3-4 Quadratic Inequalities

Graph each inequality.

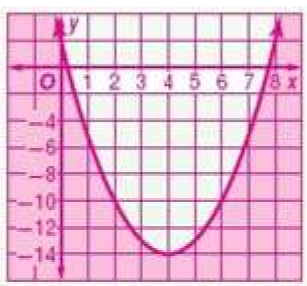
1. $y \leq x^2 - 8x + 2$

SOLUTION:

First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

$$y \leq x^2 - 8x + 2$$
$$0 \overset{?}{\leq} 0^2 - 8(0) + 2$$
$$0 \leq 2$$

So, (0, 0) is a solution of the inequality. Shade the region of the graph that contains (0, 0).



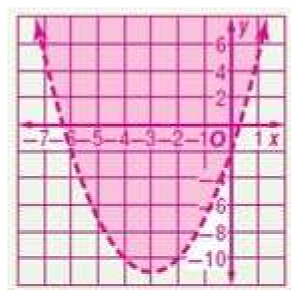
2. $y > x^2 + 6x - 2$

SOLUTION:

First graph the related function. The parabola should be dashed. Next test a point not on the graph of the parabola.

$$y > x^2 + 6x - 2$$
$$0 \overset{?}{>} 0^2 + 6(0) - 2$$
$$0 > -2$$

So, (0, 0) is a solution of the inequality. Shade the region of the graph that contains (0, 0).



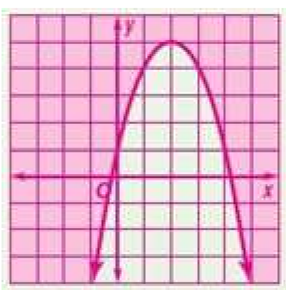
3. $y \geq -x^2 + 4x + 1$

SOLUTION:

First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

$$y \geq -x^2 + 4x + 1$$
$$1 \overset{?}{\geq} -1^2 + 4(1) + 1$$
$$1 \not\geq 6$$

So, (1, 1) is not a solution of the inequality. Shade the region of the graph that does not contain (1, 1).



CCSS SENSE-MAKING Solve each inequality by graphing.

4. $0 < x^2 - 5x + 4$

SOLUTION:

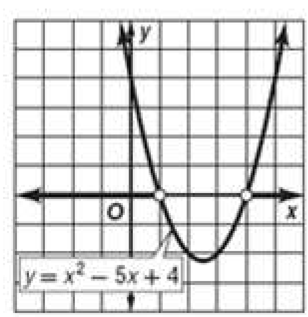
First, write the related equation and factor it.

$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0$$

By the Zero Product Property:

$$x - 1 = 0 \text{ or } \quad x - 4 = 0$$
$$x = 1 \quad \text{or} \quad x = 4$$

Sketch the graph of a parabola that has *x*-intercepts at 1 and 4. The graph should open up because $a > 0$.



The graph lies above the *x*-axis left to $x = 1$ and right to $x = 4$. Thus, the solution set of the inequality is $\{x \mid x < 1 \text{ or } x > 4\}$.

5. $x^2 + 8x + 15 < 0$

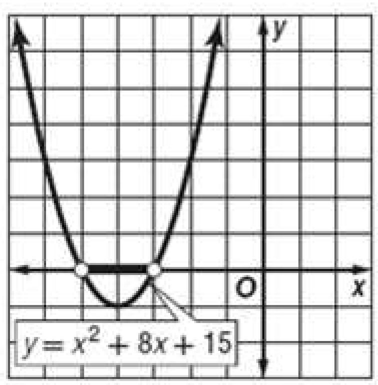
SOLUTION:
First, write the related equation and factor it.

$$x^2 + 8x + 15 = 0$$
$$(x+5)(x+3) = 0$$

By the Zero Product Property:

$$x + 5 = 0 \text{ or } \quad x + 3 = 0$$
$$x = -5 \quad \text{or} \quad x = -3$$

Sketch the graph of a parabola that has *x*-intercepts at –5 and –3. The graph should open up because $a > 0$.



The graph lies below the *x*-axis between $x = -5$ and $x = -3$. Thus, the solution set of the inequality is $\{x \mid -5 < x < -3\}$.

6. $-2x^2 - 2x + 12 \geq 0$

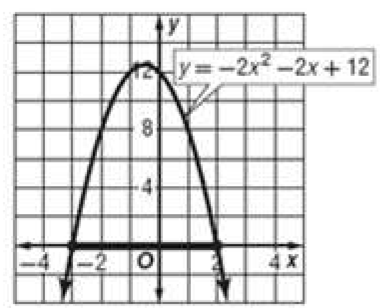
SOLUTION:
First, write the related equation and factor it.

$$-2x^2 - 2x + 12 = 0$$
$$-2(x+3)(x-2) = 0$$

By the Zero Product Property:

$$x + 3 = 0 \text{ or } \quad x - 2 = 0$$
$$x = -3 \quad \text{or} \quad x = 2$$

Sketch the graph of a parabola that has *x*-intercepts at –3 and 2. The graph should open down because $a < 0$.



The graph lies above the *x*-axis between $x = -3$ and $x = 2$ including the two endpoints. Thus, the solution set of the inequality is $\{x \mid -3 \leq x \leq 2\}$.

7. $0 \geq 2x^2 - 4x + 1$

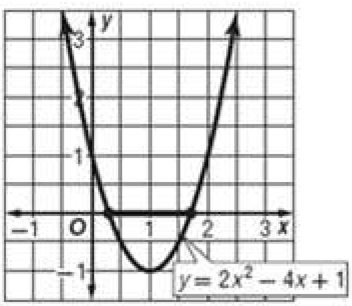
SOLUTION:

First, write the related equation and solve for x.

$$2x^2 - 4x + 1 = 0$$

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} \\ &= \frac{4 \pm \sqrt{8}}{4} \\ &= 1 \pm \frac{\sqrt{2}}{2} \\ &\approx 1.71, 0.29 \end{aligned}$$

Sketch the graph of a parabola that has x-intercepts at 0.29 and 1.71. The graph should open up because $a > 0$.



The graph lies below the x-axis between $x \approx 0.29$ and $x \approx 1.71$ including the two end points. Thus, the solution set of the inequality is $\{x \mid 0.29 \leq x \leq 1.71\}$.

8. **SOCCER** A midfielder kicks a ball toward the goal during a match. The height of the ball in feet above the ground $h(t)$ at time t can be represented by $h(t) = -0.1t^2 + 2.4t + 1.5$. If the height of the goal is 8 feet, at what time during the kick will the ball be able to enter the goal?

SOLUTION:

The height of the goal is 8 ft. So, the ball will enter the goal when $h(t)$ less than or equal to 8.

$$\begin{aligned} -0.1t^2 + 2.4t + 1.5 &\leq 8 \\ -0.1t^2 + 2.4t - 6.5 &\leq 0 \end{aligned}$$

First, write the related equation and solve it.

$$-0.1t^2 + 2.4t - 6.5 = 0$$

$$\begin{aligned} t &= \frac{-2.4 \pm \sqrt{(2.4)^2 - 4(-0.1)(-6.5)}}{2(-0.1)} \\ &= \frac{-2.4 \pm \sqrt{3.16}}{-0.2} \\ &\approx 12 \pm 8.89 \\ &\approx 3.11, 20.89 \end{aligned}$$

That is, at about 3.11 s and 20.89 s the ball will be at the height 8 ft, so it can be inside the goal.
Find the value of t for which the ball will hit the ground, that is, when the height is zero.

$$-0.1t^2 + 2.4t + 1.5 = 0$$

$$\begin{aligned} t &= \frac{-2.4 \pm \sqrt{(2.4)^2 - 4(-0.1)(1.5)}}{2(-0.1)} \\ &= \frac{-2.4 \pm \sqrt{3.16}}{-0.2} \\ &\approx 12 \pm 12.61 \\ &\approx -0.61, 24.61 \end{aligned}$$

Since a negative value for time has no meaning, discard the negative solution. So, the ball hits the ground in about 24.61 seconds. The ball will be inside the goal when $0 < t < 3.11$ or $20.89 < t \leq 24.61$.

Solve each inequality algebraically.

9. $x^2 + 6x - 16 < 0$

SOLUTION:
First, write the related equation and factor it.]

$$x^2 + 6x - 16 = 0$$
$$(x+8)(x-2) = 0$$

By the Zero Product Property:

$$x + 8 = 0 \text{or} \qquad x - 2 = 0$$
$$x = -8 \quad \text{or} \qquad x = 2$$

The two numbers divide the number line into three regions: $x \le -8$, $-8 < x < 2$ and $x \ge 2$. Test a value from each interval to see if it satisfies the original inequality.

