5-2 Inverse Functions and Relations

Find the inverse of each relation.

1. {(–9, 10), (1, –3), (8, –5)}

SOLUTION:

To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is $\{(10,-9),(-3,1),(-5,8)\}$.

2. {(–2, 9), (4, –1), (–7, 9), (7, 0)}

SOLUTION:

To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is $\{(9,-2),(-1,4),(9,-7),(0,7)\}$.

Find the inverse of each function. Then graph the function and its inverse.

3. $f(x) = -3x$

SOLUTION:

Rewrite the function as an equation relating x and y.

$y = -3x$

Exchange x and y in the equation.

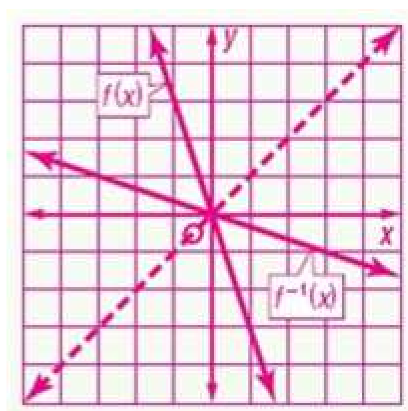
$x = -3y$

Solve the equation for y.

$y = -\frac{1}{3}x$

Replace y with $f^{-1}(x)$.

$f^{-1}(x) = -\frac{1}{3}x$



4. $g(x) = 4x - 6$

SOLUTION:

Rewrite the function as an equation relating x and y.

$y = 4x - 6$

Exchange x and y in the equation.

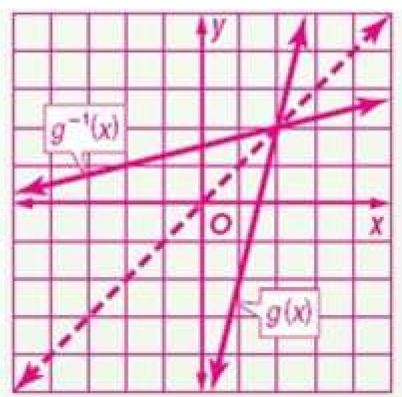
$x = 4y - 6$

Solve for y.

$\frac{x+6}{4} = y$

Replace y with $g^{-1}(x)$.

$g^{-1}(x) = \frac{x+6}{4}$



5. $h(x) = x^2 - 3$

SOLUTION:

Rewrite the function as an equation relating x and y.

$y = x^2 - 3$

Exchange x and y in the equation.

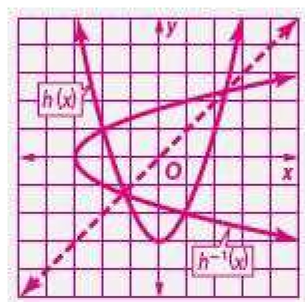
$x = y^2 - 3$

Solve for y.

$y = \pm\sqrt{x+3}$

Replace y with $h^{-1}(x)$.

$h^{-1} = \pm\sqrt{x+3}$



Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

6. $f(x) = x - 7$
$g(x) = x + 7$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f[x+7] \\ &= (x+7) - 7 \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(x-7) \\ &= x - 7 + 7 \\ &= x\end{aligned}$$

Yes, $f(x)$ and $g(x)$ are inverse functions.

7. $f(x) = \frac{1}{2}x + \frac{3}{4}$
$g(x) = 2x - \frac{4}{3}$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(2x - \frac{4}{3}\right) \\ &= \frac{1}{2}\left(2x - \frac{4}{3}\right) + \frac{3}{4} \\ &= x + \frac{1}{12}\end{aligned}$$

$$[f \circ g](x) \neq x$$

No, $f(x)$ and $g(x)$ are not inverse functions.

8. $f(x) = 2x^3$
$g(x) = \frac{1}{3}\sqrt{x}$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{3}\sqrt{x}\right) \\ &= 2\left(\frac{1}{3}\sqrt{x}\right)^3 \\ &= \frac{2}{27}x^{3/2}\end{aligned}$$

$[f \circ g](x) \neq x$

No, $f(x)$ and $g(x)$ are not inverse functions.

Find the inverse of each relation.

9. {(–8, 6), (6, –2), (7, –3)}

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is $\{(6,-8),(-2,6),(-3,7)\}$.

10. {(7, 7), (4, 9), (3, –7)}

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is $\{(7,7),(9,4),(-7,3)\}$.

11. {(8, –1), (–8, –1), (–2, –8), (2, 8)}

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is $\{(-1,8),(-1,-8),(-8,-2),(8,2)\}$.

12. {(4, 3), (–4, –4), (–3, –5), (5, 2)}

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is $\{(3,4),(-4,-4),(-5,-3),(2,5)\}$.

13. {(1, –5), (2, 6), (3, –7), (4, 8), (5, –9)}

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is $\{(-5,1),(6,2),(-7,3),(8,4),(-9,5)\}$.

14. {(3, 0), (5, 4), (7, –8), (9, 12), (11, 16)}

SOLUTION:
To find the inverse, exchange the coordinates of the ordered pairs.

The inverse of the relation is $\{(0,3),(4,5),(-8,7),(12,9),(16,11)\}$.

CCSS SENSE-MAKING Find the inverse of each function. Then graph the function and its inverse.

15. $f(x) = x + 2$

SOLUTION:
Rewrite the function as an equation relating x and y.

$y = x + 2$

Exchange x and y in the equation.

