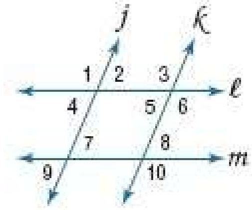
5-6 Proving Lines Parallel

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



1. $\angle 1 \cong \angle 3$

SOLUTION:

$\angle 1$ and $\angle 3$ are corresponding angles of lines *j* and *k*. Since $\angle 1 \cong \angle 3$, *j* || *k* by the Converse of Corresponding Angles Postulate.

2. $\angle 2 \cong \angle 5$

SOLUTION:

$\angle 2$ and $\angle 5$ are alternate interior angles of lines *j* and *k*. Since $\angle 2 \cong \angle 5$, *j* || *k* by the Converse of Alternate Interior Angles Theorem.

3. $\angle 3 \cong \angle 10$

SOLUTION:

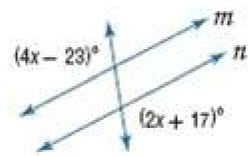
$\angle 3$ and $\angle 10$ are alternate exterior angles of lines ℓ and *m*. Since $\angle 3 \cong \angle 10$, ℓ || *m* by the Converse of Alternate Exterior Angles Theorem.

4. $m\angle 6 + m\angle 8 = 180$

SOLUTION:

$\angle 6$ and $\angle 8$ are consecutive interior angles of lines ℓ and *m*. Since $m\angle 3 + m\angle 8 = 180$, ℓ|| *m* by the Converse of Consecutive Interior Angles Theorem.

5. **SHORT RESPONSE** Find *x* so that *m* || *n*. Show your work.



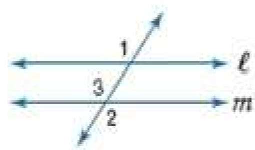
SOLUTION:

$(4x - 23)°$ angle and $(2x + 17)°$ angle are alternate exterior angles of lines *m* and *n*. Since *m* || *n*, $4x - 23 = 2x + 17$ by the Converse of Alternate Exterior Angles Theorem.

Solve for *x*.

$$4x - 23 = 2x + 17$$
$$4x - 2x - 23 = 2x - 2x + 17$$
$$2x - 23 = 17$$
$$2x - 23 + 23 = 17 + 23$$
$$2x = 40$$
$$\frac{2x}{2} = \frac{40}{2}$$
$$x = 20$$

6. **PROOF** Copy and complete the proof of Theorem 5.18.



Given: $\angle 1 \cong \angle 2$
Prove: $\ell \parallel m$
Proof:

Statements	Reasons
a. $\angle 1 \cong \angle 2$	**a.** Given
b. $\angle 2 \cong \angle 3$	**b.** ___?___
c. $\angle 1 \cong \angle 3$	**c.** Transitive Property
d. ___?___	**d.** ___?___

SOLUTION:

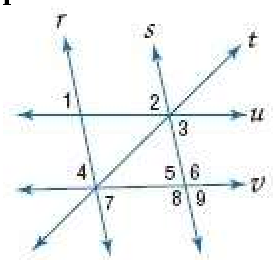
Statements	Reasons
a. $\angle 1 \cong \angle 2$	**a.** Given
b. $\angle 2 \cong \angle 3$	**b.** Vertical $\angle$s are $\cong$.
c. $\angle 1 \cong \angle 3$	**c.** Transitive Property
d. $\ell \parallel m$	**d.** If corr. $\angle$s are $\cong$, then lines are $\parallel$.

7. **RECREATION** Is it possible to prove that the backrest and footrest of the lounging beach chair are parallel? If so, explain how. If not, explain why not.

SOLUTION:

Sample answer: Yes; since the alternate exterior angles are congruent, the backrest and footrest are parallel.

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



8. $\angle 1 \cong \angle 2$

SOLUTION:

$\angle 1$ and $\angle 2$ are corresponding angles of lines *r* and *s*. Since $\angle 1 \cong \angle 2$,
r || *s* by the Converse of Corresponding Angles Postulate.

