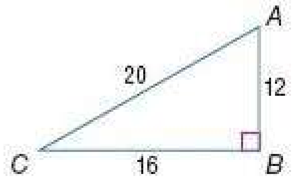
Express each ratio as a fraction and as a decimal to the nearest hundredth.



1. sin *A*

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$\sin A = \frac{BC}{AC} = \frac{16}{20} = 0.80.$

2. tan *C*

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$\tan C = \frac{AB}{BC} = \frac{12}{16} = 0.75.$

3. cos *A*

SOLUTION:

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

$\cos A = \frac{AB}{AC} = \frac{12}{20} = 0.60.$

4. tan *A*

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$\tan A = \frac{BC}{AB} = \frac{16}{12} = 1.33.$

5. cos *C*

SOLUTION:

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

$\cos C = \frac{BC}{AC} = \frac{16}{20} = 0.80.$

6. sin *C*

SOLUTION:

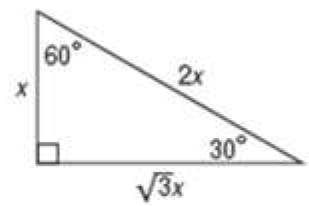
The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$\sin C = \frac{AB}{AC} = \frac{12}{20} = 0.60.$

7. Use a special right triangle to express sin $60°$ as a fraction and as a decimal to the nearest hundredth.

SOLUTION:

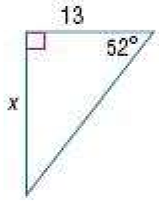
Draw and label the side lengths of a 30°-60°-90° right triangle, with *x* as the length of the shorter leg. Then the longer leg measures $x\sqrt{3}$ and the hypotenuse has a measure 2*x*.



The side opposite the 60° angle has a measure of $x\sqrt{3}$ and the hypotenuse is 2*x* units long.

$$\sin 60° = \frac{\text{Opp.}}{\text{Hyp.}} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$

Find x. Round to the nearest hundredth.

8. 

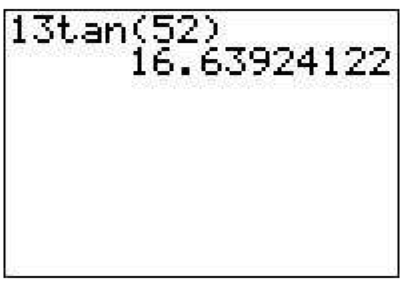
SOLUTION:
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

$$\tan 52° = \frac{x}{13}.$$

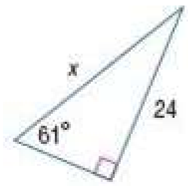
Multiply both sides by 13.

$$13 \cdot \tan 52° = x$$

Use a calculator to find the value of x. Make sure the mode of the calculator is in degrees.



$$x \approx 16.64$$

9. 

SOLUTION:
The sine of an angle is defined as the ratio of the opposite side to the hypotenuse.

$$\sin 61° = \frac{24}{x}$$

Multiply both sides by x.

$$x \cdot \sin 61° = 24$$

Divide both sides by sin 61°.

$$x = \frac{24}{\sin 61°}$$

Use a calculator to find the value of x. Make sure the mode of the calculator is in degrees.

$$x \approx 27.44$$

10. 

SOLUTION:

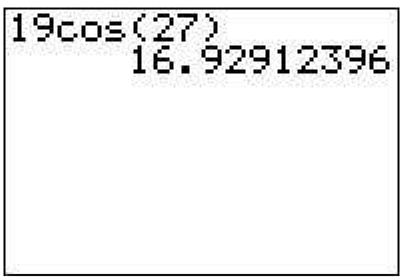
The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse.

$$\cos 27° = \frac{x}{19}$$

Multiply both sides by 19.