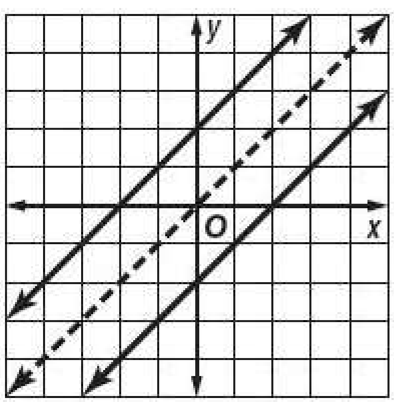
$x = y + 2$

Solve for y.

$y = x - 2$

Replace y with $f^{-1}(x)$.

Therefore:
$f^{-1}(x) = x - 2$



16. $g(x) = 5x$

SOLUTION:
Rewrite the function as an equation relating x and y.

$y = 5x$

Exchange x and y in the equation.

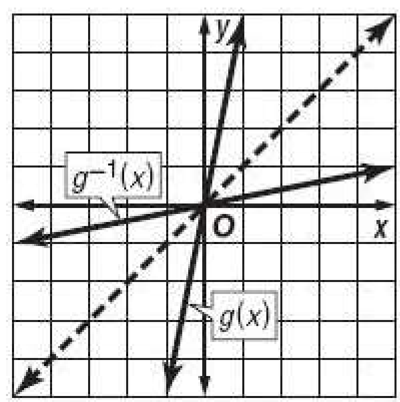
$x = 5y$

Solve for y.

$$y = \frac{1}{5}x$$

Replace y with $g^{-1}(x)$.

Therefore:
$$g^{-1}(x) = \frac{1}{5}x$$



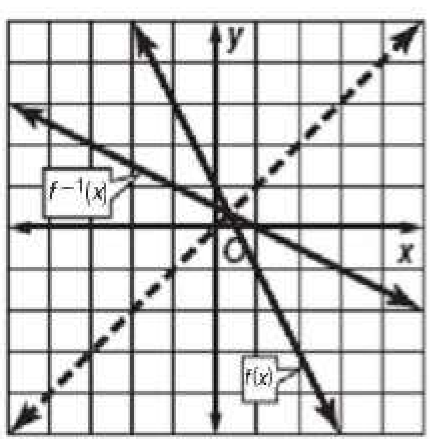
17. $f(x) = -2x + 1$

SOLUTION:
Rewrite the function with x and y.

$y = -2x + 1$

Interchange x and y and solve.

$$x = -2y + 1$$
$$x - 1 = -2y$$
$$\frac{x-1}{-2} = y$$
$$-\frac{x}{2} + \frac{1}{2} = y$$
$$f^{-1}(x) = -\frac{x}{2} + \frac{1}{2}$$



18. $h(x) = \frac{x-4}{3}$

SOLUTION:
Rewrite the function as an equation relating x and y.

$y = \frac{x-4}{3}$

Exchange x and y in the equation

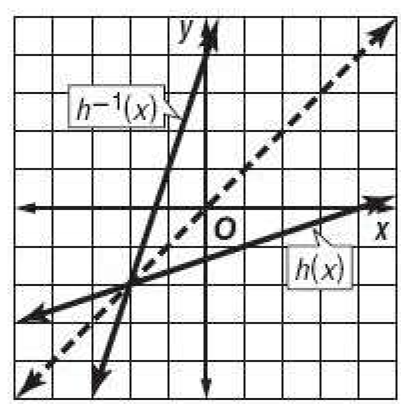
$x = \frac{y-4}{3}$

Solve for y.

$y = 3x + 4$

Replace y with $h^{-1}(x)$.

Therefore:
$h^{-1}(x) = 3x + 4$



19. $f(x) = -\frac{5}{3}x - 8$

SOLUTION:

Rewrite using x and y.

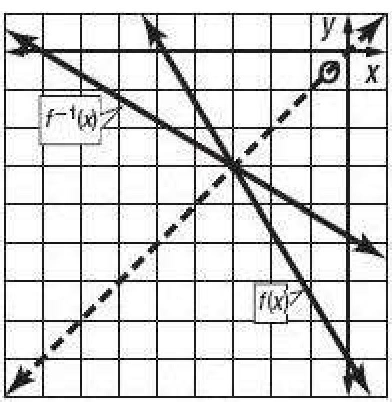
$$y = -\frac{5}{3}x - 8$$

Exchange x and y in the equation and solve for y.

$$\begin{aligned} x &= -\tfrac{5}{3}y - 8 \\ x + \tfrac{5}{3}y &= -8 \\ \tfrac{5}{3}y &= -x - 8 \\ y &= -\tfrac{3}{5}x - \tfrac{24}{5} \\ f^{-1}(x) &= -\tfrac{3}{5}x - \tfrac{24}{5} \end{aligned}$$

or

$$f^{-1}(x) = -\frac{3}{5}(x + 8)$$



20. $g(x) = x + 4$

SOLUTION:

Rewrite the function as an equation relating x and y.

$$y = x + 4$$

Exchange x and y in the equation.

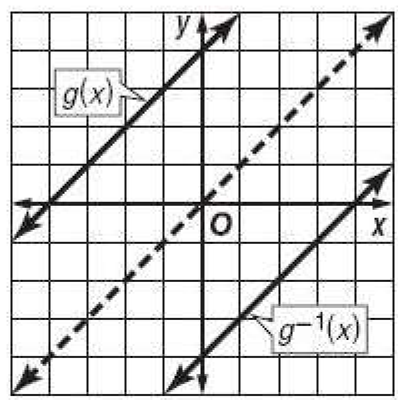
$$x = y + 4$$

Solve for y.