9. $\angle 2 \cong \angle 9$

SOLUTION:

$\angle 2$ and $\angle 9$ are alternate exterior angles of lines ℓ and *m*. Since $\angle 2 \cong \angle 9$, ℓ || *m* by the Converse of Alternate Exterior Angles Theorem.

10. $\angle 5 \cong \angle 7$

SOLUTION:

$\angle 5$ and $\angle 7$ are alternate interior angles of lines *r* and *s*. Since $\angle 5 \cong \angle 7$,
r || *s* by the Converse of Alternate Interior Angles Theorem.

11. $m\angle 7 + m\angle 8 = 180$

SOLUTION:

$\angle 7$ and $\angle 8$ are consecutive interior angles of lines *r* and *s*. Since $m\angle 7 + m\angle 8 = 180$,
r || *s* by the Converse of Consecutive Interior Angles Theorem.

12. $m\angle 3 + m\angle 6 = 180$

SOLUTION:

$\angle 3$ and $\angle 6$ are consecutive interior angles of lines *r* and *s*. Since $m\angle 3 + m\angle 6 = 180$,
r || *s* by the Converse of Consecutive Interior Angles Theorem.

13. $\angle 3 \cong \angle 5$

SOLUTION:

$\angle 3$ and $\angle 5$ are alternate interior angles of lines *u* and *v*. Since $\angle 3 \cong \angle 5$,
u || *v* by the Converse of Alternate Interior Angles Theorem.

14. $\angle 3 \cong \angle 7$

SOLUTION:

No lines can be proven parallel.

15. $\angle 4 \cong \angle 5$

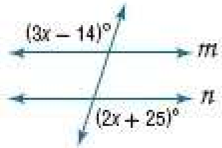
SOLUTION:

$\angle 4$ and $\angle 5$ are corresponding angles of lines *r* and *s*. Since $\angle 4 \cong \angle 5$,
r || *s* by the Converse of Corresponding Angles Postulate.

5-6 Proving Lines Parallel

Find *x* so that *m* || *n*. Identify the postulate or theorem you used.

16.



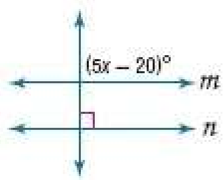
SOLUTION:

By the Alternate Exterior Angles Converse, if $3x - 14 = 2x + 25$, then $m \parallel n$.

Solve for x.

$$\begin{aligned} 3x - 14 &= 2x + 25 \\ 3x - 2x - 14 &= 2x - 2x + 25 \\ x - 14 + 14 &= 25 + 14 \\ x &= 39 \end{aligned}$$

17.



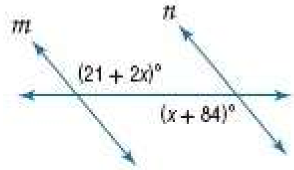
SOLUTION:

By the Converse of Corresponding Angles Postulate, if $5x - 20 = 90$, then $m \parallel n$.

Solve for x.

$$\begin{aligned} 5x - 20 &= 90 \\ 5x - 20 + 20 &= 90 + 20 \\ 5x &= 110 \\ \frac{5x}{5} &= \frac{110}{5} \\ x &= 22 \end{aligned}$$

18.



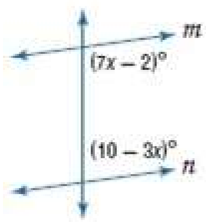
SOLUTION:

By the Alternate Interior Angles Converse, if $21 + 2x = x + 84$, then $m \parallel n$.

Solve for x.

$$\begin{aligned} 21 + 2x &= x + 84 \\ 21 + 2x - 21 &= x + 84 - 21 \\ 2x &= x + 63 \\ 2x - x &= x - x + 63 \\ x &= 63 \end{aligned}$$

19.

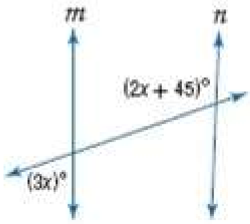


SOLUTION:

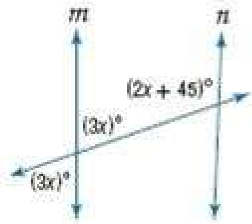
By the Consecutive Interior Angles Converse, if $7x - 2 + 10 - 3x = 180$, then $m \parallel n$.

