2003 BC Answer Key All correct
No Calculator

The Calculus BC Exam

CALCULUS BC

A CALCULATOR **CANNOT BE USED ON PART A OF SECTION I**. A GRAPHING CALCULATOR FROM THE APPROVED LIST **IS REQUIRED FOR PART B OF SECTION I AND FOR PART A OF SECTION II** OF THE EXAMINATION. CALCULATOR MEMORIES NEED NOT BE CLEARED. COMPUTERS, NONGRAPHING SCIENTIFIC CALCULATORS, CALCULATORS WITH QWERTY KEYBOARDS, AND ELECTRONIC WRITING PADS ARE NOT ALLOWED. CALCULATORS MAY NOT BE SHARED AND COMMUNICATION BETWEEN CALCULATORS IS PROHIBITED DURING THE EXAMINATION. ATTEMPTS TO REMOVE TEST MATERIALS FROM THE ROOM BY ANY METHOD WILL RESULT IN THE INVALIDATION OF TEST SCORES.

SECTION I

Time— 1 hour and 45 minutes

All questions are given equal weight.

Percent of total grade—50

Part A: 55 minutes, 28 multiple-choice questions
A calculator is NOT allowed.

Part B: 50 minutes, 17 multiple-choice questions
A graphing calculator is required.

Parts A and B of Section I are in this examination booklet; Parts A and B of Section II, which consist of longer problems, are in a separate, sealed package.

<u>General Instructions</u>

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.

INDICATE YOUR ANSWERS TO QUESTIONS IN PART A ON PAGE 2 OF THE SEPARATE ANSWER SHEET. THE ANSWERS TO QUESTIONS IN PART B SHOULD BE INDICATED ON PAGE 3 OF THE ANSWER SHEET. No credit will be given for anything written in this examination booklet, but you may use the booklet for notes or scratchwork. After you have decided which of the suggested answers is best, COMPLETELY fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

<u>Example:</u>

What is the arithmetic mean of the numbers 1, 3, and 6 ?

(A) 1

(B) $\frac{7}{3}$

(C) 3

(D) $\frac{10}{3}$

(E) $\frac{7}{2}$

<u>Sample Answer</u>

Ⓐ Ⓑ Ⓒ ● Ⓔ

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. In this section of the examination, as a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. It is improbable, therefore, that mere guessing will improve your score significantly; it may even lower your score, and it does take time. If, however, you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices as wrong, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

Use your time effectively, working as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult. Go on to other questions and come back to the difficult ones later if you have time. It is not expected that everyone will be able to answer all the multiple-choice questions.

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. If $y = \sin(3x)$, then $\frac{dy}{dx} =$

(A) $-3\cos(3x)$ (B) $-\cos(3x)$ (C) $-\frac{1}{3}\cos(3x)$ (D) $\cos(3x)$ (E) $3\cos(3x)$

$y' = 3\cos(3x)$

---

2. $\lim\limits_{x \to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x}$ is Apply l'hopital

(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

$\lim\limits_{x \to 0} \frac{e^x + \sin x - 2}{2x - 2} = \frac{1 + 0 - 2}{-2} = \frac{-1}{-2}$

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3. $\int (3x + 1)^5 dx =$

$u = 3x+1$

$du = 3dx$

$\frac{1}{3}\int u^5 du$

$\frac{1}{3} \cdot \frac{u^6}{6}$

$\frac{(3x+1)^6}{18} + C$

(A) $\frac{(3x + 1)^6}{18} + C$

(B) $\frac{(3x + 1)^6}{6} + C$

(C) $\frac{(3x + 1)^6}{2} + C$

(D) $\frac{\left(\frac{3x^2}{2} + x\right)^6}{2} + C$

(E) $\left(\frac{3x^2}{2} + x\right)^5 + C$

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4. For $0 \le t \le 13$, an object travels along an elliptical path given by the parametric equations $x = 3\cos t$ and $y = 4\sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

(A) $-\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $-\frac{4\tan 13}{3}$ (D) $-\frac{4}{3\tan 13}$ (E) $-\frac{3}{4\tan 13}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos t}{-3\sin t} = \frac{-4}{3\tan t}$

---

5. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5 ?