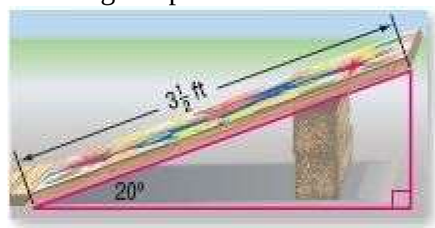
$$19 \cdot \cos 27° = x$$

Use a calculator to find the value of *x*. Make sure the mode of the calculator is in degrees.



$x \approx 16.93$

11. **SPORTS** David is building a bike ramp. He wants the angle that the ramp makes with the ground to be $20°$. If the board he wants to use for his ramp is $3\frac{1}{2}$ feet long, about how tall will the ramp need to be at the highest point?



SOLUTION:

Let *x* be the height of the ramp. The length of the ramp is $3\frac{1}{2}$ ft or 42 in.

$$\sin 20° = \frac{opp}{hyp} = \frac{x}{42}$$

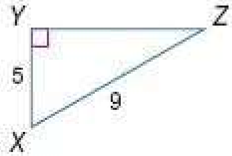
Multiply both sides by 42.

$$42 \cdot \sin 20° = x$$

Use a calculator to find the value of *x*. Make sure your calculator is in degree mode.

$x \approx 14.36$ in. or 1.2 ft.

CCSS TOOLS Use a calculator to find the measure of $\angle Z$ to the nearest tenth.

12. 

SOLUTION:

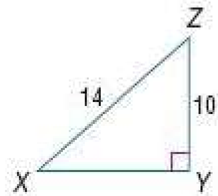
The measures given are those of the leg opposite $\angle Z$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin Z = \frac{XY}{XZ} = \frac{5}{9}$$

If $\sin Z = \frac{5}{9}$, then $Z = \sin^{-1}\left(\frac{5}{9}\right)$.

Use a calculator to solve for the measure of angle Z. Make sure your calculator is in degree mode.

$m\angle Z \approx 33.7°$

13. 

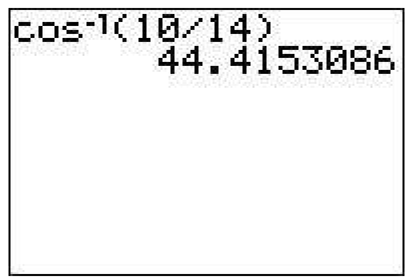
SOLUTION:

The measures given are those of the leg adjacent to $\angle Z$ and the hypotenuse, so write an equation using the cosine ratio.

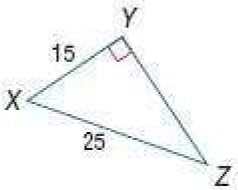
$$\cos Z = \frac{ZY}{XZ} = \frac{10}{14}$$

If $\cos Z = \frac{10}{14}$, then $Z = \cos^{-1}\left(\frac{10}{14}\right)$.

Use a calculator to solve for the measure of angle Z. Make sure your calculator is in degree mode.



$m\angle Z \approx 44.4°$

14. 

SOLUTION:

The measures given are those of the leg opposite $\angle Z$ and the hypotenuse, so write an equation using the sine ratio.

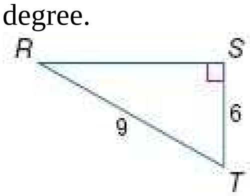
$$\sin Z = \frac{XY}{XZ} = \frac{15}{25} = \frac{3}{5}$$

$$\text{If } \sin Z = \frac{3}{5}, \text{ then } Z = \sin^{-1}\left(\frac{3}{5}\right).$$

Use a calculator to solve for the measure of angle Z. Make sure your calculator is in degree mode.