$x \le -8$	$-8 < x < 2$	$x \ge 2$
$(-10)^2 + 6(-10) - 16 \overset{?}{<} 0$	$(0)^2 + 6(0) - 16 \overset{?}{<} 0$	$(5)^2 + 6(5) - 16 \overset{?}{<} 0$
$100 - 60 - 16 \overset{?}{<} 0$	$-16 < 0$ ✓	$25 + 30 - 16 \overset{?}{<} 0$
$24 \not< 0$ ✗		$39 \not< 0$ ✗

Note that, the points $x = -8$ and $x = 2$ are not included in the solution. Therefore, the solution set is $\{x \mid -8 < x < 2\}$.

10. $x^2 - 14x > -49$

SOLUTION:
First, write the related equation and factor it.

$$x^2 - 14x + 49 = 0$$
$$(x-7)(x-7) = 0$$

By the Zero Product Property:

$$x - 7 = 0$$
$$x = 7$$

The number 7 divides the number line into three regions: $x < 7$, $x = 7$ and $x > 7$. Test a value from each interval to see if it satisfies the original inequality.

$x < 7$	$x = 7$	$x > 7$
$(2)^2 - 14(2) + 49 \overset{?}{>} 0$	$(7)^2 - 14(7) + 49 \overset{?}{>} 0$	$(10)^2 - 14(10) + 49 \overset{?}{>} 0$
$4 - 28 + 49 \overset{?}{>} 0$	$49 - 98 + 49 \overset{?}{>} 0$	$100 - 140 + 49 \overset{?}{>} 0$
$25 > 0$ ✓	$0 \not> 0$ ✗	$9 > 0$ ✓

Therefore, the solution set is $\{x \mid x < 7 \text{ or } x > 7\}$.

11. $-x^2 + 12x \geq 28$

SOLUTION:
First, write the related equation and factor it.

$$
\begin{aligned}
&-x^2 + 12x - 28 = 0 \\
x &= \frac{-12 \pm \sqrt{(12)^2 - 4(-28)(-1)}}{2(-1)} \\
&= \frac{-12 \pm \sqrt{32}}{-2} \\
&= 6 \pm 2\sqrt{2} \\
&\approx 8.83, 3.17
\end{aligned}
$$

The two numbers divide the number line into three regions $x \leq 3.17$, $3.17 \leq x \leq 8.83$ and $x \geq 8.83$. Test a value from each interval to see if it satisfies the original inequality.

$x \leq 3.17$	$3.17 \leq x \leq 8.83$	$x \geq 8.83$
$-(2)^2 + 12(2) - 28 \overset{?}{\geq} 0$	$-(4)^2 + 12(4) - 28 \overset{?}{\geq} 0$	$-(9)^2 + 12(9) - 28 \overset{?}{\geq} 0$
$-4 + 24 - 28 \overset{?}{\geq} 0$	$-16 + 48 - 28 \overset{?}{\geq} 0$	$-81 + 108 - 28 \overset{?}{\geq} 0$
$-8 \not\geq 0$	$4 \geq 0$	$-1 \not\geq 0$

Note that, the points $x = 3.17$ and $x = 8.83$ are also included in the solution.
Therefore, the solution set is $\{x \mid 3.17 \leq x \leq 8.83\}$.

12. $x^2 - 4x \leq 21$

SOLUTION:
First, write the related equation and factor it.

$$
\begin{aligned}
&x^2 - 4x - 21 = 0 \\
&(x - 7)(x + 3) = 0 \\
&x = 7 \quad \text{or} \quad x = -3
\end{aligned}
$$

The two numbers divide the number line into three regions $x \leq -3$, $-3 \leq x \leq 7$ and $x \geq 7$. Test a value from each interval to see if it satisfies the original inequality.

$x \leq -3$	$-3 \leq x \leq 7$	$x \geq 7$
$(-4)^2 - 4(-4) - 21 \overset{?}{\leq} 0$	$(4)^2 - 4(4) - 21 \overset{?}{\leq} 0$	$(8)^2 - 4(8) - 21 \overset{?}{\leq} 0$
$16 + 16 - 21 \overset{?}{\leq} 0$	$16 - 16 - 21 \overset{?}{\leq} 0$	$64 - 32 - 21 \overset{?}{\leq} 0$
$11 \not\leq 0$	$-21 \leq 0$	$11 \not\leq 0$

Note that, the points $x = -3$ and $x = 7$ are also included in the solution.
Therefore, the solution set is $\{x \mid -3 \leq x \leq 7\}$.

Graph each inequality.

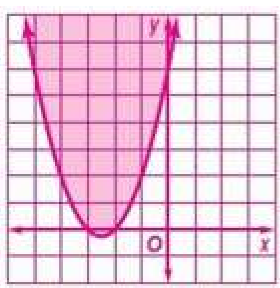
13. $y \geq x^2 + 5x + 6$

SOLUTION:

First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

$$y \geq x^2 + 5x + 6$$
$$0 \overset{?}{\geq} 0^2 + 5(0) + 6$$
$$0 \ngeq 6$$

So, (0, 0) is not a solution of the inequality. Shade the region of the graph that does not contain (0, 0).



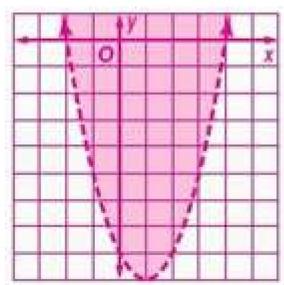
14. $x^2 - 2x - 8 < y$

SOLUTION:

First graph the related function. The parabola should be dashed. Next test a point not on the graph of the parabola.

$$y > x^2 - 2x - 8$$
$$0 \overset{?}{>} 0^2 - 2(0) - 8$$
$$0 > -8$$

So, (0, 0) is a solution of the inequality. Shade the region of the graph that contains (0, 0).



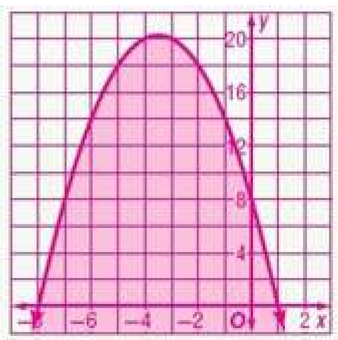
15. $y \leq -x^2 - 7x + 8$

SOLUTION:

First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

$$y \leq -x^2 - 7x + 8$$
$$0 \overset{?}{\leq} -0^2 - 7(0) + 8$$
$$0 \leq 8$$

So, (0, 0) is a solution of the inequality. Shade the region of the graph that contains (0, 0).



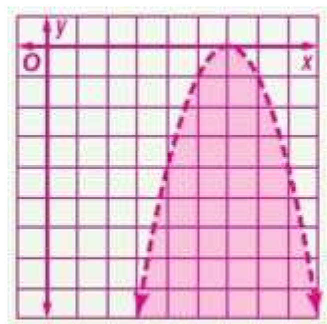
16. $-x^2 + 12x - 36 > y$

SOLUTION:

First graph the related function. The parabola should be dashed. Next test a point not on the graph of the parabola.

$$-x^2 + 12x - 36 > y$$
$$-0^2 + 12(0) - 36 \overset{?}{>} 0$$
$$-36 \ngtr 0$$

So, (0, 0) is not a solution of the inequality. Shade the region of the graph that does not contain (0, 0).



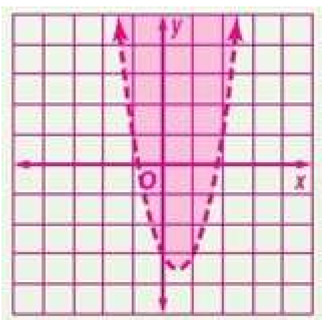
17. $y > 2x^2 - 2x - 3$

SOLUTION:

First graph the related function. The parabola should be dashed. Next test a point not on the graph of the parabola.

$$y > 2x^2 - 2x - 3$$
$$0 \overset{?}{>} 2(0)^2 - 2(0) - 3$$
$$0 > -3$$

So, (0, 0) is a solution of the inequality. Shade the region of the graph that contains (0, 0).