Solve for x.

$$\begin{aligned} 7x - 2 + 10 - 3x &= 180 \\ 4x + 8 &= 180 \\ 4x + 8 - 8 &= 180 - 8 \\ 4x &= 172 \\ \frac{4x}{4} &= \frac{172}{4} \\ x &= 43 \end{aligned}$$

20. 

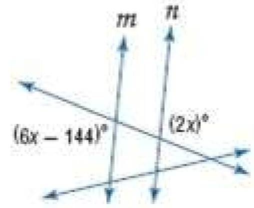
SOLUTION:
Use the Vertical Angle Theorem followed by Consecutive Interior Angles Converse to find *x*.



Then by Consecutive Interior Angles Converse, if $3x + 2x + 45 = 180$, then $m \parallel n$.

Solve for *x*.

$$
\begin{aligned}
3x + 2x + 45 &= 180 \\
6x + 45 &= 180 \\
6x + 45 - 45 &= 180 - 45 \\
6x &= 135 \\
\frac{6x}{6} &= \frac{135}{6} \\
x &= 27
\end{aligned}
$$

21. 

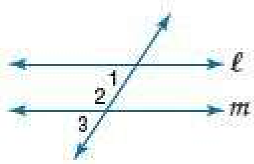
SOLUTION:
By the Alternate Exterior Angles Converse, if $6x - 144 = 2x$, then $m \parallel n$.
Solve for *x*.

$$
\begin{aligned}
6x - 144 &= 2x \\
6x - 144 - 2x &= 2x - 2x \\
4x - 144 &= 0 \\
4x - 144 + 144 &= 144 \\
4x &= 144 \\
\frac{4x}{4} &= \frac{144}{4} \\
x &= 36
\end{aligned}
$$

22. **CCSS SENSE-MAKING** Wooden picture frames are often constructed using a miter box or miter saw. These tools allow you to cut at an angle of a given size. If each of the four pieces of framing material is cut at a 45° angle, will the sides of the frame be parallel? Explain your reasoning.

SOLUTION:
Yes; when two pieces are put together, they form a 90° angle. Two lines that are perpendicular to the same line are parallel.

23. **PROOF** Copy and complete the proof of Theorem 5.19.



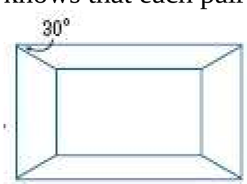
Given: $\angle 1$ and $\angle 2$ are supplementary.
Prove: $\ell \parallel m$

Statements	Reasons
a. ___?___	a. Given
b. $\angle 2$ and $\angle 3$ form a linear pair.	b. ___?___
c. ___?___	c. ___?___
d. $\angle 1 \cong \angle 3$	d. ___?___
e. $\ell \parallel m$	e. ___?___

SOLUTION:

Statements	Reasons
a. $\angle 1$ and $\angle 2$ are supplementary.	a. Given
b. $\angle 2$ and $\angle 3$ form a linear pair.	b. Def. of linear pair.
c. $\angle 2$ and $\angle 3$ are supplementary.	c. Suppl. Thm.
d. $\angle 1 \cong \angle 3$	d. $\cong$ Suppl. Thm.
e. $\ell \parallel m$	e. Converse of Corr. $\angle$s Post.

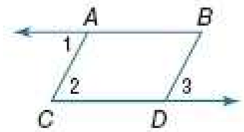
24. **CRAFTS** Jacqui is making a stained glass piece. She cuts the top and bottom pieces at a 30° angle. If the corners are right angles, explain how Jacqui knows that each pair of opposite sides are parallel.



SOLUTION:
Since the corners are right angles, each pair of opposite sides is perpendicular to the same line. Therefore, each pair of opposite sides is parallel.

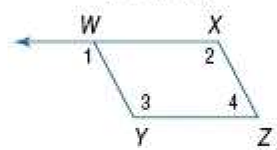
PROOF Write a two-column proof for each of the following.

