21 = m\angle 2$ Simplify.

That is, $m\angle 2 = 21.$

20. $m\angle 4$



SOLUTION:

By the Exterior Angle Theorem, $123 = m\angle 4 + 90.$

Solve for $m\angle 4$.

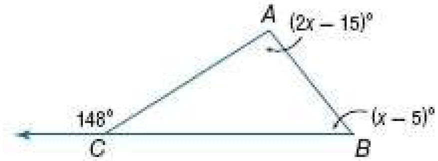
$123 = m\angle 4 + 90$ Exterior Angle Thm.

$123 - 90 = m\angle 4 + 90 - 90$ –90 from each side.

$33 = m\angle 4$ Simplify.

That is, $m\angle 4 = 33.$

21. $m\angle ABC$



SOLUTION:

By the Exterior Angle Theorem,

$148 = 2x - 15 + x - 5.$

Find x.

$148 = 2x - 15 + x - 5$ Exterior Angle Thm.

$148 = 3x - 20$ Simplify.

$148 + 20 = 3x - 20 + 20$ +20 from each side.

$168 = 3x$ Simplify.

$56 = x$ ÷ each side by 3.

That is, $x = 56$.

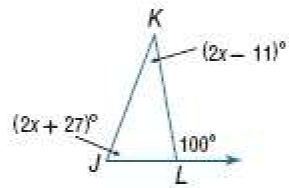
Substitute $x = 56$ in $m\angle ABC.$

$m\angle ABC = x - 5$

$= 56 - 5$

$= 51$

22. $m\angle JKL$



SOLUTION:

By the Exterior Angle Theorem,

$100 = 2x - 11 + 2x + 27$.

Find x.

$100 = 2x - 11 + 2x + 27$ Exterior Angle Theorem

$100 = 4x + 16$ Simplify.

$100 - 16 = 4x + 16 - 16$ –16 from each side.

$84 = 4x$ Simplify.

$21 = x$ ÷ each side by 4.

That is, $x = 21$.

Substitute $x = 21$ in $m\angle JKL.$

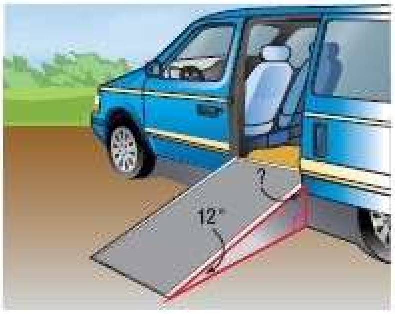
$m\angle JKL = 2x - 11$

$= 2(21) - 11$ $x = 21$

$= 42 - 11$ Multiply.

$= 31$ Subtract.

23. **WHEELCHAIR RAMP** Suppose the wheelchair ramp shown makes a 12° angle with the ground. What is the measure of the angle the ramp makes with the van door?



SOLUTION:

The sum of the measures of the angles of a triangle is 180.

Let *x* be the measure of the angle the ramp makes with the van door.

$$\begin{aligned} x + 90 + 12 &= 180 && \text{Triangle Angle-Sum Thm.} \\ x + 102 &= 180 && \text{Simplify.} \\ x + 102 - 102 &= 180 - 102 && -102 \text{ from each side.} \\ x &= 78 && \text{Simplify.} \end{aligned}$$

CCSS REGULARITY Find each measure.

24. $m\angle 1$

SOLUTION:

The sum of the measures of the angles of a triangle is 180.

In the figure, $m\angle 1 + 90 + 28 = 180$.

$$\begin{aligned} m\angle 1 + 90 + 28 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 1 + 118 &= 180 && \text{Simplify.} \\ m\angle 1 + 118 - 118 &= 180 - 118 && -118 \text{ from each side.} \\ m\angle 1 &= 62 && \text{Simplify.} \end{aligned}$$

25. $m\angle 2$

SOLUTION:

The sum of the measures of the angles of a triangle is 180.