(A) 3 (B) 5 (C) 6 (D) 10 (E) 12

$y_1 = y_0 + h \cdot F(x_0, y_0)$

$x_0 = 1$ $y_0 = 2$

$x_1 = 1.5$ $y_1 = 2 + (.5)(1+2) = 3.5$

$x_2 = 2$ $y_2 = 3.5 + (.5)(1.5 + 3.5) = 3.5 + 2.5 = 6$

---

6. What are all values of p for which $\int_1^{\infty} \frac{1}{x^{2p}}\,dx$ converges?

p-series - converges for $p > 1$

(A) $p < -1$

(B) $p > 0$

(C) $p > \frac{1}{2}$

(D) $p > 1$

(E) There are no values of p for which this integral converges.

$2p > 1$

$p > \frac{1}{2}$

---

7. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

(A) −1 only (B) 0 only (C) 2 only (D) −1 and 2 only (E) −1, 0, and 2

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t^2-6t-12}{3t^2-6t} = \frac{6(t^2-t-2)}{3t(t-2)} = \frac{6(t-2)(t+1)}{3t(t-2)} = \frac{0}{0}$$

dy/dt $\underline{\text{and}}$ dx/dt must be 0 for the particle to be at rest

$t=2,\ t=-1$

$t=0 \quad t=2$

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8. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

$u = x^3$

$du = 3x^2$

$\frac{1}{3}\int \cos u\, du$

$\frac{1}{3}\sin u$

$\frac{1}{3}\sin x^3 + C$

---

9. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

$$f'(x) = \frac{1 + e^{-3x}(-3)}{x+4+e^{-3x}}$$

$$f'(0) = \frac{1-3(1)}{0+4+1} = \frac{-2}{5}$$

10. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1 (B) 2 (C) 4 (D) 6 (E) The series diverges.

$$\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^n$$

$$\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^{n+1} = \sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^n$$

$a = \frac{4}{3}$

$r = \frac{2}{3}$

$$S = \frac{\frac{4}{3}}{1-\frac{2}{3}} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4$$

11. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$?

(A) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$

(B) $x^2 + x^3 + x^4 + x^5 + \cdots$

(C) $x^2 + 2x^3 + 3x^4 + 4x^5 + \cdots$

(D) $x^2 + x^4 + x^6 + x^8 + \cdots$

(E) $x^2 - x^4 + x^6 - x^8 + \cdots$

$$\frac{1}{1-x^2} = \sum_{n=0}^{\infty} (x^2)^n$$

$$\frac{x^2}{1-x^2} = \sum_{n=0}^{\infty} x^2(x^{2n}) =$$

$$x^2 + x^4 + x^6 + \cdots$$

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12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$

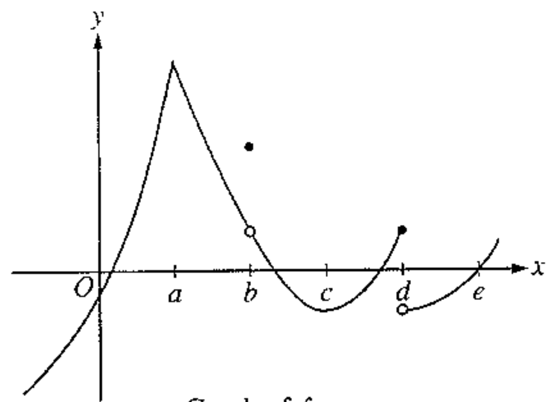
(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$