25. **Given:** $\angle 1 \cong \angle 3$
$\overline{AC} \parallel \overline{BD}$
Prove: $\overline{AB} \parallel \overline{CD}$



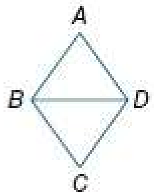
SOLUTION:
Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 3$, $\overline{AC} \parallel \overline{BD}$ (Given)
2. $\angle 2 \cong \angle 3$ (Corr. $\angle s$ postulate)
3. $\angle$ $\angle 1 \cong \angle 2$ (Trans. Prop.)
4. $\overline{AB} \parallel \overline{CD}$ (If alternate $\angle s$ are $\cong$, then lines are $\parallel$.)

26. **Given:** $\overline{WX} \parallel \overline{YZ}$
$\angle 2 \cong \angle 3$
Prove: $\overline{WY} \parallel \overline{XZ}$



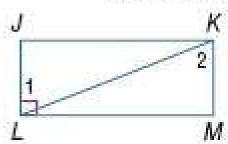
SOLUTION:
Proof:
Statements (Reasons)
1. $\overline{WX} \parallel \overline{YZ}$, $\angle 2 \cong \angle 3$ (Given)
2. $\angle 2$ and $\angle 4$ are supplementary. (Cons. Int. $\angle$s)
3. $m\angle 2 + m\angle 4 = 180$ (Def. of suppl. $\angle$s)
4. $m\angle 3 + m\angle 4 = 180$ (Substitution)
5. $\angle 3$ and $\angle 4$ are supplementary. (Def. of suppl. $\angle$s)
6. $\overline{WY} \parallel \overline{XZ}$ (If cons. int. $\angle$s are suppl., then lines are $\parallel$.)

27. **Given:** $\angle ABC \cong \angle ADC$
$m\angle A + m\angle ABC = 180$
Prove: $\overline{AB} \parallel \overline{CD}$



SOLUTION:
Proof:
Statements (Reasons)
1. $\angle ABC \cong \angle ADC$, $m\angle A + m\angle ABC = 180$ (Given)
2. $m\angle ABC = m\angle ADC$ (Def. of $\cong$ $\angle s$)
3. $m\angle A + m\angle ADC = 180$ (Substitution)
4. $\angle A$ and $\angle ABC$ are supplementary. (Def. of suppl. $\angle s$)
5. $\overline{AB} \parallel \overline{CD}$ (If consec. int. $\angle s$ are suppl., then lines are $\parallel$.)

28. **Given:** $\angle 1 \cong \angle 2$
$\overline{LJ} \perp \overline{ML}$
Prove: $\overline{KM} \perp \overline{ML}$



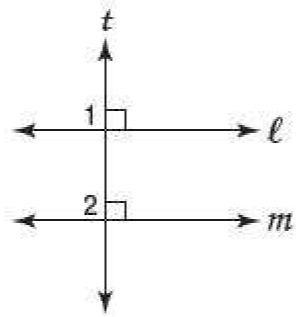
SOLUTION:
Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$, $\overline{LJ} \perp \overline{ML}$ (Given)
2. $\overline{LJ} \perp \overline{KM}$ (If alt. int. $\angle s$ are $\cong$, then lines are $\parallel$.)
3. $\overline{KM} \perp \overline{ML}$ (Perpendicular Transversal Theorem)

29. **MAILBOXES** Mail slots are used to make the organization and distribution of mail easier. In the mail slots shown, each slot is perpendicular to each of the sides. Explain why you can conclude that the slots are parallel.

SOLUTION:
The Converse of the Perpendicular Transversal Theorem states that two coplanar lines perpendicular to the same line are parallel. Since the slots are perpendicular to each of the sides, the slots are parallel. Since any pair of slots is perpendicular the sides, they are also parallel.

30. **PROOF** Write a paragraph proof of Theorem 5.21.

SOLUTION:
Given: $\ell \perp t, m \perp t$
Prove: $\ell \parallel m$

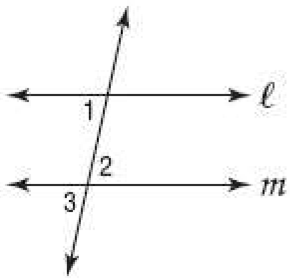


Proof:
Since $\ell \perp t$ and $m \perp t$, the measures of angle 1 and angle 2 are 90. Since $\angle 1$ and $\angle 2$ have the same measure, they are congruent. By the converse of Corresponding Angles Postulate, $\ell \parallel m$.