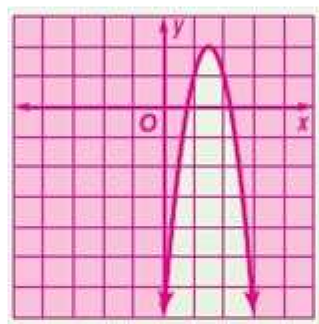
18. $y \geq -4x^2 + 12x - 7$

SOLUTION:

First graph the related function. The parabola should be solid. Next test a point not on the graph of the parabola.

$$y \geq -4x^2 + 12x - 7$$
$$0 \overset{?}{\geq} -4(0)^2 + 12(0) - 7$$
$$0 \geq -7$$

So, (0, 0) is a solution of the inequality. Shade the region of the graph that contains (0, 0).



Solve each inequality by graphing.

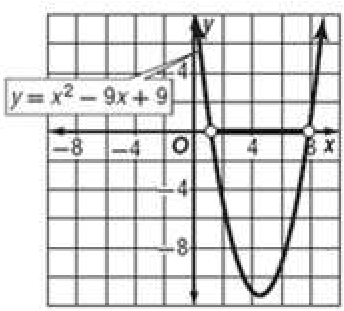
19. $x^2 - 9x + 9 < 0$

SOLUTION:

First, write the related equation and solve it.

$$x^2 - 9x + 9 = 0$$
$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(9)}}{2(1)}$$
$$= \frac{9 \pm \sqrt{45}}{2}$$
$$\approx 1.1 \text{ or } 7.9$$

Sketch the graph of a parabola that has *x*-intercepts at 1.1 and 7.9. The graph should open up because *a* > 0.



The graph lies below the *x*-axis between *x* = 1.1 and *x* = 7.9. Thus, the solution set of the inequality is $\{x \mid 1.1 < x < 7.9\}$.

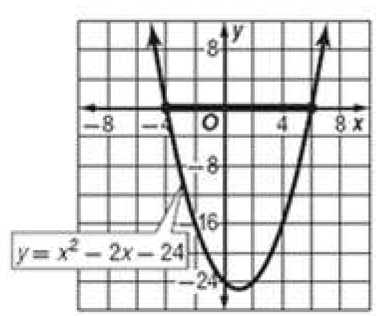
20. $x^2 - 2x - 24 \le 0$

SOLUTION:

First, write the related equation and factor it.

$$x^2 - 2x - 24 = 0$$
$$(x - 6)(x + 4) = 0$$
$$x = 6 \quad \text{or} \quad x = -4$$

Sketch the graph of a parabola that has *x*-intercepts at –4 and 6. The graph should open up because *a* > 0.



The graph lies below the *x*-axis between *x* = –4 and *x* = 6 including the two end points. Thus, the solution set of the inequality is $\{x \mid -4 \le x \le 6\}$.

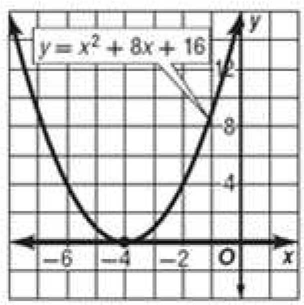
21. $x^2+8x+16\geq 0$

SOLUTION:

First, write the related equation and factor it.

$$x^2+8x+16=0$$
$$(x+4)^2=0$$
$$x+4=0$$
$$x=-4$$

The equation has only one real root.
The graph should open up because $a > 0$.



The graph lies completely above the *x*-axis. Thus, the solution set of the inequality is {*x* | all real numbers}.

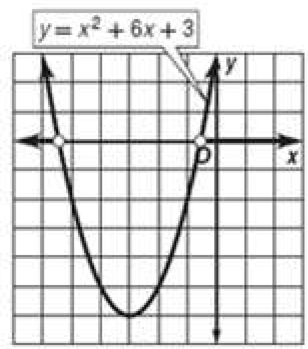
22. $x^2+6x+3>0$

SOLUTION:

First, write the related equation and solve it.

$$x^2+6x+3=0$$

Sketch the graph of a parabola that has *x*-intercepts at –5.45 and –0.55. The graph should open up because $a > 0$.



The graph lies above the *x*-axis left to $x = -5.45$ and right to $x = -0.55$. Thus, the solution set of the inequality is $\{x \mid x < -5.45 \text{ or } x > -0.55\}$.

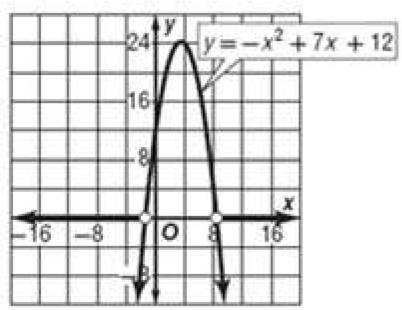
23. $0 > -x^2 + 7x + 12$

SOLUTION:

First, write the related equation and solve it.

$$-x^2 + 7x + 12 = 0$$
$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(-1)(12)}}{2(-1)}$$
$$\approx -1.42 \text{ or } 8.42$$

Sketch the graph of a parabola that has x-intercepts at –1.42 and 8.42. The graph should open down because $a < 0$.



The graph lies below the x-axis left to $x = -1.42$ and right to $x = 8.42$. Thus, the solution set of the inequality is $\{x \mid x < -1.42 \text{ or } x > 8.42\}$.

24. $-x^2 + 2x - 15 < 0$

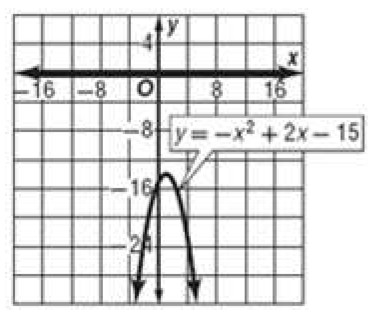
SOLUTION:

First, write the related equation and factor it.

$$-x^2 + 2x - 15 = 0$$
$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(-1)(-15)}}{2(-1)}$$
$$= \frac{-2 \pm \sqrt{-56}}{2}$$
$$\approx -1 + 7.48i, -1 - 7.48i$$

The equation does not have real roots. The parabola does not intersect the x-axis.

The graph should open down because $a < 0$.



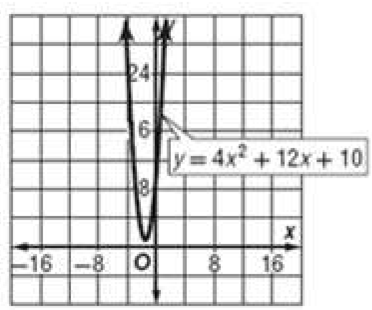
The graph lies completely below the x-axis. Thus, the solution set of the inequality is $\{x \mid \text{all real numbers}\}$.

25. $4x^2 + 12x + 10 \leq 0$

SOLUTION:
First, write the related equation and factor it.

$$4x^2 + 12x + 10 = 0$$
$$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(4)(10)}}{2(4)}$$
$$= \frac{-12 \pm \sqrt{-16}}{8}$$
$$\approx -\frac{3}{2} + \frac{1}{2}i, -\frac{3}{2} - \frac{1}{2}i$$

The equation does not have real roots. The parabola does not intersect the x-axis.
The graph should open up because $a > 0$.



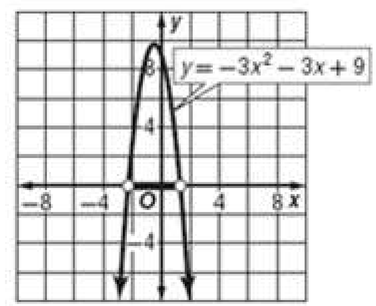
No part of the graph lies below the x-axis. Thus, the solution set of the inequality is $\varnothing$.

26. $-3x^2 - 3x + 9 > 0$

SOLUTION:
First, write the related equation and solve it.

$$-3x^2 - 3x + 9 = 0$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-3)(9)}}{2(-3)}$$
$$\approx 1.30 \text{ or } -2.30$$

Sketch the graph of a parabola that has x-intercepts at 1.30 and –2.30. The graph should open down because $a < 0$.



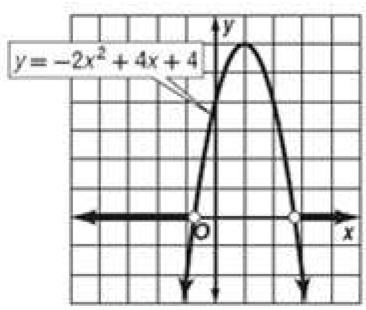
The graph lies above the x-axis between $x \approx -2.30$ and $x \approx 1.30$. Thus, the solution set of the inequality is $\{x \mid -2.30 < x < 1.30\}$.