$y = x - 4$

Replace y with $g^{-1}(x)$. Therefore:

$$g^{-1}(x) = x - 4$$



21. $f(x) = 4x$

SOLUTION:

Rewrite the function as an equation relating x and y.

$y = 4x$

Exchange x and y in the equation.

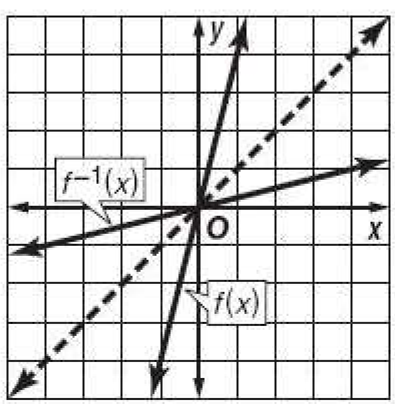
$x = 4y$

Solve for y.

$$y = \frac{1}{4}x$$

Replace y with $f^{-1}(x)$. Therefore:

$$f^{-1}(x) = \frac{1}{4}x$$



22. $f(x) = –8x + 9$

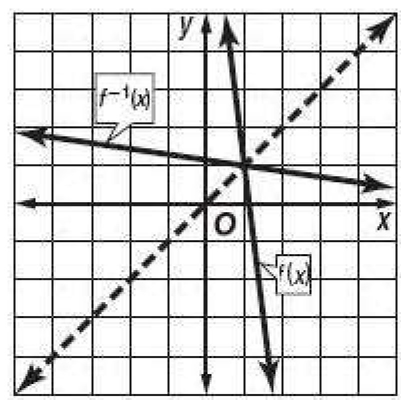
SOLUTION:

Replace $f(x)$ with y.

$y = –8x + 9$

Exchange x and y in the equation and solve for y.

$$\begin{aligned} x &= -8y + 9 \\ x + 8y &= 9 \\ 8y &= -x + 9 \\ y &= -\frac{x}{8} + \frac{9}{8} \\ f^{-1}(x) &= -\frac{x}{8} + \frac{9}{8} \end{aligned}$$



23. $f(x) = 5x^2$

SOLUTION:
Rewrite the function as an equation relating x and y.

$y = 5x^2$

Exchange x and y in the equation.

$x = 5y^2$

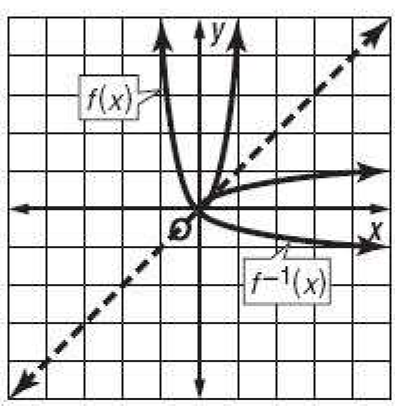
Solve for y.

$y = \pm\sqrt{\frac{1}{5}x}$

Replace y with $f^{--1}(x)$.

Therefore:

$f^{-1}(x) = \pm\sqrt{\frac{1}{5}x}$



24. $h(x) = x^2 + 4$

SOLUTION:
Rewrite the function as an equation relating x and y.

$y = x^2 + 4$

Exchange x and y in the equation.

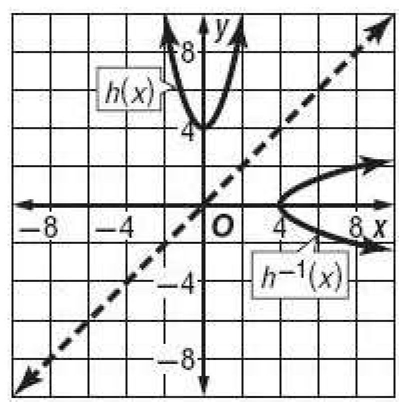
$x = y^2 + 4$

Solve for y.

$y = \pm\sqrt{x - 4}$

Replace y with $h^{-1}(x)$.

$h^{-1}(x) = \pm\sqrt{x - 4}$



25. $f(x) = \frac{1}{2}x^2 - 1$

SOLUTION:

Rewrite the function as an equation relating x and y.

$$y = \frac{1}{2}x^2 - 1$$

Exchange x and y in the equation.

$$x = \frac{1}{2}y^2 - 1$$

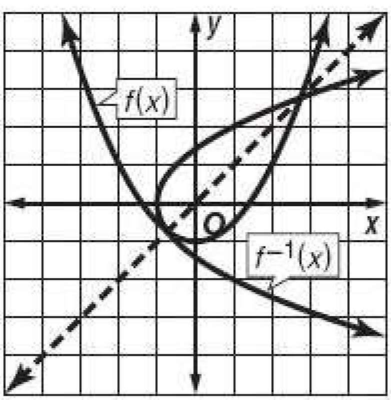
Solve for y.

$$y = \pm\sqrt{2x + 2}$$

Replace y with $f^{--1}(x)$.