31. **PROOF** Write a two-column proof of Theorem 5.20.

SOLUTION:

Given: $\angle 1 \cong \angle 2$
Prove: $\ell \parallel m$



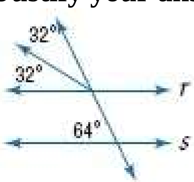
Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$ (Given)
2. $\angle 2 \cong \angle 3$ (Vertical $\angle$s are $\cong$)
3. $\angle 1 \cong \angle 3$ (Transitive Prop.)
4. $\ell \parallel m$ (If corr $\angle s$ are $\cong$, then lines are $\parallel$.)

32. **CCSS REASONING** Based upon the information given in the photo of the staircase, what is the relationship between each step? Explain your answer.

SOLUTION:

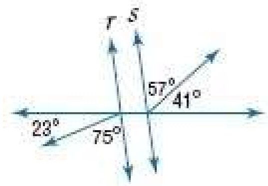
Each step is parallel to each other because the corresponding angles are congruent.

Determine whether lines *r* and *s* are parallel. Justify your answer.

33. 

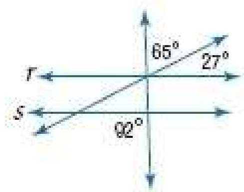
SOLUTION:

r ∥ s**;** Sample answer: The corresponding angles are congruent. Since the measures of angles are equal, the lines are parallel.

34. 

SOLUTION:

r || *s*; Sample answer: The alternate exterior angles are congruent. Since the measures of angles are equal, the lines are parallel.

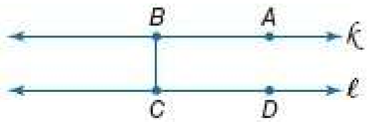
35. 

SOLUTION:

r || *s*; Sample answer: The alternate exterior angles are congruent. Since the measures of angles are equal, the lines are parallel.

36. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the shortest distance between two parallel lines.

a. GEOMETRIC Draw three sets of parallel lines k and ℓ, s and t, and x and y. For each set, draw the shortest segment $\overline{BC}$ and label points A and D as shown below.



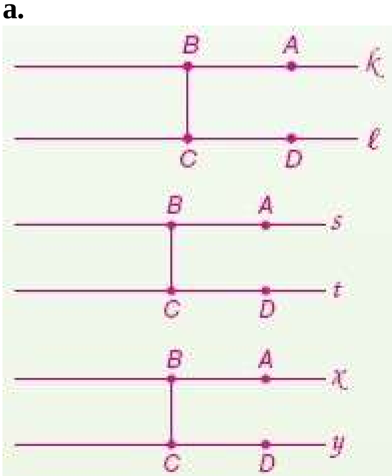
b. TABULAR Copy the table below, measure $\angle ABC$ and $\angle BCD$, and complete the table.

Set of Parallel Lines	$m\angle ABC$	$m\angle BCD$
k and ℓ		
s and t		
x and y		

c. VERBAL Make a conjecture about the angle the shortest segment forms with both parallel lines.

SOLUTION:

a.

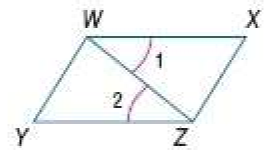


b.

Set of Parallel Lines	$m\angle ABC$	$m\angle BCD$
k and ℓ	90	90
s and t	90	90
x and y	90	90

c. Sample answer: The angle that the segment forms with the parallel lines will always measure 90.

37. **ERROR ANALYSIS** Sumi and Daniela are determining which lines are parallel in the figure at the right. Sumi says that since $\angle 1 \cong \angle 2$, $\overline{WY} \parallel \overline{XZ}$. Daniela disagrees and says that since $\angle 1 \cong \angle 2$, $\overline{WX} \parallel \overline{YZ}$. Is either of them correct? Explain.