27. $0 > -2x^2 + 4x + 4$

SOLUTION:
First, write the related equation and solve it.

$$-2x^2 + 4x + 4 = 0$$
$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(-2)(4)}}{2(-2)}$$
$$= 1 \pm \sqrt{3}$$
$$\approx -0.73 \text{ or } 2.73$$

Sketch the graph of a parabola that has *x*-intercepts at –0.73 and 2.73. The graph should open down because $a < 0$.



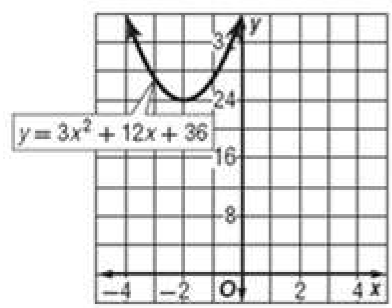
The graph lies below the *x*-axis left to $x \approx -0.73$ and right to $x \approx 2.73$. Thus, the solution set of the inequality is $\{x \mid x < -0.73 \text{ or } x > 2.73\}$.

28. $3x^2 + 12x + 36 \le 0$

SOLUTION:
First, write the related equation and solve it.

$$3x^2 + 12x + 36 = 0$$
$$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(3)(36)}}{2(3)}$$
$$= \frac{-12 \pm i12\sqrt{2}}{6}$$

The equation does not have any real roots. The parabola does not intersect the *x*-axis.
The graph should open up because $a > 0$.



The graph lies entirely above the *x*-axis. Thus, the solution set of the inequality is $\varnothing$.

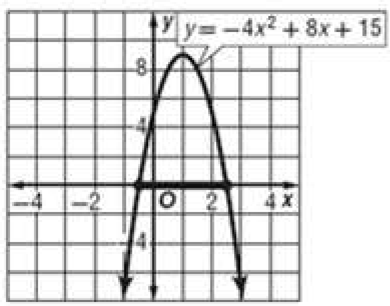
29. $0 \le -4x^2 + 8x + 5$

SOLUTION:

First, write the related equation and solve it.

$$-4x^2 + 8x + 5 = 0$$
$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(-4)(5)}}{2(-4)}$$
$$= -0.5 \text{ or } 2.5$$

Sketch the graph of a parabola that has x-intercepts at –0.5 and 2.5. The graph should open down because $a < 0$.



The graph lies above the x-axis between $x = -0.5$ and $x = 2.5$ including the two endpoints.. Thus, the solution set of the inequality is $\{x \mid -0.5 \le x \le 2.5\}$.

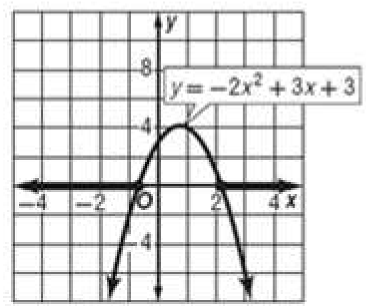
30. $-2x^2 + 3x + 3 \le 0$

SOLUTION:

First, write the related equation and solve it.

$$-2x^2 + 3x + 3 = 0$$
$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(-2)(3)}}{2(-2)}$$
$$= \frac{-3 \pm \sqrt{33}}{-4}$$
$$\approx -0.69 \text{ or } 2.19$$

Sketch the graph of a parabola that has x-intercepts at –0.69 and 2.19. The graph should open down because $a < 0$.



The graph lies below the x-axis left to $x \approx -0.69$ and right to $x \approx 2.19$ including the two end points. Thus, the solution set of the inequality is $\{x \mid x \le -0.69 \text{ or } x \le 2.19\}$.

31. **ARCHITECTURE** An arched entry of a room is shaped like a parabola that can be represented by the equation $f(x) = -x^2 + 6x + 1$. How far from the sides of the arch is its height at least 7 feet?

SOLUTION:
The height is at least 7 feet for all the places where the value of $f(x)$ is greater than or equal to 7. That is,

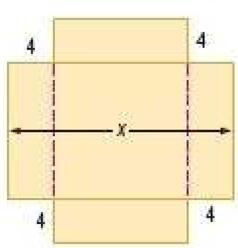
$$-x^2 + 6x + 1 \geq 7 \text{ or } -x^2 + 6x - 6 \geq 0.$$

Write the related equation and solve it.

$$\begin{aligned}-x^2 + 6x - 6 &= 0\\ x &= \frac{-(6) \pm \sqrt{(6)^2 - 4(-1)(-6)}}{2(-1)}\\ &= \frac{-6 \pm \sqrt{12}}{-2}\\ &\approx 1.27 \text{ or } 4.73\end{aligned}$$

The graph of the parabola representing the equation $y = -x^2 + 6x - 6$ lies above the x-axis between $x \approx 1.27$ and $x \approx 4.73$ including the two end points. Thus, the solution set of the inequality is $\{x \mid 1.27 \leq x \leq 4.73\}$.
Therefore, from 1.27 ft to 4.73 ft, the height will be at least 7 feet.

32. **MANUFACTURING** A box is formed by cutting 4-inch squares from each corner of a square piece of cardboard and then folding the sides. If $V(x) = 4x^2 - 64x + 256$ represents the volume of the box, what should the dimensions of the original piece of cardboard be if the volume of the box cannot exceed 750 cubic inches?



SOLUTION:

Substitute 0 for $V(x)$ and solve for x.

$$\begin{aligned}4x^2 - 64x + 256 &= 0\\ 4(x^2 - 16x + 64) &= 0\\ 4(x-8)^2 &= 0\\ x - 8 &= 0\\ x &= 8\end{aligned}$$

Therefore, x must be greater than 8 in.
The volume of the box should be less than or equal to 750 cu. in. That is,

$$4x^2 - 64x + 256 \leq 750 \text{ or } 4x^2 - 64x - 494 \leq 0.$$

Write the related equation and solve it.

$$\begin{aligned}4x^2 - 64x - 494 &= 0\\ x &= \frac{-(-64) \pm \sqrt{(-64)^2 - 4(4)(-494)}}{2(4)}\\ &= \frac{64 \pm \sqrt{12000}}{8}\\ &\approx -5.69 \text{ or } 21.69\end{aligned}$$

The value of x should be positive.
So, the maximum width should be 21.69 in.
The dimensions of the original card board is greater than 8 in. but no more than 21.69 in.

Solve each inequality algebraically.

33. $x^2 - 9x < -20$

SOLUTION:
First, write the related equation and solve it.

$$\begin{aligned} x^2 - 9x &= -20 \\ x^2 - 9x + 20 &= 0 \\ (x-5)(x-4) &= 0 \end{aligned}$$

By the Zero Product Property:

$$\begin{aligned} x - 5 = 0 \quad &\text{or} \quad x - 4 = 0 \\ x = 5 \quad &\text{or} \quad x = 4 \end{aligned}$$

The two numbers divide the number line into three regions $x \le 4$, $4 < x < 5$ and $x \ge 5$. Test a value from each interval to see if it satisfies the original inequality.

$x \le 4$	$4 < x < 5$	$x \ge 5$
Test $x = 2$	Test $x = 4.5$	Test $x = 6$
$x^2 - 9x + 20 < 0$	$x^2 - 9x + 20 < 0$	$x^2 - 9x + 20 < 0$
$2^2 - 9(2) + 20 \overset{?}{<} 0$	$(4.5)^2 - 9(4.5) + 20 \overset{?}{<} 0$	$6^2 - 9(6) + 20 \overset{?}{<} 0$
$6 \not< 0$ ✗	$-0.25 < 0$ ✓	$2 \not< 0$ ✗

Note that the points $x = 4$ and $x = 5$ are not included in the solution. Therefore, the solution set is $\{x \mid 4 < x < 5\}$.

34. $x^2 + 7x \ge -10$

SOLUTION:
First, write the related equation and solve it.

$$\begin{aligned} x^2 + 7x &= -10 \\ x^2 + 7x + 10 &= 0 \\ (x+5)(x+2) &= 0 \end{aligned}$$

By the Zero Product Property:

$$\begin{aligned} x + 5 = 0 \quad &\text{or} \quad x + 2 = 0 \\ x = -5 \quad &\text{or} \quad x = -2 \end{aligned}$$

The two numbers divide the number line into three regions $x \le -5$, $-5 \le x \le -2$ and $x \ge -2$. Test a value from each interval to see if it satisfies the original inequality.

$x \le -5$	$-5 \le x \le -2$	$x \ge -2$
Test $x = -7$	Test $x = -3$	Test $x = 0$
$x^2 + 7x \ge -10$	$x^2 + 7x \ge -10$	$x^2 + 7x \ge -10$
$(-7)^2 + 7(-7) \overset{?}{\ge} -10$	$(-3)^2 + 7(-3) \overset{?}{\ge} -10$	$0^2 + 7(0) \overset{?}{\ge} -10$
$0 \ge -10$ ✓	$-12 \not\ge -10$ ✗	$0 \ge -10$ ✓

Therefore, the solution set is $\{x \mid x \le -5 \text{ or } x \ge -2\}$.