In the figure, $m\angle 2 + 90 + 51 = 180$.

$$\begin{aligned} m\angle 2 + 90 + 51 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 2 + 141 &= 180 && \text{Simplify.} \\ m\angle 2 + 134 - 141 &= 180 - 141 && -141 \text{ from each side.} \\ m\angle 2 &= 39 && \text{Simplify.} \end{aligned}$$

26. $m\angle 3$

SOLUTION:

By the Exterior Angle Theorem, $51 = m\angle 3 + 25$.

$$\begin{aligned} 51 &= m\angle 3 + 25 && \text{Exterior Angle Thm.} \\ 51 - 25 &= m\angle 3 + 25 - 25 && -25 \text{ from each side.} \\ 26 &= m\angle 3 && \text{Simplify.} \end{aligned}$$

That is, $m\angle 3 = 26$.

27. $m\angle 4$

SOLUTION:

In the figure, $m\angle 5 + m\angle 6 = 90$.

The sum of the measures of the angles of a triangle is 180.

In the figure, $m\angle 5 + m\angle 6 + 35 + m\angle 4 = 180$.

Substitute.

$$\begin{aligned} m\angle 5 + m\angle 6 + 35 + m\angle 4 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 90 + 35 + m\angle 4 &= 180 && \text{Substitute.} \\ 125 + m\angle 4 &= 180 && \text{Simplify.} \\ 125 + m\angle 4 - 125 &= 180 - 125 && -125 \text{ from each side.} \\ m\angle 4 &= 55 && \text{Simplify.} \end{aligned}$$

28. $m\angle 5$

SOLUTION:

The sum of the measures of the angles of a triangle is 180.

In the figure, $m\angle 5 + 90 + 35 = 180$.

$$\begin{aligned} 90 + 35 + m\angle 5 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 125 + m\angle 5 &= 180 && \text{Simplify.} \\ 125 - 125 + m\angle 5 &= 180 - 125 && -125 \text{ from each side.} \\ m\angle 5 &= 55 && \text{Simplify.} \end{aligned}$$

29. $m\angle 6$

SOLUTION:

The sum of the measures of the angles of a triangle is 180.

In the figure, $m\angle 5 + 90 + 35 = 180$.

$$\begin{aligned} 90 + 35 + m\angle 5 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 125 + m\angle 5 &= 180 && \text{Simplify.} \\ 125 + m\angle 5 - 125 &= 180 - 125 && -125 \text{ from each side.} \\ m\angle 5 &= 55 && \text{Simplify.} \end{aligned}$$

In the figure, $m\angle 5 + m\angle 6 = 90$.

Substitute.

$$\begin{aligned} 55 + m\angle 6 &= 90 && \text{Def. of complementary angles} \\ 55 + m\angle 6 - 55 &= 90 - 55 && -55 \text{ from each side.} \\ m\angle 6 &= 35 && \text{Simplify.} \end{aligned}$$

ALGEBRA Find the value of *x*. Then find the measure of each angle.

30. 

SOLUTION:

The sum of the measures of the angles of a triangle is 180.

In the figure, $2x + 4x + 3x = 180$.

Solve for *x*.

$$\begin{aligned} 2x + 4x + 3x &= 180 && \text{Triangle Angle-Sum Thm.} \\ 9x &= 180 && \text{Simplify.} \\ x &= 20 && \div \text{ each side by 9.} \end{aligned}$$

Substitute $x = 20$ in each measure.

$$\begin{aligned} 2x &= 2(20) \\ &= 40 \end{aligned}$$
$$\begin{aligned} 3x &= 3(20) \\ &= 60 \end{aligned}$$
$$\begin{aligned} 4x &= 4(20) \\ &= 80 \end{aligned}$$

31. 

SOLUTION:

The sum of the measures of the angles of a triangle is 180.

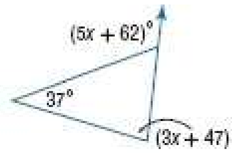
In the figure, $2x + x + 90 = 180$.

Solve for *x*.

$$\begin{aligned} 2x + x + 90 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 3x + 90 &= 180 && \text{Simplify.} \\ 3x + 90 - 90 &= 180 - 90 && -90 \text{ from each side.} \\ 3x &= 90 && \text{Simplify.} \\ x &= 30 && \div \text{ each side by 3.} \end{aligned}$$

Substitute $x = 30$ in $2x$.

$$\begin{aligned} 2x &= 2(30) \\ &= 60 \end{aligned}$$

32. 

SOLUTION:

By the Exterior Angle Theorem, $5x + 62 = 37 + 3x + 47$.