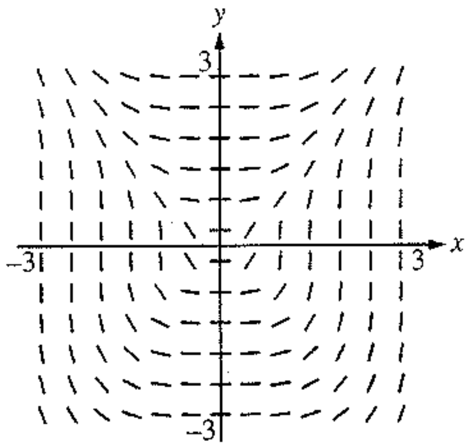
$$\frac{dV}{dt} = k\sqrt{V}$$



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e



14. Shown above is a slope field for which of the following differential equations?

(A) $\cancel{\dfrac{dy}{dx} = \dfrac{x}{y}}$ (B) $\cancel{\dfrac{dy}{dx} = \dfrac{x^2}{y^2}}$ (C) $\cancel{\dfrac{dy}{dx} = \dfrac{x^3}{y}}$ (D) $\dfrac{dy}{dx} = \dfrac{x^2}{y}$ (E) $\dfrac{dy}{dx} = \dfrac{x^3}{y^2}$

(A) Would require + slopes in QIII

(B) All positive slopes

(C) Positive slope in Q3

(D) Positive slope in Q2

15. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1 + 9x^4}\,dx$. If the curve contains the point $(1, 6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$

(B) $y = 5 + x^3$

(C) $y = 6 + x^3$

(D) $y = 6 - x^3$

(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

From the arc length formula,

$$9x^4 = [f'(x)]^2$$

$$3x^2 = f'(x)$$

$$\int 3x^2\,dx = \int f'(x)$$

$$C + \frac{3x^3}{3} = f(x)$$

$$C + x^3 = y$$

$$C + 1 = 6$$

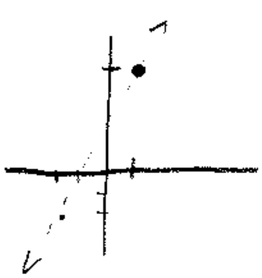
$$C = 5$$

16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

$$m = \frac{-2-7}{-2-1} = \frac{-9}{-3} = 3$$

$$f'(1) = 3$$



17. A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3, 8)$?

(A) $x = -3$

(B) $x = 2$

(C) $y = 8$

(D) $y = -\frac{27}{10}(x + 3) + 8$

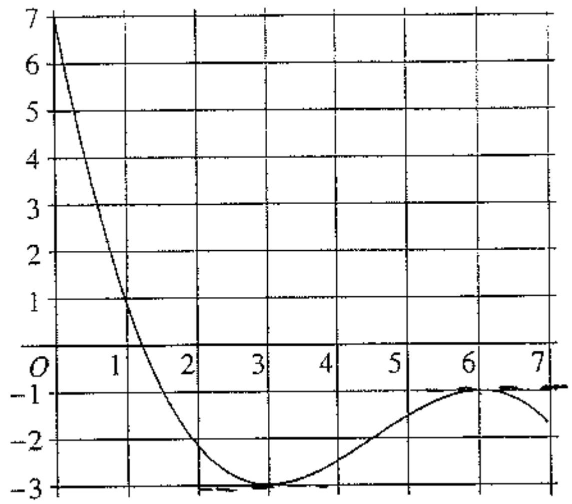
(E) $y = 12(x + 3) + 8$

$\frac{dy}{dx} = \frac{3t^2}{2t-4}$

at $C(-3,8)$ $8=t^3$ $t=2$

$\frac{dy}{dx} = \frac{3(2^2)}{2(2)-4} = \frac{12}{0}$

$m = \emptyset$



Graph of f

18. The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$, what is the value of $g'(3)$?

(A) 0 (B) −1 (C) −2 (D) −3 (E) −6

$$g' = \frac{d}{dx}\int_0^{2x} f(t)\,dx = f(2x)\cdot 2$$

$$g'(3) = f(2\cdot 3)\cdot 2 = (-1)(2) = -2$$

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$
(B) $y = x^2 + 1$
(C) $y = x^2 + 3x$
(D) $y = x^2 + 3x - 2$
(E) $y = 2x^2 + 3x - 3$