3-4 Quadratic Inequalities

35. $2 > x^2 - x$

SOLUTION:

First, write the related equation and solve it.

$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$

By the Zero Product Property:

$$x - 2 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 2 \quad \text{or} \quad x = -1$$

The two numbers divide the number line into three regions $x \le -1$, $-1 < x < 2$ and $x \ge 2$. Test a value from each interval to see if it satisfies the original inequality.

$x < -1$	$-1 < x < 2$	$x > 2$
Test $x = -2$	Test $x = 1$	Test $x = 4$
$2 > x^2 - x$	$2 > x^2 - x$	$2 > x^2 - x$
$2 \overset{?}{>} (-2)^2 - (-2)$	$2 \overset{?}{>} 1^2 - (1)$	$2 \overset{?}{>} 4^2 - 4$
$2 \not> 6$ ✗	$2 > 0$ ✓	$2 \not> 12$ ✗

Note that the points $x = -1$ and $x = 2$ are not included in the solution. Therefore, the solution set is $\{x \mid -1 < x < 2\}$.

36. $-3 \le -x^2 - 4x$

SOLUTION:

First, write the related equation and solve it.

$$x^2 + 4x - 3 = 0$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{28}}{2}$$
$$= -2 \pm \sqrt{7}$$
$$\approx -4.65 \text{ or } 0.65$$

The two numbers divide the number line into three regions $x \le -4.65$, $-4.65 \le x \le 0.65$ and $x \ge 0.65$. Test a value from each interval to see if it satisfies the original inequality.

$x \le -4.65$	$-4.65 \le x \le 0.65$	$x \ge 0.65$
Test $x = -5$	Test $x = 0$	Test $x = 1$
$-3 \le -x^2 - 4x$	$-3 \le -x^2 - 4x$	$-3 \le -x^2 - 4x$
$-3 \overset{?}{\le} -(-5)^2 - 4(-5)$	$-3 \overset{?}{\le} -(0)^2 - 4(0)$	$-3 \overset{?}{\le} -(1)^2 - 4(1)$
$-3 \not\le -5$ ✗	$-3 \le 0$ ✓	$-3 \not\le -5$ ✗

Therefore, the solution set is $\{x \mid -4.65 \le x \le 0.65\}$.

3-4 Quadratic Inequalities

37. $-x^2+2x\le -10$

SOLUTION:

First, write the related equation and solve it.

$$\begin{aligned}x^2-2x-10&=0\\x&=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-10)}}{2(1)}\\&=\frac{2\pm\sqrt{44}}{2}\\&=1\pm\sqrt{11}\\&\approx -2.32 \text{ or } 4.32\end{aligned}$$

The two numbers divide the number line into three regions $x \le -2.32$, $-2.32 \le x \le 4.32$ and $x \ge 4.32$. Test a value from each interval to see if it satisfies the original inequality.

$x\le -2.32$	$-2.32\le x\le 4.32$	$x\ge 4.32$
Test $x=-3$	Test $x=0$	Test $x=5$
$-x^2+2x\le -10$	$-x^2+2x\le -10$	$-x^2+2x\le -10$
$-(-3)^2+2(-3)\overset{?}{\le}-10$	$-(0)^2+2(0)\overset{?}{\le}-10$	$-(5)^2+2(5)\overset{?}{\le}-10$
$-15\le -10$ ✓	$0\not\le -10$ ✗	$-15\le -10$ ✓

Therefore, the solution set is $\{x \mid x \le -2.32$ or $x \ge 4.32\}$.

38. $-6>x^2+4x$

SOLUTION:

$$\begin{aligned}-6&>x^2+4x\\0&>x^2+4x+6\\0&<-(x^2+4x+6)\end{aligned}$$

The solution is empty set, as no real value for x will satisfy the inequality.

39. $2x^2+4\ge 9$

SOLUTION:

First, write the related equation and solve it.

$$\begin{aligned}2x^2-5&=0\\2x^2&=5\\x^2&=2.5\\x&=\pm\sqrt{2.5}\\x&\approx -1.58 \text{ or } 1.58\end{aligned}$$

The two numbers divide the number line into three regions $x \le -1.58$, $-1.58 \le x \le 1.58$ and $x \ge 1.58$. Test a value from each interval to see if it satisfies the original inequality.

$x\le -1.58$	$-1.58\le x\le 1.58$	$x\ge 1.58$
Test $x=-2$	Test $x=0$	Test $x=2$
$2x^2+4\ge 9$	$2x^2+4\ge 9$	$2x^2+4\ge 9$
$2(-2)^2+4\overset{?}{\ge}9$	$2(0)^2+4\overset{?}{\ge}9$	$2(2)^2+4\overset{?}{\ge}9$
$12\ge 9$ ✓	$4\not\ge 9$ ✗	$12\ge 9$ ✓

Therefore, the solution set is $\{x \mid x \le -1.58$ or $x \ge 1.58\}$.

40. $3x^2+x\ge -3$

SOLUTION:

$$\begin{aligned}3x^2+x&\ge -3\\3x^2+x+3&\ge 0\end{aligned}$$

The solution set of the quadratic inequality is all real numbers, as all the real numbers satisfy the inequality.

Solution set: $\{x \mid$ all real numbers$\}$

3-4 Quadratic Inequalities

41. $-4x^2 + 2x < 3$

SOLUTION:

$$\begin{aligned} -4x^2 + 2x &< 3 \\ -4x^2 + 2x - 3 &< 0 \end{aligned}$$

The solution set of the quadratic inequality is all real numbers, as all the real numbers satisfy the inequality.
Solution set: {x | all real numbers}

42. $-11 \geq -2x^2 - 5x$

SOLUTION:
First, write the related equation and solve it.

$$\begin{aligned} &2x^2 + 5x - 11 = 0 \\ &x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-11)}}{2(2)} \\ &\quad = \frac{-5 \pm \sqrt{113}}{4} \\ &\quad \approx -3.91 \text{ or } 1.41 \end{aligned}$$

The two numbers divide the number line into three regions $x \leq -3.91$, $-3.91 \leq x \leq 1.41$ and $x \geq 1.41$. Test a value from each interval to see if it satisfies the original inequality.

$x \leq -3.91$	$-3.91 \leq x \leq 1.41$	$x \geq 1.41$
Test $x = -4$	Test $x = 1$	Test $x = 2$
$-11 \geq -2x^2 - 5x$	$-11 \geq -2x^2 - 5x$	$-11 \geq -2x^2 - 5x$
$-11 \overset{?}{\geq} -2(-4)^2 - 5(-4)$	$-11 \overset{?}{\geq} -2(1)^2 - 5(1)$	$-11 \overset{?}{\geq} -2(2)^2 - 5(2)$
$-11 \geq -12$ ✓	$-11 \ngeq 0$ ✗	$-11 \geq -18$ ✓

Therefore, the solution set is {x | $x \leq -3.91$ or $x \geq 1.41$}.

43. $-12 < -5x^2 - 10x$

SOLUTION:
First, write the related equation and solve it.

$$\begin{aligned} &5x^2 + 10x - 12 = 0 \\ &x = \frac{-10 \pm \sqrt{10^2 - 4(5)(-12)}}{2(5)} \\ &\quad = \frac{-10 \pm \sqrt{340}}{10} \\ &\quad = \frac{-5 \pm \sqrt{85}}{5} \\ &\quad \approx -2.84 \text{ or } 0.84 \end{aligned}$$

The two numbers divide the number line into three regions $x \leq -2.84$, $-2.84 < x < 0.84$ and $x \geq 0.84$. Test a value from each interval to see if it satisfies the original inequality.

$x \leq -2.84$	$-2.84 < x < 0.84$	$x \geq 0.84$
Test $x = -3$	Test $x = 0$	Test $x = 1$
$-12 < -5x^2 - 10x$	$-12 < -5x^2 - 10x$	$-12 < -5x^2 - 10x$
$-12 \overset{?}{<} -5(-3)^2 - 10(-3)$	$-12 \overset{?}{<} -5(0)^2 - 10(0)$	$-12 \overset{?}{<} -5(1)^2 - 10(1)$
$-12 \nless -15$ ✗	$-12 < 0$ ✓	$-12 \nless -15$ ✗

Therefore, the solution set is {x | $-2.84 < x < 0.84$}.