Solve for *x*.

$$\begin{aligned} 5x + 62 &= 37 + 3x + 47 && \text{Exterior Angle Thm.} \\ 5x + 62 &= 84 + 3x && \text{Simplify.} \\ 5x + 62 - 3x &= 84 + 3x - 3x && -3x \text{ from each side.} \\ 2x + 62 &= 84 && \text{Simplify.} \\ 2x + 62 - 62 &= 84 - 62 && -62 \text{ from each side.} \\ 2x &= 22 && \text{Simplify.} \\ x &= 11 && \text{Divide each side by 2.} \end{aligned}$$

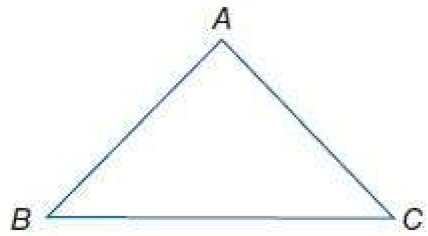
Substitute $x = 11$ in $5x + 62$.

$$\begin{aligned} 5x + 62 &= 5(11) + 62 \\ &= 55 + 62 \\ &= 117 \end{aligned}$$

Substitute $x = 11$ in $3x + 47$.

$$\begin{aligned} 3x + 47 &= 3(11) + 47 \\ &= 33 + 47 \\ &= 80 \end{aligned}$$

33. **GARDENING** A landscaper is forming an isosceles triangle in a flowerbed using chrysanthemums. She wants $m\angle A$ to be three times the measure of $\angle B$ and $\angle C$. What should the measure of each angle be?



SOLUTION:

$m\angle B = m\angle C$ and $m\angle A = 3m\angle C$

The sum of the measures of the angles of a triangle is 180. In the figure, $m\angle A + m\angle B + m\angle C = 180$.

Substitute.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180 && \text{Triangle Angle-Sum Thm.} \\ 3m\angle C + m\angle C + m\angle C &= 180 && \text{Substitute.} \\ 5m\angle C &= 180 && \text{Simplify.} \\ m\angle C &= 36 && \div \text{ each side by 5.} \end{aligned}$$

Since $m\angle C = 36$, $m\angle B = 36$.

Substitute $m\angle C = 36$ in $m\angle A = 3m\angle C$.

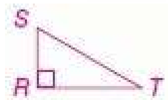
$$\begin{aligned} m\angle A &= 3(36) \\ &= 108 \end{aligned}$$

PROOF Write the specified type of proof.

34. Flow proof of Corollary 6.1

SOLUTION:

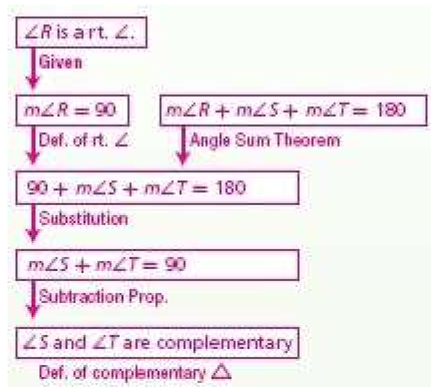
Given: ΔRST



$\angle R$ is a right angle.

Prove: $\angle S$ and $\angle T$ are complementary.

Proof:



35. Paragraph proof of Corollary 6.2

SOLUTION:

Given: ΔMNO

$\angle M$ is a right angle.

Prove: There can be at most one right angle in a triangle.

Proof: In ΔMNO, $\angle M$ is a right angle. $m\angle M + m\angle N + m\angle O = 180$. $m\angle M = 90$, so $m\angle N + m\angle O = 90$.

If $\angle N$ were a right angle, then $m\angle O = 0$. But that is impossible, so there cannot be two right angles in a triangle.

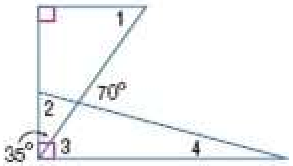
Given: ΔPQR

$\angle P$ is obtuse.

Prove: There can be at most one obtuse angle in a triangle.

Proof: In ΔPQR, $\angle P$ is obtuse. So $m\angle P > 90$. $m\angle P + m\angle Q + m\angle R = 180$. It must be that $m\angle Q + m\angle R < 90$. So, $\angle Q$ and $\angle R$ must be acute.