Therefore:

$$f^{-1}(x) = \pm\sqrt{2x + 2}$$

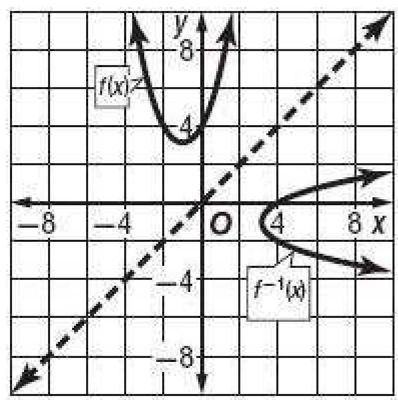


26. $f(x) = (x + 1)^2 + 3$

SOLUTION:

Replace $f(x)$ with y. Then exchange x and y in the equation and solve for y.

$$\begin{aligned} f(x) &= (x + 1)^2 + 3 \\ y &= (x + 1)^2 + 3 \\ x &= (y + 1)^2 + 3 \\ x - 3 &= (y + 1)^2 \\ \pm\sqrt{x - 3} &= y + 1 \\ \pm\sqrt{x - 3} - 1 &= y \\ \pm\sqrt{x - 3} - 1 &= f^{-1}(x) \end{aligned}$$



Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

27. $f(x) = 2x + 3$
$g(x) = 2x - 3$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(2x-3) \\ &= 2(2x-3)+3 \\ &= 4x-3\end{aligned}$$
$$f(x) \neq x$$

No, $f(x)$ and $g(x)$ are not inverse functions.

28. $f(x) = 4x + 6$
$g(x) = \frac{x-6}{4}$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{x-6}{4}\right) \\ &= 4\left(\frac{x-6}{4}\right)+6 \\ &= x \\ [g \circ f](x) &= g[f(x)] \\ &= g(4x+6) \\ &= \frac{4x+6-6}{4} \\ &= x\end{aligned}$$

Therefore:

$$[f \circ g](x) = [g \circ f](x) = x$$

Yes, $f(x)$ and $g(x)$ are inverse functions.

29. $f(x) = -\frac{1}{3}x + 3$
$g(x) = -3x + 9$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(-3x + 9) \\ &= -\frac{1}{3}(-3x + 9) + 3 \\ &= x - 3 + 3 \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g\left(-\frac{1}{3}x + 3\right) \\ &= -3\left(-\frac{1}{3}x + 3\right) + 9 \\ &= x - 9 + 9 \\ &= x\end{aligned}$$

$[f \circ g](x) = [g \circ f](x) = x$

Yes, $f(x)$ and $g(x)$ are inverse functions.

30. $f(x) = -6x$
$g(x) = \frac{1}{6}x$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{6}x\right) \\ &= -6\left(\frac{1}{6}x\right) \\ &= -x\end{aligned}$$

$[f \circ g](x) \neq x$

No, $f(x)$ and $g(x)$ are not inverse functions.

31. $f(x)=\frac{1}{2}x+5$
$g(x)=2x-10$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f\circ g](x)=[g\circ f](x)=x$.

$$\begin{aligned}[f\circ g](x)&=f[g(x)]\\&=f(2x-10)\\&=\frac{1}{2}(2x-10)+5\\&=x-5+5\\&=x\end{aligned}$$

$$\begin{aligned}[g\circ f](x)&=g[f(x)]\\&=g\left(\frac{1}{2}x+5\right)\\&=2\left(\frac{1}{2}x+5\right)-10\\&=x\end{aligned}$$

$[f\circ g](x)=[g\circ f](x)=x$

Yes, $f(x)$ and $g(x)$ are inverse functions.

32. $f(x)=\frac{x+10}{8}$
$g(x)=8x-10$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f\circ g](x)=[g\circ f](x)=x$.

$$\begin{aligned}[f\circ g](x)&=f[g(x)]\\&=f(8x-10)\\&=\frac{8x-10+10}{8}\\&=x\end{aligned}$$

$$\begin{aligned}[g\circ f](x)&=g[f(x)]\\&=g\left(\frac{x+10}{8}\right)\\&=8\left(\frac{x+10}{8}\right)-10\\&=x\end{aligned}$$

Therefore:

$[f\circ g](x)=[g\circ f](x)=x$

Yes, $f(x)$ and $g(x)$ are inverse functions.

33. $f(x) = 4x^2$
$g(x) = \frac{1}{2}\sqrt{x}$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{2}\sqrt{x}\right) \\ &= 4\left(\frac{1}{2}\sqrt{x}\right)^2 \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(4x^2) \\ &= \frac{1}{2}\sqrt{4x^2} \\ &= x\end{aligned}$$

$$[f \circ g](x) = [g \circ f](x) = x$$

Yes, $f(x)$ and $g(x)$ are inverse functions.

34. $f(x) = \frac{1}{3}x^2 + 1$
$g(x) = \sqrt{3x - 3}$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\sqrt{3x-3}\right) \\ &= \frac{1}{3}\left(\sqrt{3x-3}\right)^2 + 1 \\ &= x - 1 + 1 \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g\left(\frac{1}{3}x^2 + 1\right) \\ &= \sqrt{3\left(\frac{1}{3}x^2 + 1\right) - 3} \\ &= \sqrt{x^2} \\ &= x\end{aligned}$$

$$[f \circ g](x) = [g \circ f](x) = x$$

Yes, $f(x)$ and $g(x)$ are inverse functions.

35. $f(x) = x^2 - 9$
$g(x) = x + 3$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(x+3) \\ &= (x+3)^2 - 9 \\ &= x^2 + 6x + 9 - 9 \\ &= x^2 + 6x\end{aligned}$$
$$[f \circ g](x) \neq x$$

No, $f(x)$ and $g(x)$ are not inverse functions.