44. $-3x^2 - 10x > -1$

SOLUTION:

First, write the related equation and solve it.

$$3x^2 + 10x - 1 < 0$$
$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-1)}}{2(3)}$$
$$= \frac{-10 \pm \sqrt{112}}{6}$$
$$= \frac{-5 \pm 2\sqrt{7}}{3}$$
$$\approx -3.43 \text{ or } 0.10$$

The two numbers divide the number line into three regions $x \le -3.43$, $-3.43 < x < 0.10$ and $x \ge 0.10$. Test a value from each interval to see if it satisfies the original inequality.

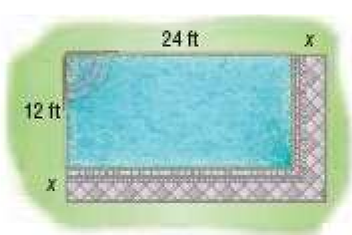
$x \le -3.43$	$-3.43 < x < 0.10$	$x \ge 0.10$
Test $x = -4$	Test $x = 0$	Test $x = 1$
$-3x^2 - 10x > -1$	$-3x^2 - 10x > -1$	$-3x^2 - 10x > -1$
$-3(-4)^2 - 10(-4) \overset{?}{>} -1$	$-3(0)^2 - 10(0) \overset{?}{>} -1$	$-3(1)^2 - 10(1) \overset{?}{>} -1$
$-8 \not> -1$ ✗	$0 > -1$ ✓	$-13 \not> -1$ ✗

Therefore, the solution set is $\{x \mid -3.43 < x < 0.10\}$.

45. **CCSS PERSEVERANCE** The Sanchez family is adding a deck along two sides of their swimming pool. The deck width will be the same on both sides and the total area of the pool and deck cannot exceed 750 square feet.

a. Graph the quadratic inequality.

b. Determine the maximum width of the deck.

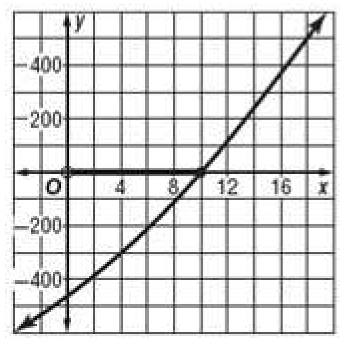


SOLUTION:

a. The area of the pool including the deck is (24 + x)(12 + x). This should be less than or equal to 750 sq. ft.

$$(24 + x)(12 + x) \le 750$$
$$x^2 + 36x + 288 \le 750$$
$$x^2 + 36x - 462 \le 0$$

Graph the inequality on a coordinate plane.

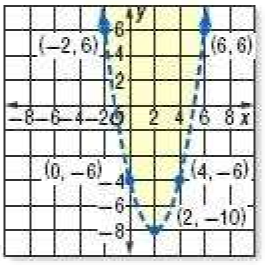


b. Write the related equation and factor it.

$$x^2 + 36x - 462 = 0$$
$$x = \frac{-(36) \pm \sqrt{(36)^2 - 4(1)(-462)}}{2(1)}$$
$$= \frac{-36 \pm \sqrt{3144}}{2}$$
$$\approx -46.04,\ 10.04$$

Here, x is a length, so it cannot be negative. So, the maximum width is 10.04 ft. For the same reason, the minimum length should be greater than 0.

Write a quadratic inequality for each graph.

46. 

SOLUTION:

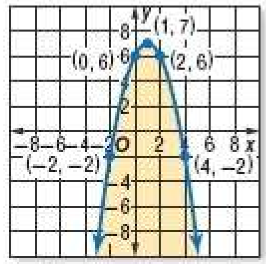
The coordinates of the vertex of the parabola is (2, –10). So, the equation of the parabola is $y = a(x - 2)^2 - 10$. Use any pair of points on the parabola to find the value of a.

$$-6 = a(4-2)^2 - 10$$
$$4 = 4a$$
$$1 = a$$

So, the equation of the parabola is

$$y = 1(x^2 - 4x + 4) - 10$$
$$y = x^2 - 4x - 6$$

The boundary line is dashed and the region above the line is shaded. So, the inequality is $y > x^2 - 4x - 6$.

47. 

SOLUTION:

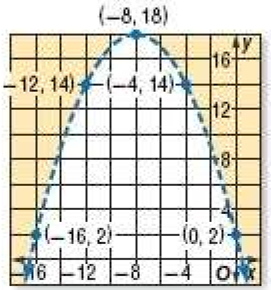
The coordinates of the vertex of the parabola is (1, 7). So, the equation of the parabola is $y = a(x - 1)^2 + 7$. Use any pair of points on the parabola to find the value of a.

$$6 = a(2-1)^2 + 7$$
$$-1 = a$$

So, the equation of the parabola is

$$y = -1(x^2 - 2x + 1) + 7.$$
$$y = -x^2 + 2x + 6$$

The boundary line is a solid line and the region below the line is shaded. So, the inequality is $y \le -x^2 + 2x + 6$.

48. 

SOLUTION:

The coordinates of the vertex of the parabola is (–8, 18). So, the equation of the parabola is $y = a(x + 8)^2 + 18$. Use any pair of points on the parabola to find the value of a.

$$14 = a(-4+8)^2 + 18$$
$$-4 = 16a$$
$$-\frac{1}{4} = a$$

So, the equation of the parabola is

$$y = -\frac{1}{4}\left(x^2 + 16x + 64\right) + 18.$$
$$y = -\frac{1}{4}x^2 - 4x + 2$$

The boundary line is dashed and the region above the line is shaded. So, the inequality is $y > -0.25x^2 - 4x + 2$.

Solve each quadratic inequality by using a graph, a table, or algebraically.

49. $-2x^2 + 12x < -15$

SOLUTION:

First, write the related equation and solve it.

$$-2x^2 + 12x + 15 = 0$$
$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(-2)(15)}}{2(-2)}$$
$$= \frac{-12 \pm \sqrt{264}}{-4}$$
$$\approx -1.06 \text{ or } 7.06$$

The two numbers divide the number line into three regions $x < -1.06$, $-1.06 \le x \le 7.06$ and $x > 7.06$. Test a value from each interval to see if it satisfies the original inequality.

$x < -1.06$	$-1.06 \le x \le 7.06$	$x > 7.06$
$-2(-2)^2 + 12(-2) \stackrel{?}{<} -15$	$-2(6)^2 + 12(6) \stackrel{?}{<} -15$	$-2(8)^2 + 12(8) \stackrel{?}{<} -15$
$-8 - 24 \stackrel{?}{<} -15$	$-72 + 72 \stackrel{?}{<} -15$	$-128 + 86 \stackrel{?}{<} -15$
$-32 < -15$ ✓	$0 \not< -15$ ✗	$-42 < -15$ ✓

Note that, the points $x = -1.06$ and $x = 7.06$ are not included in the solution. Therefore, the solution set is $\{x \mid x < -1.06 \text{ or } x > 7.06\}$.

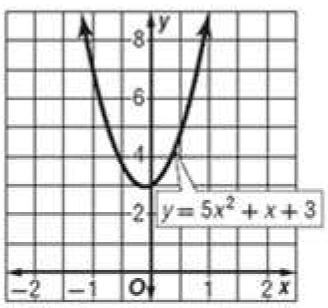
50. $5x^2 + x + 3 \geq 0$

SOLUTION:

First, write the related equation and solve it.

$$5x^2 + x + 3 = 0$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(5)(3)}}{2(5)}$$
$$= \frac{-1 \pm \sqrt{-59}}{10}$$

The equation does not have real roots. Graph the function.
The graph should open up because $a > 0$.



The graph lies completely above the x-axis. Thus, the solution set of the inequality is $\{x \mid \text{all real numbers}\}$.

51. $11 \leq 4x^2 + 7x$

SOLUTION:

First, write the related equation and solve it.

$$4x^2 + 7x - 11 = 0$$
$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(-11)}}{2(4)}$$
$$= \frac{-7 \pm 15}{8}$$
$$= -2.75 \text{ or } 1$$

The two numbers divide the number line into three regions $x \leq -2.75$, $-2.75 \leq x \leq 1$ and $x \geq 1$. Test a value from each interval to see if it satisfies the original inequality.