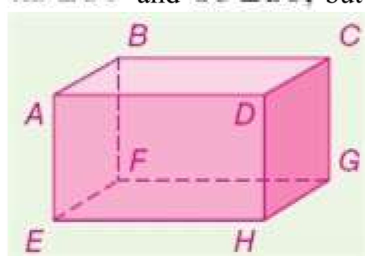
SOLUTION:

Daniela; $\angle 1$ and $\angle 2$ are alternate interior angles for $\overline{WX}$ and $\overline{YZ}$, so if alternate interior angles are congruent, then the lines are parallel.

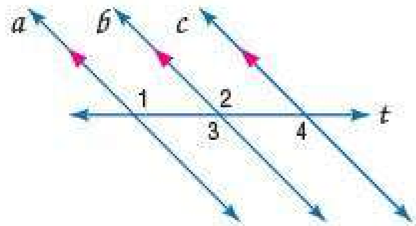
38. **CCSS REASONING** Is Theorem 5.21 still true if the two lines are not coplanar? Draw a figure to justify your answer..

SOLUTION:

No; sample answer: In the figure shown, $\overline{AB} \perp \overline{BC}$ and $\overline{GC} \perp \overline{BC}$, but $\overline{AB} \perp \overline{GC}$.



39. **CHALLENGE** Use the figure to prove that two lines parallel to a third line are parallel to each other.



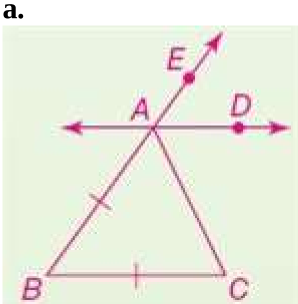
SOLUTION:

Given: $a \parallel b$ and $b \parallel c$
Prove: a $\parallel c$
Proof:
Statements (Reasons)
1. $a \parallel b$ and $b \parallel c$ (Given)
2. $\angle 1 \cong \angle 3$ (Alternate Interior $\angle$'s Theorem)
3. $\angle 3 \cong \angle 2$ (Vertical. $\angle$'s are $\cong$)
4. $\angle 2 \cong \angle 4$ (Alternate Interior. $\angle$'s Theorem)
5. $\angle 1 \cong \angle 4$ (Trans. Prop.)
6. $a \parallel c$ (Alternate Interior. $\angle$'s Converse Theorem)

40. **OPEN ENDED** Draw a triangle ABC.
a. Construct the line parallel to $\overline{BC}$ through point A.
b. Use measurement to justify that the line you constructed is parallel to $\overline{BC}$.
c. Use mathematics to justify this construction.

SOLUTION:

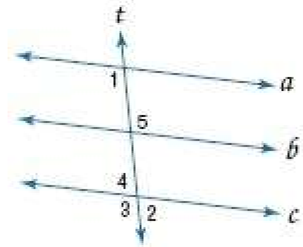
a.



b. Sample answer: Using a straightedge, the lines are equidistant. So they are parallel.

c. Sample answer: $\overleftrightarrow{AB}$ is a transversal for $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$. $\triangle ABC$ was copied to construct $\triangle EAD$. So, $\triangle ABC \cong \triangle EAD$. $\angle ABC$ and $\angle EAD$ are corresponding angles, so by the converse of corresponding angles postulate, $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.

41. **CHALLENGE** Refer to the figure at the right.
a. If $m\angle 1 + m\angle 2 = 180$, prove that $a \parallel c$.
b. Given that $a \parallel c$, if $m\angle 1 + m\angle 3 = 180$, prove that $t \perp c$.



SOLUTION:

a. We know that $m\angle 1 + m\angle 2 = 180$. Since $\angle 2$ and $\angle 3$ are linear pairs, $m\angle 2 + m\angle 3 = 180$. By substitution, $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$. By subtracting $m\angle 2$ from both sides we get $m\angle 1 = m\angle 3$. $\angle 1 \cong \angle 3$, by the definition of congruent angles. Therefore, $a \parallel c$ since the corresponding angles are congruent.
b. We know that $a \parallel c$ and $m\angle 1 + m\angle 3 = 180$. Since $\angle 1$ and $\angle 3$ are corresponding angles, they are congruent and their measures are equal. By substitution, $m\angle 3 + m\angle 3 = 180$. By dividing both sides by 2, we get $m\angle 3 = 90$. Therefore, $t \perp c$ since they form a right angle.