CCSS REGULARITY Find the measure of each numbered angle.

36. 

SOLUTION:

The sum of the measures of the angles of a triangle is 180.

In the figure, $m\angle 1 + 35 + 90 = 180$.

$$\begin{aligned} m\angle 1 + 35 + 90 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 1 + 125 &= 180 && \text{Simplify.} \\ m\angle 1 + 125 - 125 &= 180 - 125 && -125 \text{ from each side.} \\ m\angle 1 &= 55 && \text{Simplify.} \end{aligned}$$

In the figure, $m\angle 3 + 35 = 90$.

Solve for $m\angle 3$.

$$\begin{aligned} m\angle 3 + 35 &= 90 \\ m\angle 3 + 35 - 35 &= 90 - 35 \\ m\angle 3 &= 55 \end{aligned}$$

By the Exterior Angle Theorem, $m\angle 3 + m\angle 4 = 70$.

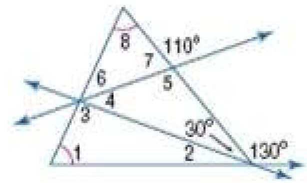
Substitute.

$$\begin{aligned} 55 + m\angle 4 &= 70 && \text{Exterior Angle Thm.} \\ 55 + m\angle 4 - 55 &= 70 - 55 && -55 \text{ from each side.} \\ m\angle 4 &= 15 && \text{Simplify.} \end{aligned}$$

Also, $m\angle 2 + m\angle 4 + 90 = 180$.

Substitute.

$$\begin{aligned} m\angle 2 + 15 + 90 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 2 + 105 &= 180 && \text{Simplify.} \\ m\angle 2 + 105 - 105 &= 180 - 105 && -105 \text{ from each side.} \\ m\angle 2 &= 75 && \text{Simplify.} \end{aligned}$$

37. 

SOLUTION:

Look for pairs of vertical angles first. Here, $110°$ angle and $\angle 5$ are vertical angles, since $110°$ angle and $\angle 5$ are vertical angles, they are congruent. By the definition of congruence, $m\angle 5 = 110$.

Look for linear pairs next. Angles 5 and 7 are a linear pair. Since $m\angle 5 = 110$, $m\angle 7 = 180 - 110$ or 70.

Next, the Triangle Angle Sum theorem can be used to find $m\angle 4$.

$$\begin{aligned} 30 + m\angle 4 + m\angle 5 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 30 + m\angle 4 + 110 &= 180 && \text{Substitution.} \\ m\angle 4 + 140 &= 180 && \text{Simplify.} \\ m\angle 4 + 140 - 140 &= 180 - 140 && -140 \text{ from each side.} \\ m\angle 4 &= 40 && \text{Simplify.} \end{aligned}$$

From the diagram, $\angle 1 \cong \angle 8$. By the Exterior Angle Theorem, $m\angle 1 + m\angle 8 = 130$.

Since congruent angles have equal measure, $m\angle 1 = m\angle 8 = \frac{130}{2} = 65$.

Using the Triangle Angle Sum Theorem we know that $m\angle 6 + m\angle 7 + m\angle 8 = 180$.

$$\begin{aligned} m\angle 6 + m\angle 7 + m\angle 8 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 6 + 70 + 65 &= 180 && \text{Substitute.} \\ m\angle 6 + 135 &= 180 && \text{Simplify.} \\ m\angle 6 &= 45 && -35 \text{ from each side.} \end{aligned}$$

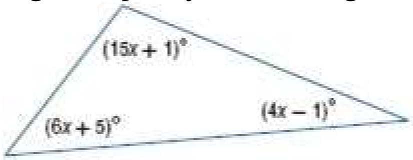
Using the Triangle Angle Sum Theorem we know that $m\angle 3 + m\angle 4 + m\angle 6 = 180$.

$$\begin{aligned} m\angle 3 + m\angle 4 + m\angle 6 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 3 + 40 + 45 &= 180 && \text{Substitute.} \\ m\angle 3 + 85 &= 180 && \text{Simplify.} \\ m\angle 3 &= 95 && -95 \text{ from each side.} \end{aligned}$$

Using the Triangle Angle Sum Theorem we know that $m\angle 1 + m\angle 2 + m\angle 3 = 180$.