$x \leq -2.75$	$-2.75 \leq x \leq 1$	$x \geq 1$
$11 \overset{?}{\leq} 4(-3)^2 + 7(-3)$	$11 \overset{?}{\leq} 4(-1)^2 + 7(-1)$	$11 \overset{?}{\leq} 4(2)^2 + 7(2)$
$11 \overset{?}{\leq} 36 - 21$	$11 \overset{?}{\leq} 4 - 7$	$11 \overset{?}{\leq} 16 + 14$
$11 \leq 15$ ✓	$11 \not\leq -3$ ✗	$11 \leq 30$ ✓

Therefore, the solution set is $\{x \mid x \leq -2.75 \text{ or } x \geq 1\}$.

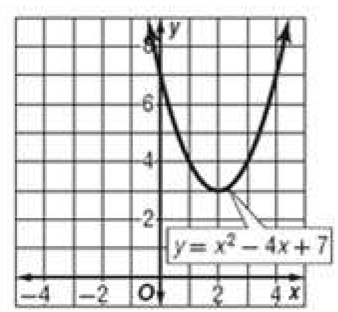
52. $x^2 - 4x \le -7$

SOLUTION:

First, write the related equation and solve it.

$$x^2 - 4x + 7 = 0$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{-12}}{2}$$

The equation does not have real roots.
The graph should open up because $a > 0$.



No part of the graph lies below the x-axis. Thus, the solution set of the inequality is $\varnothing$.

53. $-3x^2 + 10x < 5$

SOLUTION:

First, write the related equation and solve it.

$$-3x^2 + 10x - 5 = 0$$
$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(-3)(-5)}}{2(-3)}$$
$$= \frac{-10 \pm \sqrt{40}}{-6}$$
$$\approx 0.61 \text{ or } 2.72$$

The two numbers divide the number line into three regions $x < 0.61$, $0.61 \le x \le 2.72$ and $x > 2.72$. Test a value from each interval to see if it satisfies the original inequality.

$x < 0.61$	$0.61 \le x \le 2.72$	$x > 2.72$
$-3(0)^2 + 10(0) \overset{?}{<} 5$	$-3(1)^2 + 10(1) \overset{?}{<} 5$	$-3(3)^2 + 10(3) \overset{?}{<} 5$
$0 + 0 \overset{?}{<} 5$	$-3 + 10 \overset{?}{<} 5$	$-27 + 30 \overset{?}{<} 5$
$0 < 5$ ✓	$7 \not< 5$ ✗	$3 < 5$ ✓

Note that, the points $x = 0.61$ and $x = 2.72$ are not included in the solution. Therefore, the solution set is $\{x \mid x < 0.61 \text{ or } x > 2.72\}$.

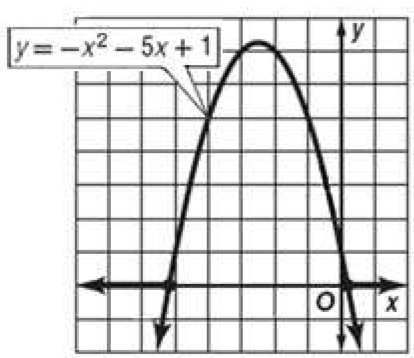
54. $-1 \geq -x^2 - 5x$

SOLUTION:

First, write the related equation and solve it.

$$-x^2 - 5x + 1 = 0$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-1)(1)}}{2(-1)}$$
$$= \frac{5 \pm \sqrt{29}}{-2}$$
$$\approx -5.19 \text{ or } 0.19$$

Sketch the graph of a parabola that has x-intercepts at –5.19 and 0.19. The graph should open down because $a < 0$.



The graph lies below the x-axis left to $x \approx -5.19$ and right to $x \approx 0.19$. Not that: The x-intercepts are also included in the solution. Thus, the solution set of the inequality is $\{x \mid x \leq -5.19 \text{ or } x \geq 0.19\}$.

55. **BUSINESS** An electronics manufacturer uses the function $P(x) = x(-27.5x + 3520) + 20{,}000$ to model their monthly profits when selling x thousand digital audio players.

a. Graph the quadratic inequality for a monthly profit of at least \$100,000.

b. How many digital audio players must the manufacturer sell to earn a profit of at least \$100,000 in a month?

c. Suppose the manufacturer has an additional monthly expense of \$25,000. Explain how this affects the graph of the profit function. Then determine how many digital audio players the manufacturer needs to sell to have at least \$100,000 in profits.

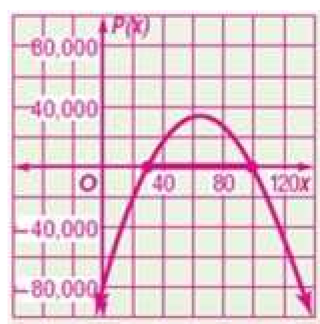
SOLUTION:

a.

The profit is at least \$100,000 if the value of $P(x)$ is greater than or equal to 100,000. That is,

$-27.5x^2 + 3520x + 20000 \geq 100000$ or $-27.5x^2 + 3520x - 80000 \geq 0$.

Graph the inequality on a coordinate plane.



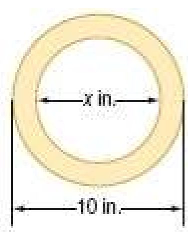
b. The graph lies above the x-axis from $x = 30$ to $x = 98$. So, the company should sell 30,000 to 98,000 digital audio players to earn a profit of at least \$100,000 in a month.

c. If the manufacturer has an additional monthly expense of \$25,000, the amount 25,000 will be subtracted from the profit function. So, the graph denoting the inequality will be shifted down 25,000 units. The manufacturer must sell from 47,000 to 81,000 digital audio players.

56. **UTILITIES** A contractor is installing drain pipes for a shopping center's parking lot. The outer diameter of the pipe is to be 10 inches. The cross sectional area of the pipe must be at least 35 square inches and should not be more than 42 square inches.

a. Graph the quadratic inequalities.

b. What thickness of drain pipe can the contractor use?



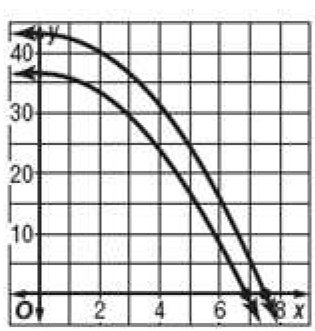
SOLUTION:

a. The inner radius is $\frac{x}{2}$ cm. The cross sectional area of the pipe must be at least 35 square inches. So,

$$\pi(5)^2 - \pi\left(\frac{x}{2}\right)^2 \geq 35.$$

Also, $\pi(5)^2 - \pi\left(\frac{x}{2}\right)^2 \leq 42.$

Graph the two inequalities on a coordinate plane.



b. Solve the inequality $35 \leq 25\pi - \left(\frac{\pi}{4}\right)x^2 \leq 42.$

$$35 - 25\pi \leq -\left(\frac{\pi}{4}\right)x^2 \leq 42 - 25\pi$$

$$\frac{-4(35-25\pi)}{\pi} \geq x^2 \geq \frac{-4(42-25\pi)}{\pi}$$

$$55.44 \geq x^2 \geq 46.52$$

Here, x is a length, it cannot be negative. So, take the positive square root.

$$7.45 \geq x \geq 6.82$$

That is, the value of x should be between 6.82 and 7.45.

When x is 6.82, the thickness of the pipe will be $\frac{10-6.82}{2} = 1.59.$

When x is 7.45, the thickness of the pipe will be $\frac{10-7.45}{2} = 1.275 \approx 1.28.$

Therefore, the thickness of the pipe should be between 1.28 in and 1.59 in.

57. **OPEN ENDED** Write a quadratic inequality for each condition.

a. The solution set is all real numbers.

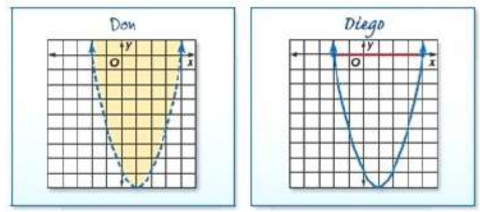
b. The solution set is the empty set.

SOLUTION:

a. Sample answer: to find an inequality with the solution set of all real numbers, choose one point on the number line such as -1 to divide the number line into two intervals. Then the write a related quadratic equation: $(x + 1)(x + 1) = 0$. Test values from each interval to determine the sign of the inequality. When $x = -1$, the equation equals 0 so the inequality sign must be *greater than or equal to* or l*ess than or equal to*. When $x = -2$, the equation equals 1 which is greater than 0. when $x = 1$, the equation equals 4 which is greater than 0. So the inequality is: $x^2 + 2x + 1 \geq 0$.

b. Sample answer: to find an inequality with a solution set of teh empty set, write an inequality that does not have any real solutions such as: $x^2 - 4x + 6 < 0$.