$m = 2x + 3$

$f'(x) = 2x + 3$

$f(x) = \int 2x + 3 \, dx$

$f(x) = \frac{2x^2}{2} + 3x + C$

$2 = 1 + 3 + C$

$-2 = C$

$f(x) = x^2 + 3x - 2$

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20. A function f has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots + \frac{x^{n+3}}{(n+1)!} + \cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x \sin x + 3x^2$
(B) $-\cos(x^2) + 1$
(C) $-x^2 \cos x + x^2$
(D) $x^2 e^x - x^3 - x^2$
(E) $e^{x^2} - x^2 - 1$

$x^2 \cdot \frac{x^2}{2!} + \frac{x^2 \cdot x^3}{3!} + \cdots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$x^2 \cdot e^x = x^2 + x^3 + \frac{x^2 \cdot x^2}{2!} + \frac{x^2 \cdot x^3}{3!} + \cdots$

$x^2 e^x - x^2 - x^3 = \frac{x^2 \cdot x^2}{2!} + \frac{x^2 + x^3}{3!} + \cdots$

$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$

L = carrying capacity

21. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\frac{dM}{dt} = 0.6M\left(1 - \frac{M}{200}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t \to \infty} M(t)$?

(A) 50 (B) 200 (C) 500 (D) 1000 (E) 2000

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22. What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges?

(A) $p > 0$ (B) $p \geq 1$ (C) $p > 1$ (D) $p \geq 2$ (E) $p > 2$

$\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ is essentially the same as $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}+1}$ for sufficiently large values of n

$p - 1 > 1$

$p > 2$

---

23. $\int x \sin(6x)dx =$

(A) $-x \cos(6x) + \sin(6x) + C$

(B) $-\frac{x}{6} \cos(6x) + \frac{1}{36} \sin(6x) + C$

(C) $-\frac{x}{6} \cos(6x) + \frac{1}{6} \sin(6x) + C$

(D) $\frac{x}{6} \cos(6x) + \frac{1}{36} \sin(6x) + C$

(E) $6x \cos(6x) - \sin(6x) + C$

	u	dV
+	x	$\sin(6x)$
−	1	$\frac{-\cos 6x}{6}$
+	0	$\frac{-\sin 6x}{36}$

$-\frac{x}{6} \cos 6x + \frac{1}{36} \sin 6x + C$

---

24. Which of the following series diverge?

I. $\sum_{n=0}^{\infty} \left(\frac{\sin 2}{\pi}\right)^n$ $\quad \sin 2 < 1 \quad \frac{\sin 2}{\pi} < 1 \quad \therefore$ convergent geometric series

II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} = \sum \frac{1}{n^{1/3}}$ divergent p-series $p < 1$

III. $\sum_{n=1}^{\infty} \left(\frac{e^n}{e^n + 1}\right)$ $\quad \lim_{n \to \infty} \frac{e^n}{e^n + 1} = 1 \quad \therefore$ divergent by nth term test

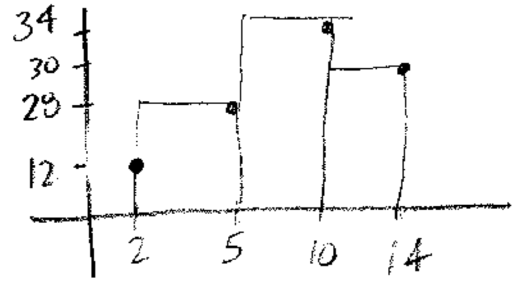
(A) III only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III



x	2	5	10	14
$f(x)$	12	28	34	30

25. The function f is continuous on the closed interval $[2, 14]$ and has values as shown in the table above. Using the subintervals $[2, 5]$, $[5, 10]$, and $[10, 14]$, what is the approximation of $\int_2^{14} f(x)\,dx$ found by using a right Riemann sum?