$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle 3 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 65 + m\angle 2 + 95 &= 180 && \text{Substitute.} \\ m\angle 2 + 160 &= 180 && \text{Simplify.} \\ m\angle 2 &= 20 && -160 \text{ from each side.} \end{aligned}$$

38. **ALGEBRA** Classify the triangle shown by its angles. Explain your reasoning.



SOLUTION:

Obtuse; the sum of the measures of the three angles of a triangle is 180. So, $(15x + 1) + (6x + 5) + (4x - 1) = 180$ and $x = 7$. Substituting 7 into the expressions for each angle, the angle measures are 106, 47, and 27. Since the triangle has an obtuse angle, it is obtuse.

39. **ALGEBRA** The measure of the larger acute angle in a right triangle is two degrees less than three times the measure of the smaller acute angle. Find the measure of each angle.

SOLUTION:

Let x and y be the measure of the larger and smaller acute angles in a right triangle respectively. Given that $x = 3y - 2$. The sum of the measures of the angles of a triangle is 180.
So, $90 + x + y = 180.$

Substitute.

$90 + 3y - 2 + y = 180$	Triangle Angle-Sum Thm.
$88 + 4y = 180$	Simplify.
$88 + 4y - 88 = 180 - 88$	−88 from each side.
$4y = 92$	Simplify.
$y = 23$	Divide each side by 4.

Substitute $y = 23$ in $x = 3y - 2$.

$$x = 3(23) - 2$$
$$= 69 - 2$$
$$= 67$$

Thus the measure of the larger acute angle is 67 and the measure of the smaller acute angle is 23.

40. Determine whether the following statement is *true* or *false*. If false, give a counterexample. If true, give an argument to support your conclusion.
If the sum of two acute angles of a triangle is greater than 90, then the triangle is acute.

SOLUTION:

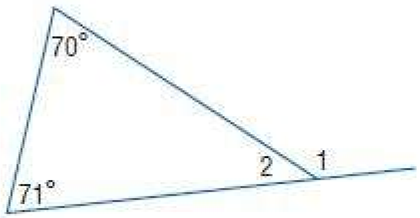
True; sample answer: Since the sum of the two acute angles is greater than 90, the measure of the third angle is a number greater than 90 subtracted from 180, which must be less than 90. Therefore, the triangle has three acute angles and is acute.

41. **ALGEBRA** In ΔXYZ, $m\angle X = 157$, $m\angle Y = y$, and $m\angle Z = z$. Write an inequality to describe the possible measures of $\angle Z$. Explain your reasoning.

SOLUTION:

$z < 23$; Sample answer: Since the sum of the measures of the angles of a triangle is 180 and $m\angle X = 157$, $m\angle X + m\angle Y + m\angle Z = 180$, so $m\angle Y + m\angle Z = 23.$ If $m\angle Y$ was 0, then $m\angle Z$ would equal 23. But since an angle must have a measure greater than 0, $m\angle Z$ must be less than 23, so $z < 23$.

42. **CARS**



a. Find $m\angle 1$ and $m\angle 2$.
b. If the support for the hood were shorter than the one shown, how would $m\angle 1$ change? Explain.
c. If the support for the hood were shorter than the one shown, how would $m\angle 2$ change? Explain.

SOLUTION:

a. By the Exterior Angle Theorem, $m\angle 1 = 70 + 71$. So, $m\angle 1 = 141$. In the figure, $m\angle 2 + 70 + 71 = 180$.

Solve for $m\angle 2$.

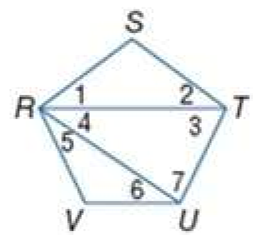
$$m\angle 2 + 70 + 71 = 180$$
$$m\angle 2 + 141 = 180$$
$$m\angle 2 + 141 - 141 = 80 - 141$$
$$m\angle 2 = 139$$

b. Sample answer: The measure of $\angle 1$ would get larger if the support were shorter because the hood would be closer to the leg of the triangle that is along the fender of the car.