58. **CCSS CRITIQUE** Don and Diego used a graph to solve the quadratic inequality $x^2 - 2x - 8 > 0$. Is either of them correct? Explain.



SOLUTION:

Don graphed the inequality in two variables, so he is not correct.

The inequality is $x^2 - 2x - 8 > 0$. So, one should consider the region of the graph of $y = x^2 - 2x - 8$ above the x-axis to find the limits for x. But Diego has marked the wrong region. So, he is not correct either.

59. **REASONING** Are the boundaries of the solution set of $x^2 + 4x - 12 \leq 0$ twice the value of the boundaries of $\frac{1}{2}x^2 + 2x - 6 \leq 0$? Explain.

SOLUTION:

The graphs of the inequalities intersect the x-axis at the same points. So, both have the same boundaries for the solution set.

60. **REASONING** Determine if the following statement is sometimes, always, or never true. Explain your reasoning.

The intersection of $y \le -ax^2 + c$ *and* $y \ge ax^2 - c$ *is the empty set.*

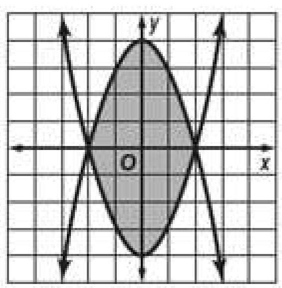
SOLUTION:

When a is positive and c is negative, there is no intersection, hence the solution is an empty set and when a is negative and c is positive there is an intersection, and hence the solution is not an empty set. Therefore, the statement is *sometimes* true.

61. **CHALLENGE** Graph the intersection of the graphs of $y \le -x^2 + 4$ and $y \ge x^2 - 4$..

SOLUTION:

Graph the two inequalities on a coordinate plane and find the intersection.



62. **WRITING IN MATH** How are the techniques used when solving quadratic inequalities and quadratic equations similar? different?

SOLUTION:

For both quadratic and linear inequalities, you must first graph the related equation. You use the inequality symbol to determine if the line is dashed or solid. Then you use test points to determine where to shade. One difference is that one related equation is a straight line while the other related equation is a curve.

63. **]GRIDDED RESPONSE** You need to seed an area that is 80 feet by 40 feet. Each bag of seed can cover 25 square yards of land. How many bags of seed will you need?

SOLUTION:

Each yard is equivalent to 3 feet.

So, $80 \text{ ft} = \left(\frac{80}{3}\right) \text{yd}$ and $40 \text{ ft} = \left(\frac{40}{3}\right) \text{yd}$.

The area that has to be seeded is

$$\begin{aligned}\frac{80}{3} \times \frac{40}{3} &= \frac{3200}{9} \\ &\approx 355.56.\end{aligned}$$

Each bag of seed can cover 25 square yards of land. Divide 355.56 by 25 to find the number of bags required to seed the area.

$$\frac{355.56}{25} = 14.2224$$

Therefore, 15 bags of seed is required for the area.

64. **SAT/ACT** The product of two integers is between 107 and 116. Which of the following cannot be one of the integers?

A 5

B 10

C 12

D 15

E 23

SOLUTION:

Identify the number which does not have multiples between 107 and 116.

The numbers 5 and 10 have 110 as a multiple, and 12 has 108 as a multiple. But 15 does not have a multiple between 107 and 116. So, it cannot be one of the numbers. Therefore, the correct choice is D.

65. **PROBABILITY** Five students are to be arranged side by side with the tallest student in the center and the two shortest students on the ends. If no two students are the same height, how many different arrangements are possible?

F 2

G 4

H 5

J 6

SOLUTION:

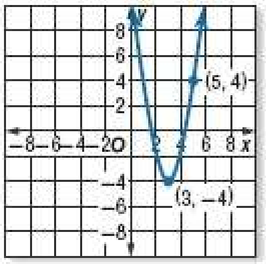
There are two choices for the first place, two for the second, and one choice for the middle position. So, the total number of arrangements is 4. Therefore, the correct choice is G.

66. **SHORT RESPONSE** Simplify $\frac{5+i}{6-3i}$.

SOLUTION:

$$\begin{aligned}\frac{5+i}{6-3i} &= \frac{5+i}{6-3i}\cdot\frac{6+3i}{6+3i}\\ &= \frac{30+15i+6i+3i^2}{36+18i-18i-9i^2}\\ &= \frac{30+21i-3}{36+9}\\ &= \frac{27+21i}{45}\\ &= \frac{3}{5}+\frac{7}{15}i\end{aligned}$$

Write an equation in vertex form for each parabola.

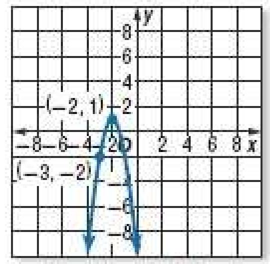
67. 

SOLUTION:

The vertex of the parabola is at (3, –4), so $h = 3$ and $k = –4$.
Since (5, 4) is a point on the parabola, let $x = 5$ and $y = 4$.
Substitute these values into the vertex form of the equation and solve for a.

$$
\begin{aligned}
y &= a(x-h)^2 + k \\
4 &= a(5-3)^2 - 4 \\
4 &= 4a - 4 \\
8 &= 4a \\
2 &= a
\end{aligned}
$$

The equation of the parabola in vertex form is $y = 2(x-3)^2 - 4$.

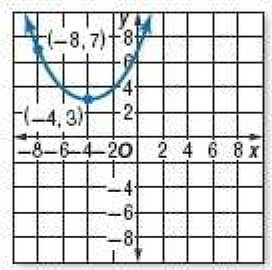
68. 

SOLUTION:

The vertex of the parabola is at (–2, 1), so $h = –2$ and $k = 1$.
Since (–3, –2) is a point on the parabola, let $x = –3$ and $y = –2$.
Substitute these values into the vertex form of the equation and solve for a.

$$
\begin{aligned}
y &= a(x-h)^2 + k \\
-2 &= a(-3-(-2))^2 + 1 \\
-2 &= a + 1 \\
-3 &= a
\end{aligned}
$$

The equation of the parabola in vertex form is $y = -3(x+2)^2 + 1$.

69. 

SOLUTION:

The vertex of the parabola is at (–4, 3), so $h = –4$ and $k = 3$.
Since (–8, 7) is a point on the parabola, let $x = –8$ and $y = 7$.
Substitute these values into the vertex form of the equation and solve for a.

$$\begin{aligned} y &= a(x-h)^2 + k \\ 7 &= a(-8-(-4))^2 + 3 \\ 7 &= 16a + 3 \\ 4 &= 16a \\ 0.25 &= a \end{aligned}$$

The equation of the parabola in vertex form is $y = 0.25(x+4)^2 + 3$.

Complete parts a and b for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

70. $4x^2 + 7x - 3 = 0$

SOLUTION:

a. Identify a, b, and c from the equation.

$a = 4$, $b = 7$ and $c = –3$.

Substitute the values in $b^2 - 4ac$ and simplify.

$$\begin{aligned} b^2 - 4ac &= 7^2 - 4(4)(-3) \\ &= 49 + 48 \\ &= 97 \end{aligned}$$

b. The discriminant is not a perfect square, so there are two irrational roots.

71. $-3x^2 + 2x - 4 = 9$

SOLUTION:

a. Write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$\begin{aligned} -3x^2 + 2x - 4 &= 9 \\ -3x^2 + 2x - 13 &= 0 \end{aligned}$$

$a = –3$, $b = 2$ and $c = –13$.

Substitute these values in $b^2 - 4ac$ and simplify.

$$\begin{aligned} b^2 - 4ac &= 2^2 - 4(-3)(-13) \\ &= 4 - 156 \\ &= -152 \end{aligned}$$

b. The discriminant is a negative, so there are two complex roots.

72. $6x^2 + x - 4 = 12$

SOLUTION:

a. Write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$\begin{aligned} 6x^2 + x - 4 &= 12 \\ 6x^2 + x - 16 &= 0 \end{aligned}$$

$a = 6$, $b = 1$ and $c = -16$.

Substitute these values in $b^2 - 4ac$ and simplify.

$$\begin{aligned} b^2 - 4ac &= 1^2 - 4(6)(-16) \\ &= 1 + 384 \\ &= 385 \end{aligned}$$

b. The discriminant is not a perfect square, so there are two irrational roots.

Use the Distributive Property to find each product.

73. $-6(x - 4)$

SOLUTION:

$-6(x - 4) = -6x + 24$

74. $8(w + 3x)$

SOLUTION:

$8(w + 3x) = 8w + 24x$