(A) 296 (B) 312 (C) 343 (D) 374 (E) 390

$$\int = 3(28) + 5(34) + 4(30)$$

$$= 84 + 170 + 120$$

$$\begin{array}{r} 120 \\ 170 \\ 84 \\ \hline 374 \end{array}$$

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26. $\displaystyle\int \frac{2x}{(x+2)(x+1)}\,dx =$

(A) $\ln|x+2| + \ln|x+1| + C$

(B) $\ln|x+2| + \ln|x+1| - 3x + C$

(C) $-4\ln|x+2| + 2\ln|x+1| + C$

(D) $4\ln|x+2| - 2\ln|x+1| + C$

(E) $2\ln|x| + \frac{2}{3}x + \frac{1}{2}x^2 + C$

$$\frac{2x}{(\ )(\ )} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$2x = Ax + A + Bx + 2B$$

$$\begin{array}{r} 2 = A + B \\ -(\ 0 = A + 2B\ ) \\ \hline 2 = -B \\ -2 = B \\ 4 = A \end{array}$$

$$\int \frac{4}{x+2}\,dx - \int \frac{2}{x+1}\,dx$$

$$4\ln|x+2| - 2\ln|x+1| + C$$

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27. $\frac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)\,dt\right) =$

(A) $\frac{2x^3}{x^6+1}$ (B) $\frac{3x^2}{x^6+1}$ (C) $\ln(x^6+1)$ (D) $2x^3\ln(x^6+1)$ (E) $3x^2\ln(x^6+1)$

$\ln((x^3)^2+1)(3x^2)$

28. What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x = 0$?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) 6

$\frac{1}{1+x} = \cancel{1 - x + x^2 - x^3 \cdots x^n(-1)^n}$ $\cancel{n=0 \to \infty}$

$\cancel{\int \frac{1}{1+x}dx = \int u^{-1}du = \frac{u^{-2}}{-2} = -\frac{1}{2(x+1)^2}}$

???

$f(x) = (1+x)^{-2}$

$f'(x) = -2(1+x)^{-3}$

$f''(x) = 6(1+x)^{-4}$

$f'''(x) = -24(1+x)^{-5}$

END OF PART A OF SECTION I

$f(0) = 1$

$f'(0) = -2$

$f''(0) = 6$

$f'''(0) = -24$

$P_2(x) = 1 + -2(x)^1 + \frac{6(x^2)}{2!}$

$= 1 - 2x + 3x^2$

2003 w. Calculator All correct

CALCULUS BC

SECTION I, Part B

Time—50 minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

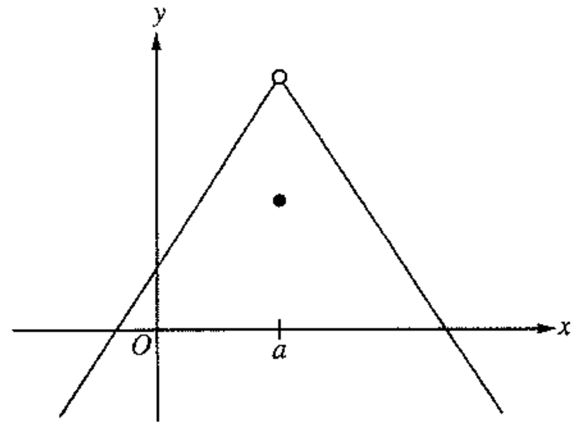
YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

76. The graph of the function f is shown above. Which of the following statements must be false?

(A) $f(a)$ exists.

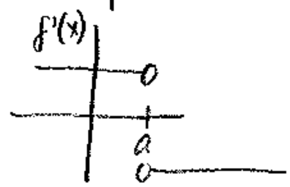
(B) $f(x)$ is defined for $0 < x < a$.

(C) f is not continuous at $x = a$.

(D) $\lim_{x \to a} f(x)$ exists.

(E) $\lim_{x \to a} f'(x)$ exists.

slope from the left is positive
slope from the right is negative



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77. Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

(A) -30 (B) -15 (C) -5 (D) $-\frac{5}{6}$ (E) $-\frac{1}{6}$

$$-5 = \frac{f'''(0)}{3!}$$

$$-30 = f'''(0)$$

---

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) 0.04π m^2/sec

(B) 0.4π m^2/sec

(C) 4π m^2/sec

(D) 20π m^2/sec

(E) 100π m^2/sec

$\frac{dr}{dt} = .2$ m/sec.

