2008 Multiple Choice

No Calculator Problems 1-28

1. $\lim_{x\to\infty}\dfrac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

2. $\int\dfrac{1}{x^2}dx=$

(A) $\ln x^2+C$ (B) $-\ln x^2+C$ (C) $x^{-1}+C$ (D) $-x^{-1}+C$ (E) $-2x^{-3}+C$

3. If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

4. $\int(\sin(2x)+\cos(2x))dx=$

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(B) $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$

(E) $-2\cos(2x)+2\sin(2x)+C$

5. $\lim_{x\to0}\dfrac{5x^4+8x^2}{3x^4-16x^2}$ is

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}+1$ (E) nonexistent

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$.

II. f is continuous at $x=2$.

III. f is differentiable at $x=2$.

(A) I only

(B) II only

(C) III only

(D) I and II only

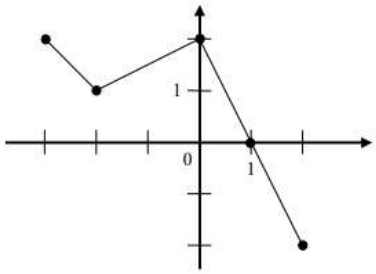
(E) I, II, and III

7. A particle moves along the x-axis with velocity given by $v(t)=3t^2+6t$ for time $t\geq 0$. If the particle is at position $x=2$ at time $t=0$, what is the position of the particle at $t=1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

8. If $f(x)=\cos(3x)$, then $f'\left(\dfrac{\pi}{9}\right)=$

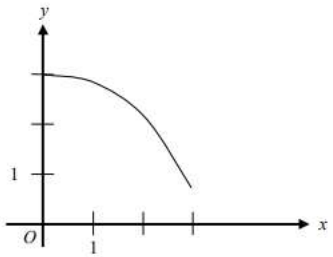
(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$



Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$



Graph of f

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

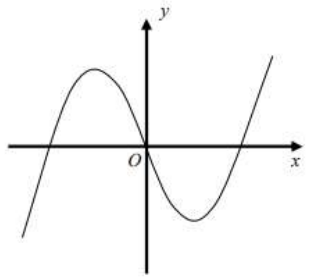
(A) $\int_{1}^{3} f(x)\,dx$

(B) Left Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

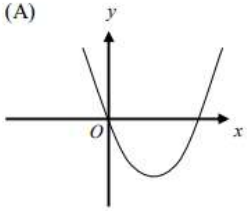
(D) Midpoint Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

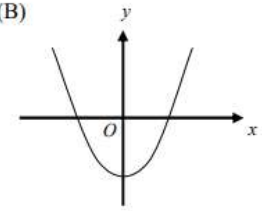
(E) Trapezoidal sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

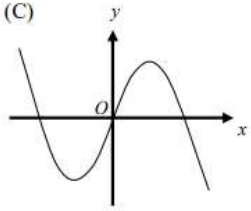


Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

(A)


(B)


(C)


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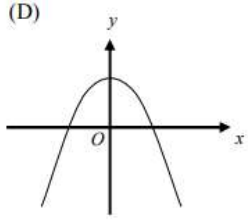
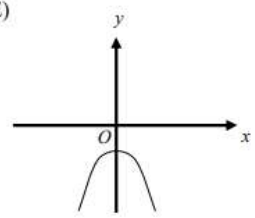


(E)


12. If $f(x)=e^{(2/x)}$, then $f'(x)=$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

13. If $f(x)=x^2+2x$, then $\frac{d}{dx}\left(f(\ln x)\right)=$

(A) $\frac{2\ln x+2}{x}$ (B) $2x\ln x+2$ (C) $2\ln x+2$ (D) $2\ln x+\frac{2}{x}$ (E) $\frac{2x+2}{x}$

x	0	1	2	3
$f''(x)$	5	0	–7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

15. $\int \frac{x}{x^2-4}\,dx =$

(A) $\frac{-1}{4(x^2-4)^2}+C$

(B) $\frac{1}{2(x^2-4)}+C$

(C) $\frac{1}{2}\ln|x^2-4|+C$

(D) $2\ln|x^2-4|+C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

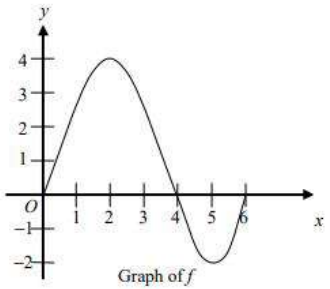
(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$



Graph of f

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) –3 (B) –2 (C) –1 (D) 0 (E) 1

19. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y = -1$ only

(B) $y = 0$ only

(C) $y = 5$ only

(D) $y = -1$ and $y = 0$

(E) $y = -1$ and $y = 5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

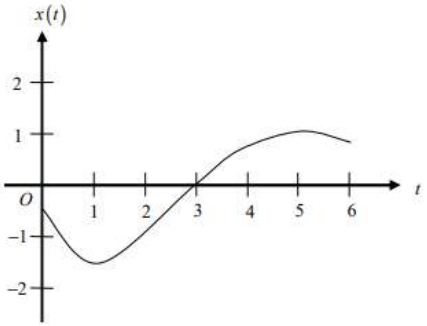
(A) 0 only

(B) 3 only

(C) 0 and 6 only

(D) 3 and 6 only

(E) 0, 3, and 6



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0<t<6$. The graph has horizontal tangents at $t=1$ and $t=5$ and a point of inflection at $t=2$. For what values of t is the velocity of the particle increasing?

(A) $0<t<2$

(B) $1<t<5$

(C) $2<t<6$

(D) $3<t<5$ only

(E) $1<t<2$ and $5<t<6$

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt}=kp$

(B) $\frac{dp}{dt}=kp(N-p)$

(C) $\frac{dp}{dt}=kp(p-N)$

(D) $\frac{dp}{dt}=kt(N-t)$

(E) $\frac{dp}{dt}=kt(t-N)$

23. Which of the following is the solution to the differential equation $\frac{dy}{dx}=\frac{x^2}{y}$ with the initial condition $y(3)=-2$?

(A) $y=2e^{-9+x^3/3}$

(B) $y=-2e^{-9+x^3/3}$

(C) $y=\sqrt{\frac{2x^3}{3}}$

(D) $y=\sqrt{\frac{2x^3}{3}-14}$

(E) $y=-\sqrt{\frac{2x^3}{3}-14}$

24. The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

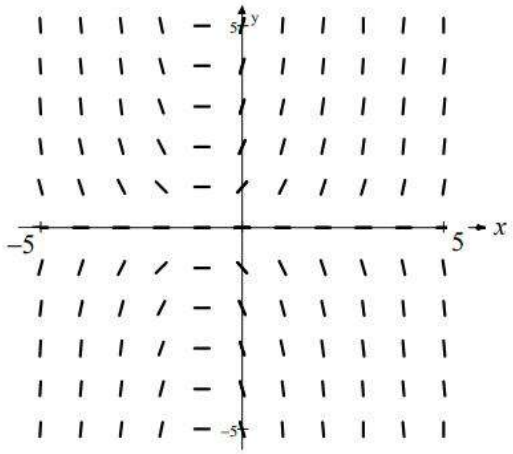
$$f(x)=\begin{cases} cx+d & \text{for } x\le 2 \\ x^2-cx & \text{for } x>2 \end{cases}$$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x=2$, what is the value of $c+d$?

(A) −4 (B) −2 (C) 0 (D) 2 (E) 4

26. What is the slope of the line tangent to the curve $y=\arctan(4x)$ at the point at which $x=\frac{1}{4}$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) −2



27. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$

28. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

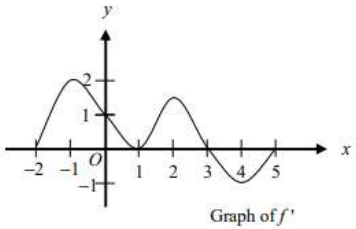
(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

Calculator Allowed Problems 76-92



Graph of f'

76. The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

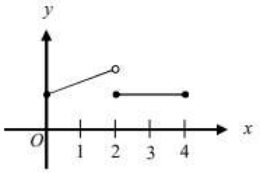
(A) $[-2, 1]$ only

(B) $[-2, 3]$

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$



Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

78. The first derivative of the function f is defined by $f'(x)=\sin\left(x^3-x\right)$ for $0\le x\le 2$. On what interval(s) is f increasing?

(A) $1\le x\le 1.445$

(B) $1\le x\le 1.691$

(C) $1.445\le x\le 1.875$

(D) $0.577\le x\le 1.445$ and $1.875\le x\le 2$

(E) $0\le x\le 1$ and $1.691\le x\le 2$

79. If $\int_{-5}^{2} f(x)\,dx=-17$ and $\int_{5}^{2} f(x)\,dx=-4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) −21 (B) −13 (C) 0 (D) 13 (E) 21

80. The derivative of the function f is given by $f'(x)=x^2\cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$

(A) $f'(4)$

(B) $-7+f'(4)$

(C) $\int_{2}^{4} f(t)\,dt$

(D) $\int_{2}^{4}\left(-7+f(t)\right)dt$

(E) $-7+\int_{2}^{4} f(t)\,dt$

82. A particle moves along a straight line with velocity given by $v(t)=7-(1.01)^{-t^2}$ at time $t\ge 0$. What is the acceleration of the particle at time $t=3$?