36. $f(x) = \frac{2}{3}x^3$
$g(x) = \sqrt{\frac{2}{3}x}$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\sqrt{\frac{2}{3}x}\right) \\ &= \frac{2}{3}\left(\sqrt{\frac{2}{3}x}\right)^3 \\ &= \frac{2}{3}\left(\frac{2}{3}x\right)^{3/2}\end{aligned}$$

$$[f \circ g](x) \neq x$$

No, $f(x)$ and $g(x)$ are not inverse functions.

37. $f(x) = (x+6)^2$
$g(x) = \sqrt{x} - 6$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\sqrt{x} - 6\right) \\ &= \left(\sqrt{x} - 6 + 6\right)^2 \\ &= x \\ [g \circ f](x) &= g[f(x)] \\ &= g\left[(x+6)^2\right] \\ &= \sqrt{(x+6)^2} - 6 \\ &= x\end{aligned}$$

$$[f \circ g](x) = [g \circ f](x) = x$$

Yes, $f(x)$ and $g(x)$ are inverse functions.

38. $f(x) = 2\sqrt{x-5}$
$g(x) = \frac{1}{4}x^2 - 5$

SOLUTION:
The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{4}x^2 - 5\right) \\ &= 2\sqrt{\left(\frac{1}{4}x^2 - 5\right) - 5} \\ &= 2\sqrt{\frac{x^2}{4} - 10} \\ &= 2\sqrt{\frac{x^2 - 40}{4}} \\ &= 2 \cdot \frac{\sqrt{x^2 - 40}}{\sqrt{4}} \\ &= 2 \cdot \frac{\sqrt{x^2 - 40}}{2} \\ &= \sqrt{x^2 - 40} \\ &\neq x\end{aligned}$$

Therefore, the functions are not inverses.

39. **FUEL** The average miles traveled for every gallon g of gas consumed by Leroy's car is represented by the function $m(g) = 28g$.

a. Find a function $c(g)$ to represent the cost per gallon of gasoline.

b. Use inverses to determine the function used to represent the cost per mile traveled in Leroy's car.

SOLUTION:
a. Let g be the number of gallons of gasoline.

The cost per gallon of gasoline is given by $c(g) = 2.95g$.

b. The average miles traveled for every gallon g of gas consumed by Leroy's car is :

$m(g) = 28g$
So:
$g = \frac{m}{28}$

Therefore, the cost per mile is given by:

$$\begin{aligned}c(m) &= 2.95 \times \frac{m}{28} \\ &\approx 0.105m\end{aligned}$$

40. **SHOES** The shoe size for the average U.S. teen or adult male can be determined using the formula $M(x) = 3x - 22$, where x is length of a foot in measured inches. The shoe size for the average U.S. teen or adult female can be found by using the formula $F(x) = 3x - 21$.

a. Find the inverse of each function.

b. If Lucy wears a size $7\frac{1}{2}$ shoe, how long are her feet?

SOLUTION:

a. $M(x) = 3x - 22$

Rewrite the function as an equation relating x and y.

$y = 3x - 22$

Exchange x and y.

$x = 3y - 22$

Solve for y.

$$M^{-1}(x) = \frac{x + 22}{3}$$

$$F(x) = 3x - 21$$

Rewrite the function as an equation relating x and y.

$y = 3x - 21$

Exchange x and y.

$x = 3y - 21$

Solve for y.

$$F^{-1}(x) = \frac{x + 21}{3}$$

b. Find $F^{-1}\left(7\frac{1}{2}\right)$.

$$F^{-1}\left(7\frac{1}{2}\right) = \frac{\frac{15}{2} + 21}{3}$$
$$= 9\frac{1}{2} \text{ in}$$

41. **GEOMETRY** The formula for the area of a circle is $A = \pi r^2$.

a. Find the inverse of the function.

b. Use the inverse to find the radius of a circle with an area of 36 square centimeters.

SOLUTION:

a.

$$A = \pi r^2$$
$$r^2 = \frac{A}{\pi}$$
$$r = \sqrt{\frac{A}{\pi}}$$

b. Substitute $A = 36$.

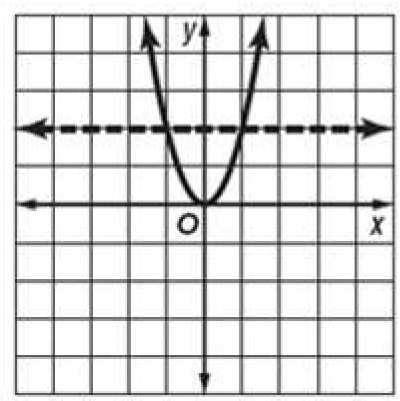
$$r = \sqrt{\frac{36}{\pi}}$$
$$\approx 3.39 \text{ cm}$$

Use the horizontal line test to determine whether the inverse of each function is also a function.

42. $f(x) = 2x^2$

SOLUTION:

Graph the function $f(x) = 2x^2$.

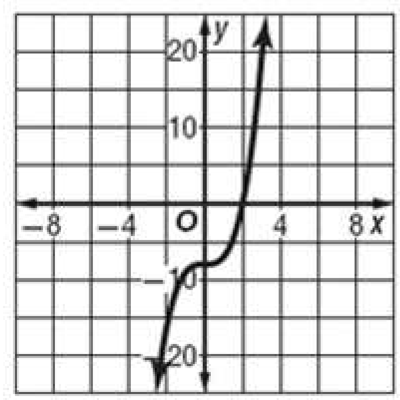


A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

43. $f(x) = x^3 - 8$

SOLUTION:

Graph the function $f(x) = x^3 - 8$.