42. **WRITING IN MATH** Summarize the five methods used in this lesson to prove that two lines are parallel.

SOLUTION:

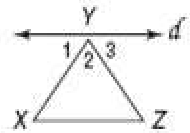
Sample answer: Use a pair of alternate exterior angles that are
congruent and cut by transversal; show that a pair of consecutive interior angles are supplementary; show that alternate interior angles are congruent; show two coplanar lines are perpendicular to same line; show corresponding angles are congruent.

43. **WRITING IN MATH** Can a pair of angles be supplementary and congruent?
Explain your reasoning.

SOLUTION:

Yes; sample answer: A pair of angles can be both supplementary and congruent if the measure of both angles is 90, since the sum of the angle measures would be 180.

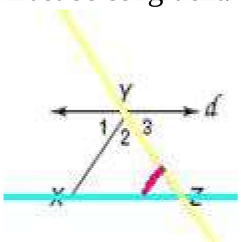
44. Which of the following facts would be sufficient to prove that line d is parallel to $\overline{XZ}$?



A $\angle 1 \cong \angle 3$
B $\angle 3 \cong \angle Z$
C $\angle 1 \cong \angle Z$
D $\angle 2 \cong \angle X$

SOLUTION:

If line d is parallel to the line through $\overleftrightarrow{XZ}$ then with the transversals of $\overleftrightarrow{YZ}$, the alternate interior angles must be congruent. Thus $\angle Z \cong \angle 3$.



Thus B is the correct choice.

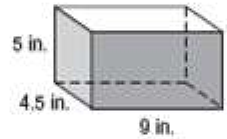
45. **ALGEBRA** The expression $\sqrt{52} + \sqrt{117}$ is equivalent to
F 13
G $5\sqrt{13}$
H $6\sqrt{13}$
J $13\sqrt{13}$

SOLUTION:

$$\begin{aligned} \sqrt{52} + \sqrt{117} &= \sqrt{4 \cdot 13} + \sqrt{9 \cdot 13} && \text{Product Property} \\ &= \sqrt{2^2 \cdot 13} + \sqrt{3^2 \cdot 13} && \text{Prime Factorization.} \\ &= \sqrt{2^2}\sqrt{13} + \sqrt{3^2}\sqrt{13} && \text{Product Property} \\ &= 2\sqrt{13} + 3\sqrt{13} && \text{Simplify.} \\ &= 5\sqrt{13} && \text{Simplify.} \end{aligned}$$

So, the correct option is G.

46. What is the approximate surface area of the figure?



A 101.3 in^2
B 108 in^2
C 202.5 in^2
D 216 in^2

SOLUTION:

The formula for finding the surface area of a prism is $S = Ph + 2B$.
S = total surface area, h = height of a solid, B = area of the base, P = perimeter of the base

Since the base of the prism is a rectangle, the perimeter P of the base is $2(9 + 4.5)$ or 27 inches. The area of the base B is $9 \cdot 4.5$ or 40.5 square inches. The height is 5 cinches.

$$\begin{aligned} S &= Ph + 2B \\ &= (27 \cdot 5) + 2(40.5) \\ &= 135 + 81 \\ &= 216 \end{aligned}$$

The surface area of the prism is 216 square inches. So, the correct option is D.

47. **SAT/ACT** If $x^2 = 25$ and $y^2 = 9$, what is the greatest possible value of $(x - y)^2$?
F 4
G 16
H 58
J 64
K 70

SOLUTION:
First solve for x and y.

$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$
$$y^2 = 9$$
$$\sqrt{y^2} = \sqrt{9}$$
$$y = \pm 3$$

Next, find the greatest possible value. Substituting -5 and 3 for x and y, respectively, leads to the greatest positive number. Another solution is to substitute 5 and -3 for x and y.

$$\begin{aligned}(x-y)^2 &= x^2 + y^2 - 2xy && \text{Factor.}\\ &= 25 + 9 - 2(-5)(3) && \text{Substitute.}\\ &= 25 + 9 + 30 && \text{Simplify.}\\ &= 64 && \text{Add.}\end{aligned}$$

The correct choice is H.