$2\pi r = 20\pi$

$r = 10$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$= 2\pi(10)(.2)$

$= 4\pi$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
−1	6	5	3	−2
1	3	−3	−1	2
3	1	−2	2	3

79. The table above gives values of f, f', g, and g' at selected values of x. If $h(x) = f(g(x))$, then $h'(1) =$

(A) 5 (B) 6 (C) 9 (D) 10 (E) 12

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1)) \cdot g'(1)$

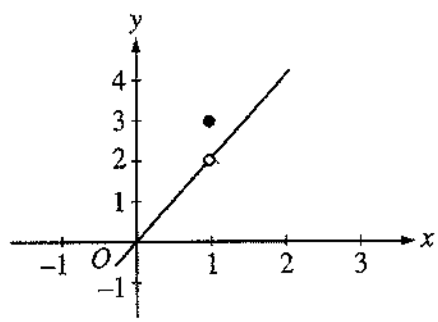
$= f'(-1) \cdot 2$

$= 5 \cdot 2 = 10$

80. Insects destroyed a crop at the rate of $\dfrac{100e^{-0.1t}}{2-e^{-3t}}$ tons per day, where time t is measured in days. To the nearest ton, how many tons did the insects destroy during the time interval $7 \le t \le 14$?

(A) 125 (B) 100 (C) 88 (D) 50 (E) 12

$$\int_7^{14} \frac{100e^{-.1t}}{2-e^{-3t}}\,dt = 124.9917$$



Graph of f

81. The graph of the function f is shown in the figure above. The value of $\lim\limits_{x\to 1} \sin(f(x))$ is

(A) 0.909 (B) 0.841 (C) 0.141 (D) -0.416 (E) nonexistent

$\sin 2 = .909297$

---

82. The rate of change of the altitude of a hot-air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

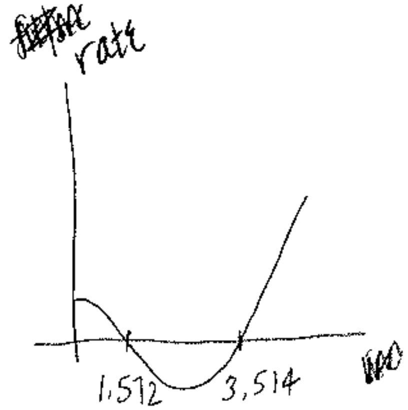
(A) $\int_{1.572}^{3.514} r(t)\,dt$

(B) $\int_{0}^{8} r(t)\,dt$

(C) $\int_{0}^{2.667} r(t)\,dt$

(D) $\int_{1.572}^{3.514} r'(t)\,dt$

(E) $\int_{0}^{2.667} r'(t)\,dt$



$r(t)$ = rate of change in alt.

$\int r(t)$ = change in alt.

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

83. The function f is continuous and differentiable on the closed interval $[0, 4]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

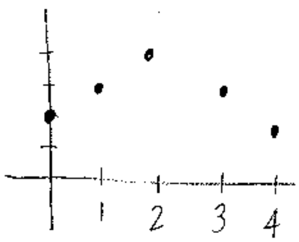
(A) The minimum value of f on $[0, 4]$ is 2. Function could drop below 2 between any of the values.

(B) The maximum value of f on $[0, 4]$ is 4. Function could go above 4 " " " "

(C) $f(x) > 0$ for $0 < x < 4$ Function could drop below the x-axis " " " "

(D) $f'(x) < 0$ for $2 < x < 4$ Not necessarily.

(E) There exists c, with $0 < c < 4$, for which $f'(c) = 0$. Meets conditions of Rolle's theorem.



$f(0) = f(4)$

---

84. A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

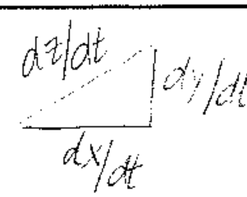
(A) 2.909 (B) 3.062 (C) 6.884 (D) 9.016 (E) 47.393

$$\cancel{\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos 4t}{2t}}$$

$$\cancel{\frac{4\cos(12)}{6}} \approx .56256$$

$$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{6^2 + (3.3754)^2} = 6.884$$