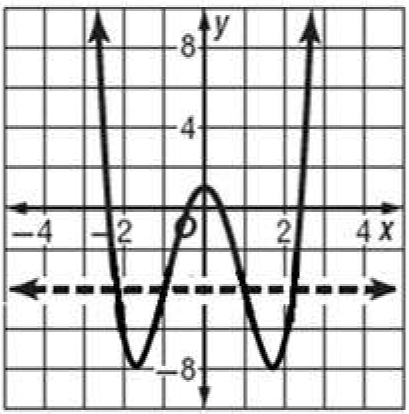
No horizontal line can be drawn such that it intersects the graph of the function at more than one point.
Therefore, the inverse of the given function is a function.

5-2 Inverse Functions and Relations

44. $g(x) = x^4 - 6x^2 + 1$

SOLUTION:

Graph the function $g(x) = x^4 - 6x^2 + 1$.

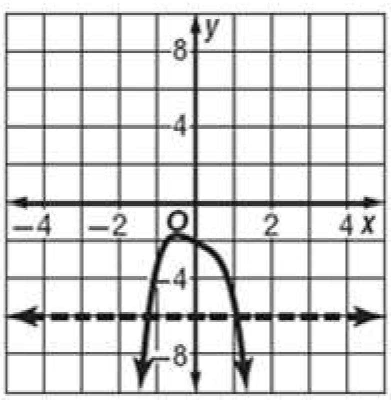


A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

45. $h(x) = -2x^4 - x - 2$

SOLUTION:

Graph the function $h(x) = -2x^4 - x - 2$.

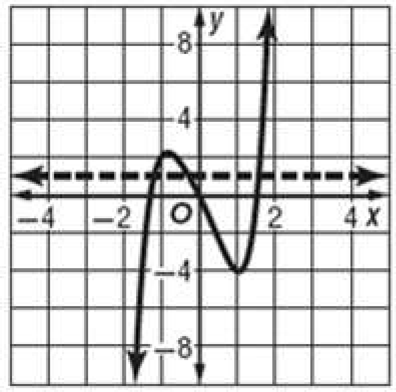


A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

46. $g(x) = x^5 + x^2 - 4x$

SOLUTION:

Graph the function $g(x) = x^5 + x^2 - 4x$.

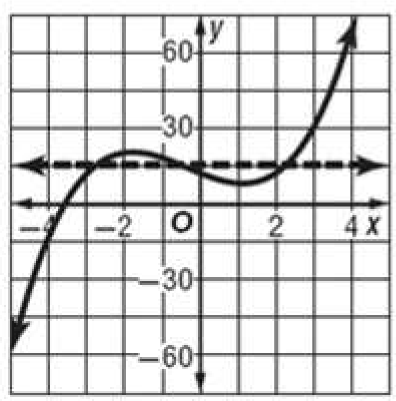


A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

47. $h(x) = x^3 + x^2 - 6x + 12$

SOLUTION:

Graph the function $h(x) = x^3 + x^2 - 6x + 12$.



A horizontal line can be drawn such that it intersects the graph of the function at more than one point. Therefore, the inverse of the given function is not a function.

48. **SHOPPING** Felipe bought a used car. The sales tax rate was 7.25% of the selling price, and he paid $350 in processing and registration fees. Find the selling price if Felipe paid a total of $8395.75.

SOLUTION:
Let x be the selling price of the car.

$$\begin{aligned} x + 7.25\%(x) + 350 &= 8395.75 \\ x + 0.0725x + 350 &= 8395.75 \\ 1.0725x &= 8395.75 - 350 \\ 1.0725x &= 8045.75 \\ x &\approx 7501.86 \end{aligned}$$

The selling price is $7501.86.

49. **TEMPERATURE** A formula for converting degrees Celsius to Fahrenheit is $F(x) = \frac{9}{5}x + 32$.

a. Find the inverse $F^{-1}(x)$. Show that $F(x)$ and $F^{-1}(x)$ are inverses.

b. Explain what purpose $F^{-1}(x)$ serves.

SOLUTION:
a. Rewrite the function as an equation relating x and y.

$$y = \frac{9}{5}x + 32$$

Exchange x and y in the equation.

$$x = \frac{9}{5}y + 32$$

Solve for y.

$$y = \frac{5}{9}(x - 32)$$

Therefore:

$$F^{-1}(x) = \frac{5}{9}(x - 32)$$

The functions $f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = [g \circ f](x) = x$.

$$\begin{aligned} F\left[F^{-1}(x)\right] &= F\left[\frac{5}{9}(x - 32)\right] \\ &= \frac{9}{5}\left[\frac{5}{9}(x - 32)\right] + 32 \\ &= x \end{aligned}$$

$$\begin{aligned} F^{-1}\left[F(x)\right] &= F^{-1}\left[\frac{9}{5}x + 32\right] \\ &= \frac{5}{9}\left[\frac{9}{5}x + 32 - 32\right] \\ &= x \end{aligned}$$

Therefore, $F(x)$ and $F^{-1}(x)$ are inverses.

b. $F^{-1}(x)$ can be used to convert Fahrenheit to Celsius.