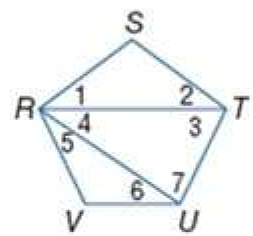
c. Sample answer: The measure of $\angle 2$ would get smaller if the support were shorter because $\angle 1$ would get larger and they are a linear pair.

PROOF Write the specified type of proof.

43. two-column proof
Given: *RSTUV* is a pentagon.
Prove: $m\angle S + m\angle STU + m\angle TUV + m\angle V + m\angle VRS = 540$



SOLUTION:

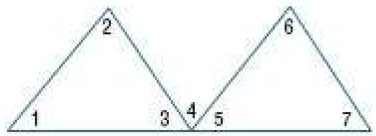


Proof: Statements (Reasons)
1. *RSTUV* is a pentagon. (Given)
2. $m\angle S + m\angle 1 + m\angle 2 = 180$; $m\angle 3 + m\angle 4 + m\angle 7 = 180$; $m\angle 6 + m\angle V + m\angle 5 = 180$ ($\angle$ Sum Thm.)
3. $m\angle S + m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 7 + m\angle 6 + m\angle V + m\angle 5 = 540$ (Add. Prop.)
4. $m\angle VRS = m\angle 1 + m\angle 4 + m\angle 5$; $m\angle TUV = m\angle 7 + m\angle 6$; $m\angle STU = m\angle 2 + m\angle 3$ ($\angle$ Addition)
5. $m\angle S + m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle STU + m\angle TUV + m\angle V + m\angle VRS = 540$ (Subst.)

44. flow proof

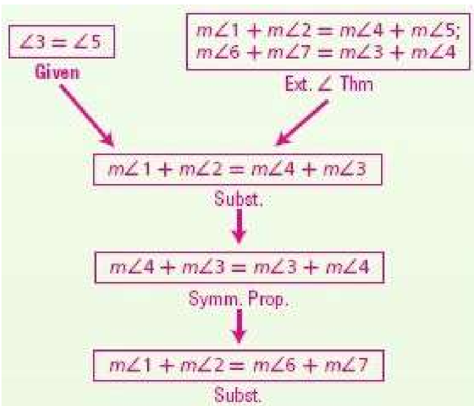
Given: $\angle 3 \cong \angle 5$

Prove: $m\angle 1 + m\angle 2 = m\angle 6 + m\angle 7$



SOLUTION:

Proof:



45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the sum of the measures of the exterior angles of a triangle.

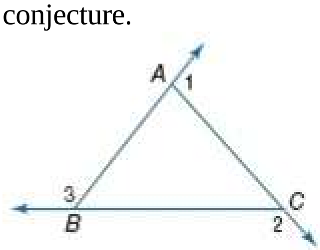
a. GEOMETRIC Draw five different triangles, extending the sides and labeling the angles as shown. Be sure to include at least one obtuse, one right, and one acute triangle.

b. TABULAR Measure the exterior angles of each triangle. Record the measures for each triangle and the sum of these measures in a table.

c. VERBAL Make a conjecture about the sum of the exterior angles of a triangle. State your conjecture using words.

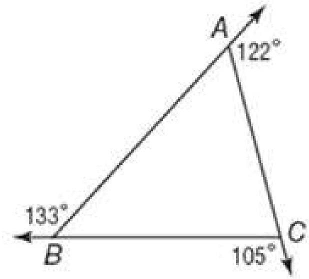
d. ALGEBRAIC State the conjecture you wrote in part *c* algebraically.

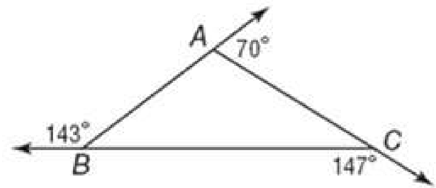
e. ANALYTICAL Write a paragraph proof of your conjecture.