(A) −0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

83. What is the area enclosed by the curves $y=x^3-8x^2+18x-5$ and $y=x+5$?

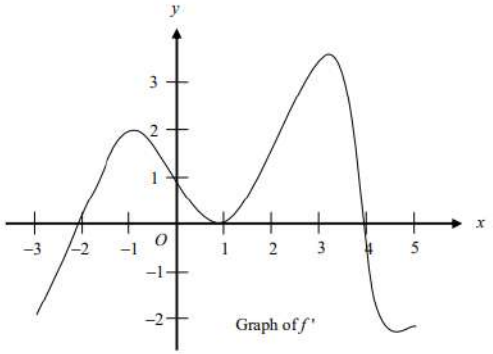
(A) 10.667

(B) 11.833

(C) 14.583

(D) 21.333

(E) 32



84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x=-1$, $x=1$, and $x=3$. At which of the following values of x does f have a relative maximum?

(A) −2 only

(B) 1 only

(C) 4 only

(D) −1 and 3 only

(E) −2, 1, and 4

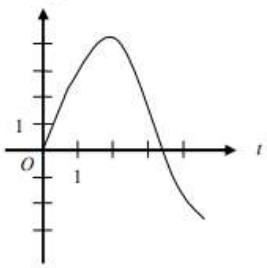
x	−4	−3	−2	−1
$f(x)$	0.75	−1.5	−2.25	−1.5
$f'(x)$	−3	−1.5	0	1.5

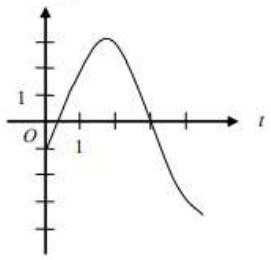
85. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

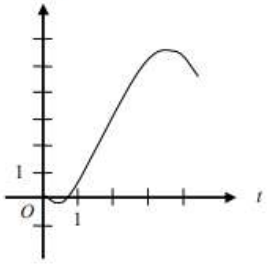
(A) −4.5 (B) −2.25 (C) 0 (D) 2.25 (E) 4.5

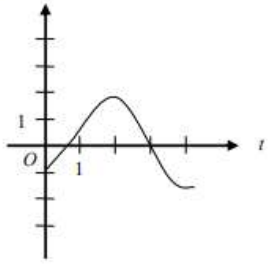
t	0	1	2	3	4
$v(t)$	–1	2	3	0	–4

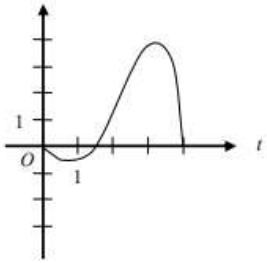
86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t=0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

(A) 

(B) 

(C) 

(D) 

(E) 

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time $t=3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S=4\pi r^2$)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

89. The function f is continuous for $-2 \le x \le 2$ and $f(-2)=f(2)=0$. If there is no c, where $-2<c<2$, for which $f'(c)=0$, which of the following statements must be true?

(A) For $-2<k<2$, $f'(k)>0$.

(B) For $-2<k<2$, $f'(k)<0$.

(C) For $-2<k<2$, $f'(k)$ exists.

(D) For $-2<k<2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2<k<2$, $f'(k)$ does not exist.

90. The function f is continuous on the closed interval $[2, 4]$ and twice differentiable on the open interval $(2, 4)$. If $f'(3)=2$ and $f''(x)<0$ on the open interval $(2,4)$, which of the following could be a table of values for f?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

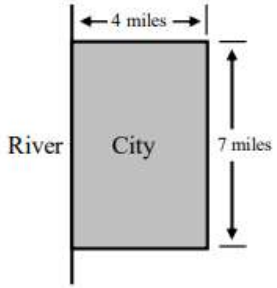
x	$f(x)$
2	3
3	5
4	7

(E)

x	$f(x)$
2	3.5
3	5
4	7.5

91. What is the average value of $y=\dfrac{\cos x}{x^2+x+2}$ on the closed interval $[-1, 3]$?

(A) −0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732



92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)dx$

(B) $7\int_0^4 f(x)dx$

(C) $28\int_0^4 f(x)dx$

(D) $\int_0^7 f(x)dx$

(E) $4\int_0^7 f(x)dx$