50. **MEASUREMENT** There are approximately 1.852 kilometers in a nautical mile.

a. Write a function that converts nautical miles to kilometers.

b. Find the inverse of the function that converts kilometers back to nautical miles.

c. Using composition of functions, verify that these two functions are inverses.

SOLUTION:

a. Let *m* be the number of miles.

The number of kilometers in *m* miles is given by $K(m) = 1.852m$.

b. Inverse is given by $K^{-1}(m) = \frac{1}{1.852}m$.

c. $K\left[K^{-1}(m)\right] = K^{-1}\left[K(m)\right] = m$.
Therefore, the functions are inverses.

51. **MULTIPLE REPRESENTATIONS** Consider the functions $y = x^n$ for $n = 0, 1, 2, \ldots$
.

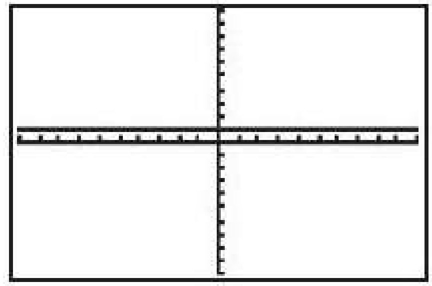
a. GRAPHING Use a graphing calculator to graph $y = x^n$ for $n = 0, 1, 2, 3,$ and 4.

b. TABULAR For which values of *n* is the inverse a function? Record your results in a table.

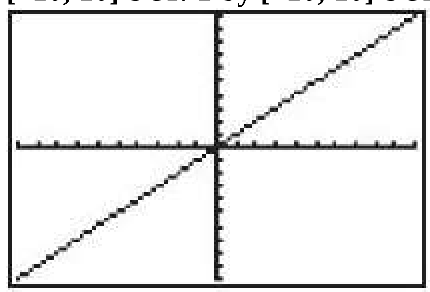
c. ANALYTICAL Make a conjecture about the values of *n* for which the inverse of $f(x) = x^n$ is a function. Assume that *n* is a whole number.

SOLUTION:

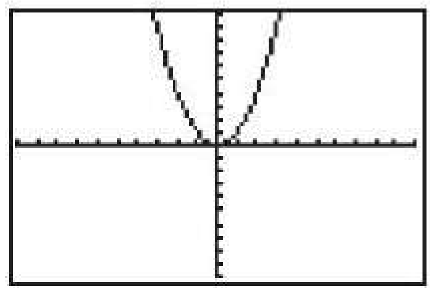
a.



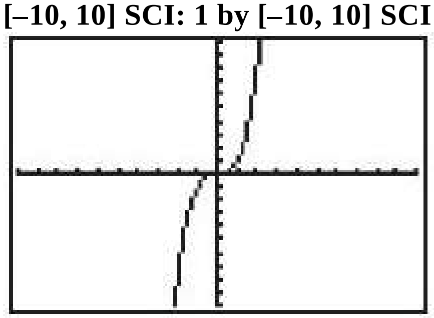
[–10, 10] SCI: 1 by [–10, 10] SCI: 1



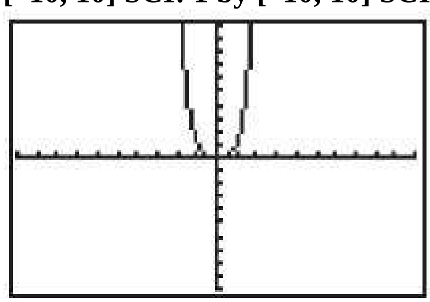
[–10, 10] SCI: 1 by [–10, 10] SCI: 1



[–10, 10] SCI: 1 by [–10, 10] SCI: 1



[–10, 10] SCI: 1 by [–10, 10] SCI: 1



[–10, 10] SCI: 1 by [–10, 10] SCI: 1

b.

Function	Inverse a function?
$y = x^0$ or $y = 1$	no
$y = x^1$ or $y = x$	yes
$y = x^2$	no
$y = x^3$	yes
$y = x^4$	no

c. When n is odd, the function passes the horizontal line test. Therefore, when n is odd, the inverse of the function is a function.

52. **REASONING** If a relation is not a function, then its inverse is sometimes, always, or never a function. Explain your reasoning.

SOLUTION:

Sample answer: Sometimes; $y = \pm\sqrt{x}$ is an example of a relation that is not a function, with an inverse being a function. A circle is an example of a relation that is not a function with an inverse not being a function.

53. **OPEN ENDED** Give an example of a function and its inverse. Verify that the two functions are inverses.

SOLUTION:

Sample answer:
$f(x) = 2x, f^{-1}(x) = 0.5x; f[f^{-1}(x)] = f^{-1}[f(x)] = x$

54. **CHALLENGE** Give an example of a function that is its own inverse.

SOLUTION:

Sample answer: $f(x) = x$ and $f^{-1}(x) = x$ or $f(x) = -x$ and $f^{-1}(x) = -x$

55. **CCSS ARGUMENTS** Show that the inverse of a linear function $y = mx + b$, where $m \neq 0$ and $x \neq b$, is also a linear function.

SOLUTION:

$y = mx + b$

To find the inverse, exchange x and y, and solve for y.

$$x = my + b$$
$$x - b = my$$
$$\frac{x-b}{m} = y$$
$$y = \frac{x-b}{m}$$
$$y = \frac{1}{m}x - \frac{b}{m}$$

56. **WRITING IN MATH** Suppose you have a composition of two functions that are inverses. When you put in a value of 5 for x, why is the result always 5?