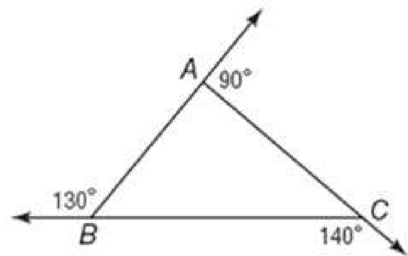


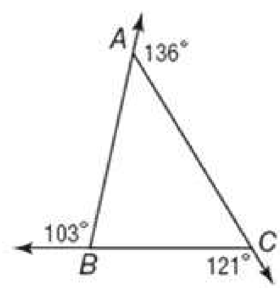
SOLUTION:

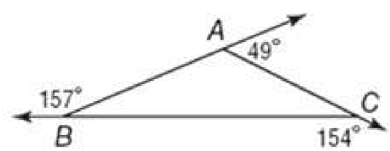
a. Sample answer:











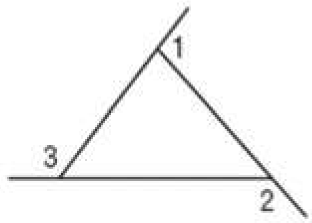
b. Sample answer:

∠1	∠2	∠3	Sum
122	105	133	360
70	147	143	360
90	140	130	360
136	121	103	360
49	154	157	360

c. Sample answer:

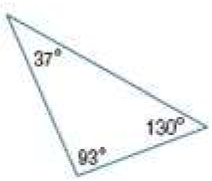
The sum of the measures of the exterior angles of a triangle is 360.

d. $m\angle 1 + m\angle 2 + m\angle 3 = 360$



e. The Exterior Angle Theorem tells us that $m\angle 3 = m\angle BAC + m\angle BCA$, $m\angle 2 = m\angle BAC + m\angle CBA$, $m\angle 1 = m\angle CBA + m\angle BCA$.
Through substitution,
$m\angle 1 + m\angle 2 + m\angle 3 = m\angle CBA + m\angle BCA + m\angle BAC + m\angle CBA + m\angle BAC + m\angle BCA$. Which can be simplified to $m\angle 1 + m\angle 2 + m\angle 3 = 2m\angle BAC + 2m\angle BCA + 2m\angle CBA$.
The Distributive Property can be applied and gives $m\angle 1 + m\angle 2 + m\angle 3 = 2(m\angle BAC + m\angle BCA + m\angle CBA)$. The Triangle Angle-Sum Theorem tells us that
$m\angle BAC + m\angle BCA + m\angle CBA = 180$. Through substitution we have $m\angle 1 + m\angle 2 + m\angle 3 = 2(180) = 360$.

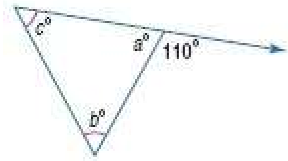
46. **CCSS CRITIQUE** Curtis measured and labeled the angles of the triangle as shown. Arnoldo says that at least one of his measures is incorrect. Explain in at least two different ways how Arnoldo knows that this is true.



SOLUTION:

Sample answer: Corollary 4.2 states that there can be at most one right or obtuse angle in a triangle. Since this triangle is labeled with two obtuse angle measures, 93 and 130, at least one of these measures must be incorrect. Since by the Triangle Angle Sum Theorem the sum of the interior angles of the triangle must be 180 and $37 + 93 + 130 \neq 180$, at least one of these measures must be incorrect.

47. **WRITING IN MATH** Explain how you would find the missing measures in the figure shown.



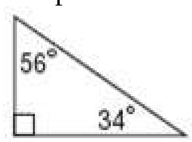
SOLUTION:

The measure of $\angle a$ is the supplement of the exterior angle with measure 110, so $m\angle a = 180 - 110$ or 70. Because the angles with measures b and c are congruent, $b = c$. Using the Exterior Angle Theorem, $b + c = 110$. By substitution, $b + b = 110$, so $2b = 110$ and $b = 55$. Because $b = c$, $c = 55$.

48. **OPEN ENDED** Construct a right triangle and measure one of the acute angles. Find the measure of the second acute angle using calculation and explain your method. Confirm your result using a protractor.

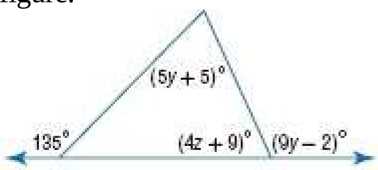
SOLUTION:

Sample answer:



I found the measure of the second angle by subtracting the first angle from 90° since the acute angles of a right triangle are complementary.

49. **CHALLENGE** Find the values of y and z in the figure.