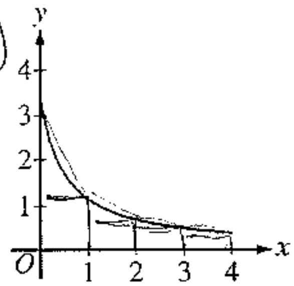


concave up

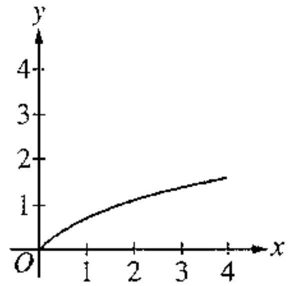
decreasing

85. If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

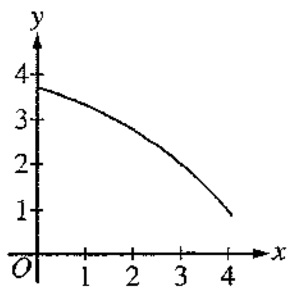
(A)



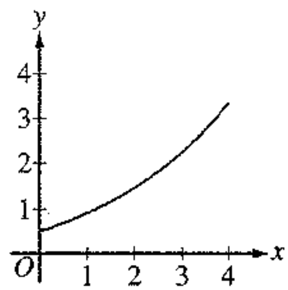
(B)



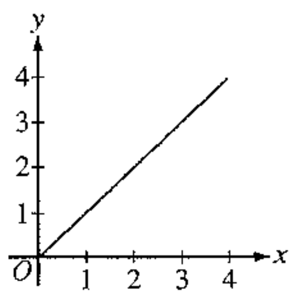
(C)



(D)



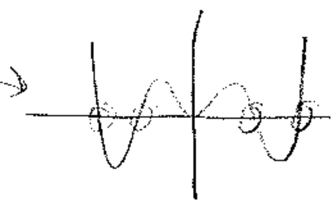
(E)



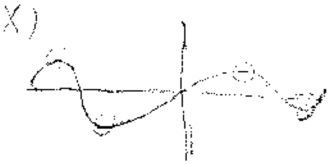
86. Let f be the function with derivative defined by $f'(x) = \sin(x^3)$ on the interval $-1.8 < x < 1.8$. How many points of inflection does the graph of f have on this interval?

(A) Two (B) Three (C) Four (D) Five (E) Six

$f''(x) = \cos(x^3) \cdot 3x^2$



$f'(x)$



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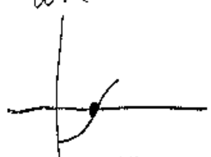
87. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \cos(2 - t^2)$. The position of the particle is 3 at time $t = 0$. What is the position of the particle when its velocity is first equal to 0 ?

(A) 0.411 (B) 1.310 (C) 2.816 (D) 3.091 (E) 3.411

$\cos(2-t^2)$



$X = .65513638$

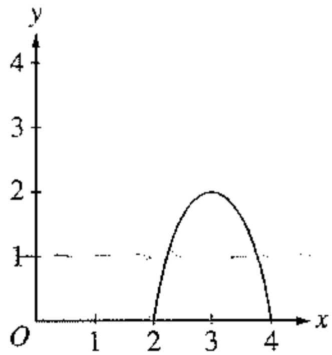
$x(t) = \int_0^{.65513638} \cos(2-t^2)\, dt = -.1835406$

$3 - .1835406 =$

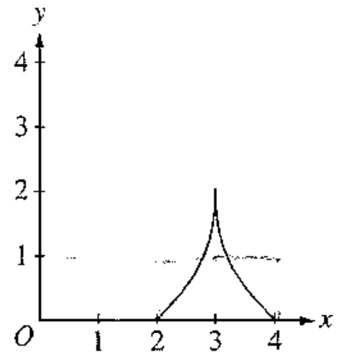
---

88. On the closed interval [2, 4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$? Average value of function = 1

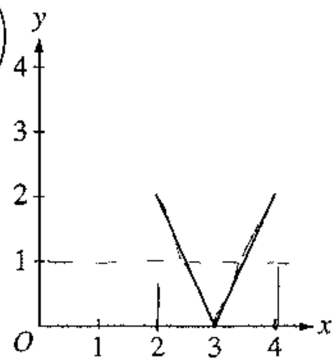
(A)