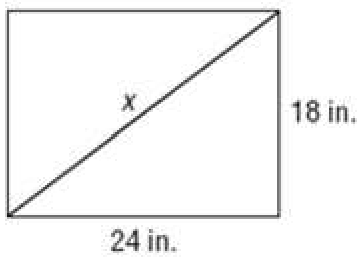
SOLUTION:

Sample answer: One of the functions carries out an operation on 5. Then the second function that is an inverse of the first function reverse the operation on 5. Thus, the result is 5.

57. **SHORT RESPONSE** If the length of a rectangular television screen is 24 inches and its height is 18 inches, what is the length of its diagonal in inches?

SOLUTION:

Let x be the length of the diagonal.



$$x = \sqrt{24^2 + 18^2} = \sqrt{900} = 30$$

The length of the diagonal is 30 inches.

58. **GEOMETRY** If the base of a triangle is represented by $2x + 5$ and the height is represented by $4x$, which expression represents the area of the triangle?

A $(2x + 5) + (4x)$

B $(2x + 5)(4x)$

C $\frac{1}{2}(2x+5)+(4x)$

D $\frac{1}{2}(2x+5)(4x)$

SOLUTION:

The area of a triangle is given by $A = \frac{1}{2}bh$, where b is the base and h is the height.

$$A = \frac{1}{2}(2x+5)(4x)$$

The correct choice is **D.**

59. Which expression represents $f[g(x)]$ if $f(x) = x^2 + 3$ and $g(x) = -x + 1$?

F $x^2 - x + 2$

G $-x^2 - 2$

H $-x^3 + x^2 - 3x + 3$

J $x^2 - 2x + 4$

SOLUTION:

$$\begin{aligned} f[g(x)] &= f(-x+1) \\ &= (-x+1)^2 + 3 \\ &= x^2 + 1 - 2x + 3 \\ &= x^2 - 2x + 4 \end{aligned}$$

The correct choice is **J.**

60. **SAT/ACT** Which of the following is the inverse of $f(x) = \frac{3x-5}{2}$?

A $g(x) = \frac{2x+5}{3}$

B $g(x) = \frac{2x-5}{3}$

C $g(x) = \frac{3x+5}{2}$

D $g(x) = 2x + 5$

E $g(x) = \frac{3x-5}{2}$

SOLUTION:
Rewrite the function as an equation relating x and y.

$$y = \frac{3x-5}{2}$$

Exchange x and y in the equation.

$$x = \frac{3y-5}{2}$$

Solve for y.

$$y = \frac{2x+5}{3}$$

Therefore:

$$g(x) = \frac{2x+5}{3}$$

The correct choice is **A.**

If $f(x) = 3x + 5$, $g(x) = x - 2$, and $h(x) = x^2 - 1$, find each value.

61. $g[f(3)]$

SOLUTION:

$$\begin{aligned} g[f(x)] &= g(3x+5) \\ &= 3x+5-2 \\ &= 3x+3 \end{aligned}$$

Substitute $x = 3$.

$$\begin{aligned} g[f(3)] &= 3(3)+3 \\ &= 12 \end{aligned}$$

62. $f\ [h(-2)]$

SOLUTION:

$$\begin{aligned} f[h(x)] &= f(x^2-1) \\ &= 3(x^2-1)+5 \\ &= 3x^2+2 \end{aligned}$$

Substitute $x = -2$.

$$\begin{aligned} f[h(-2)] &= 3(-2)^2+2 \\ &= 14 \end{aligned}$$

63. $h[g(1)]$

SOLUTION:

$$\begin{aligned} h[g(x)] &= h(x-2) \\ &= (x-2)^2-1 \\ &= x^2-4x+3 \end{aligned}$$

Substitute $x = 1$.

$$h[g(1)] = 0$$

64. **CONSTRUCTION** A picnic area has the shape of a trapezoid. The longer base is 8 more than 3 times the length of the shorter base, and the height is 1 more than 3 times the shorter base. What are the dimensions if the area is 4104 square feet?

SOLUTION:

Let *x* be the length of the shorter base.

Therefore, the length of the longer base is 3*x* + 8 and height is 3*x* + 1.

The area of a trapezoid is given by

$A = \frac{1}{2}h(b_1 + b_2)$.

$$4104 = \frac{1}{2}(3x+1)(x+3x+8)$$
$$8208 = (3x+1)(4x+8)$$
$$8208 = 12x^2 + 24x + 4x + 8$$
$$8208 = 12x^2 + 28x + 8$$
$$12x^2 + 28x - 8200 = 0$$

Use the Quadratic formula to solve for *x*.

$$x = \frac{-28 \pm \sqrt{28^2 - 4(12)(-8200)}}{2(12)}$$
$$= \frac{-28 \pm \sqrt{784 + 393600}}{24}$$
$$= \frac{-28 \pm \sqrt{394384}}{24}$$
$$= \frac{-28 \pm 628}{24}$$
$$x = \frac{-28 + 628}{24} \text{ or } x = \frac{-28 - 628}{24}$$
$$x = \frac{600}{24} \text{ or } x = \frac{-656}{24}$$
$$x = 25 \text{ or } x = -\frac{656}{24}$$

Since *x* is the length of the shorter base, it cannot be negative. Therefore, *x* = 25 feet.
The length of the longer base is 83 feet, and the height is 76 feet.