SOLUTION:

In the figure, $(4z+9)+(9y-2)=180$ because they are a linear pair and $(5y+5)+(4z+9)=135$ because of the External Angle Theorem.
Simplify the equations and name them.

$$(4z+9)+(9y-2)=180$$
$$4z+9y+7=180$$
$$4z+9y=173 \rightarrow (1)$$
$$(5y+5)+(4z+9)=135$$
$$5y+4z+14=135$$
$$5y+4z=121 \rightarrow (2)$$

Subtract the equation (2) from (1).

$$4y=52$$
$$y=13$$

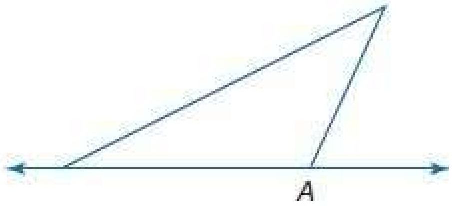
Substitute $y=13$ in (1).

$$4z+9(13)=173$$
$$4z+117=173$$
$$4z+117-117=173-117$$
$$4z=56$$
$$z=14$$

50. **REASONING** If an exterior angle adjacent to $\angle A$ is acute, is ΔABC acute, right, obtuse, or can its classification not be determined? Explain your reasoning.

SOLUTION:

Obtuse; since the exterior angle is acute, the sum of the remote interior angles must be acute, which means the third angle must be obtuse. Therefore, the triangle must be obtuse. Also, since the exterior angle forms a linear pair with $\angle A$, $\angle A$ must be obtuse since two acute angles cannot be a linear pair.



51. **WRITING IN MATH** Explain why a triangle cannot have an obtuse, acute, and a right exterior angle.

SOLUTION:

Sample answer: Since an exterior angle is acute, the adjacent angle must be obtuse. Since another exterior angle is right, the adjacent angle must be right. A triangle cannot contain both a right and an obtuse angle because it would be more than 180 degrees. Therefore, a triangle cannot have an obtuse, acute, and a right exterior angle.

52. **PROBABILITY** Mr. Glover owns a video store and wants to survey his customers to find what type of movies he should buy. Which of the following options would be the best way for Mr. Glover to get accurate survey results?

A surveying customers who come in from 9 p.m. until 10 p.m.
B surveying customers who come in on the weekend
C surveying the male customers
D surveying at different times of the week and day

SOLUTION:

The most accurate survey would ask a random sampling of customers. Choices A, B, and C each survey a specific group of customers. Choice D is a random sample of customers so it will give Mr. Glover the most accurate result.

53. **SHORT RESPONSE** Two angles of a triangle have measures of 35° and 80°. Find the values of the exterior angle measures of the triangle.

SOLUTION:

Sample answer: Since the sum of the measures of the angles of a triangle is 180, the measure of the third angle is 180 – (35 + 80) or 60. To find the measures of the exterior angles, subtract each angle measure from 180. The values for the exterior angle of the triangle are 100°, 115°, and 145°.

54. **ALGEBRA** Which equation is equivalent to $7x - 3(2 - 5x) = 8x$?

F $2x - 6 = 8$
G $22x - 6 = 8x$
H $-8x - 6 = 8x$
J $22x + 6 = 8x$

SOLUTION:

$7x - 3(2 - 5x) = 8x$	Original equation
$7x - 6 + 15x = 8x$	Distributive Property
$22x - 6 = 8x$	Simplify.

So, the correct option is G.

55. **SAT/ACT** Joey has 4 more video games than Solana and half as many as Melissa. If together they have 24 video games, how many does Melissa have?

A 7
B 9
C 12
D 13
E 14

SOLUTION:

Let j, s, and m be the number of video games with Joey, Solana, and Melissa respectively. Given that $j = s + 4$, $j = \frac{1}{2}m$, and $j + s + m = 24$.

Substitute $s = j - 4$ in $j + s + m = 24$.

$j + j - 4 + m = 24$

Substitute $j = \frac{1}{2}m$ in $j + s + m = 24$.

$$\frac{1}{2}m + \frac{1}{2}m - 4 + m = 24$$
$$\frac{1}{2}m + \frac{1}{2}m - 4 + m + 4 = 24 + 4$$
$$\frac{1}{2}m + \frac{1}{2}m + m = 28$$
$$2m = 28$$
$$m = 14$$

So, Melissa has 14 video games. The correct option is E.