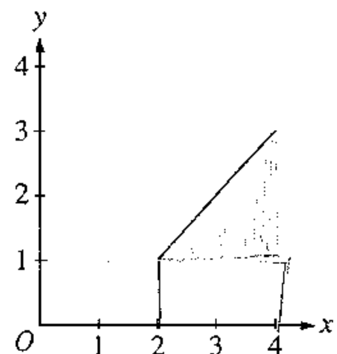
(B)



(C)

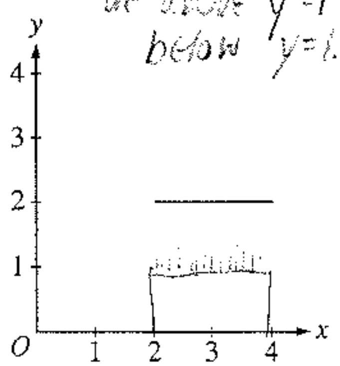


(D)



Half of the y-coordinates are above $y = 1$, half are below $y = 1$.

(E)

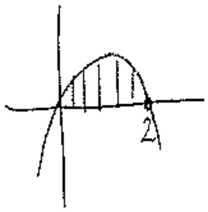


$$\frac{1}{2}\int_2^4 f(t)\,dt =$$

$$\frac{1}{2}\left[\frac{1}{2}\cdot 1\cdot 2 + \frac{1}{2}\cdot 1\cdot 2\right] = 1$$

89. The region bounded by the graph of $y = 2x - x^2$ and the x-axis is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. What is the volume of the solid?

(A) 1.333 (B) 1.067 (C) 0.577 (D) 0.462 (E) 0.267

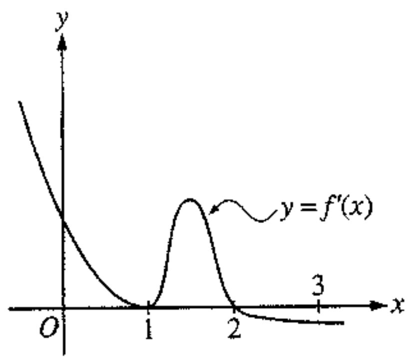


$h = \frac{1}{2}(2x-x^2)\sqrt{3}$

$A = \frac{1}{2}\underbrace{(2x-x^2)}_{b}\underbrace{\frac{\sqrt{3}}{2}(2x-x^2)}_{h}$

$\text{Volume} = \int_0^2 \frac{\sqrt{3}}{4}(2x-x^2)^2\,dx$

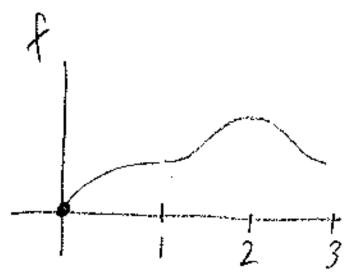
$= .46188$



90. The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?

NO I. $f(0) > f(1)$
II. $f(2) > f(1)$
III. $f(1) > f(3)$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only



Integrate f' to get back to f.

f is the accumulated area under the curve f'

91. The height h, in meters, of an object at time t is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum upward velocity?

(A) 2.545 meters
(B) 10.263 meters
(C) 34.125 meters
(D) 54.889 meters
(E) 89.005 meters

$h'(t) = 24 + 24\left(\frac{3}{2}\right)t^{1/2} - 32t$

$= 24 + 36t^{1/2} - 32t$

Max at $X = .31640706$

$h(X) = 10.263\text{m}$

92. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over $[1, 4]$?

(A) 0.456 (B) 1.244 (C) 2.164 (D) 2.342 (E) 2.452

Avg rate of change $\dfrac{f(4)-f(1)}{4-1} = \dfrac{(4+\ln 4)-(1+\ln 1)}{3}$

$= 1.46209812$

Inst. Rate: $1 + \frac{1}{x}$

$1 + \frac{1}{x} - 1.46209812 = 0$

$x = 2.164$

END OF SECTION I