$m\angle Z \approx 36.9°$

15. Solve the right triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.



SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

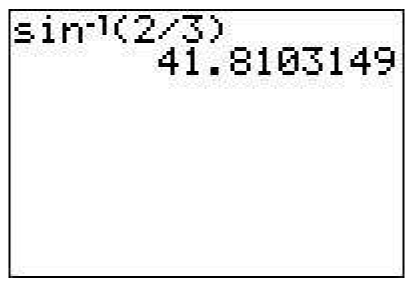
$$RS^2 + TS^2 = RT^2$$
$$RS^2 + 6^2 = 9^2$$
$$RS^2 + 36 = 81$$
$$RS^2 = 45$$
$$RS = \sqrt{45} \approx 6.7$$

The measures given are those of the leg opposite $\angle R$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin R = \frac{ST}{RT} = \frac{6}{9} = \frac{2}{3}$$

$$\text{If } \sin R = \frac{2}{3}, \text{ then } R = \sin^{-1}\left(\frac{2}{3}\right).$$

Use a calculator to find the measure of angle R. Make sure your calculator is in degree mode.

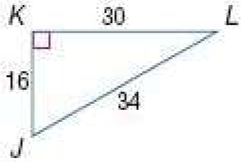


$m\angle R \approx 42°$

The sum of the measures of the angles of a triangle is 180. Therefore,

$$m\angle T = 180 - (90 + 42) = 48.$$

Find *sin J*, *cos J*, *tan J*, *sin L*, *cos L*, and *tan L*. Express each ratio as a fraction and as a decimal to the nearest hundredth.

16. 

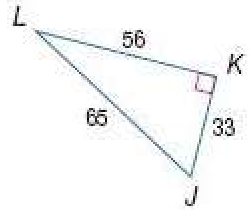
SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse.

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse.

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

$$\sin J = \frac{30}{34} \approx 0.88 \qquad \sin L = \frac{16}{34} \approx 0.47$$
$$\cos J = \frac{16}{34} \approx 0.47 \qquad \cos L = \frac{30}{34} \approx 0.88$$
$$\tan J = \frac{30}{16} \approx 1.88 \qquad \tan L = \frac{16}{30} \approx 0.53$$

17. 

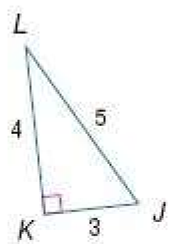
SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse.

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse.

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

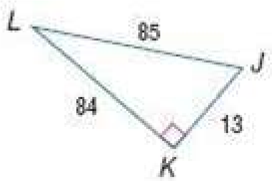
$$\sin J = \frac{56}{65} \approx 0.86 \qquad \sin L = \frac{33}{65} \approx 0.51$$
$$\cos J = \frac{33}{65} \approx 0.51 \qquad \cos L = \frac{56}{65} \approx 0.86$$
$$\tan J = \frac{56}{33} \approx 1.70 \qquad \tan L = \frac{33}{56} \approx 0.59$$

18. 

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse.
The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse.
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

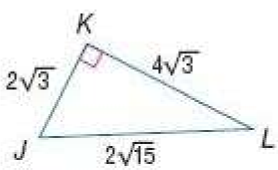
$$\sin J = \frac{4}{5} = 0.80 \qquad \sin L = \frac{3}{5} = 0.60$$
$$\cos J = \frac{3}{5} = 0.60 \qquad \cos L = \frac{4}{5} = 0.80$$
$$\tan J = \frac{4}{3} \approx 1.33 \qquad \tan L = \frac{3}{4} = 0.75$$

19. 

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse.
The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse.
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

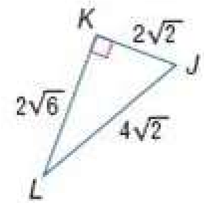
$$\sin J = \frac{84}{85} \approx 0.99 \qquad \sin L = \frac{13}{85} \approx 0.15$$
$$\cos J = \frac{13}{85} \approx 0.15 \qquad \cos L = \frac{84}{85} \approx 0.99$$
$$\tan J = \frac{84}{13} \approx 6.46 \qquad \tan L = \frac{13}{84} \approx 0.15$$

20. 

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse.
The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse.
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

$$\sin J = \frac{4\sqrt{3}}{2\sqrt{15}} = \frac{2\sqrt{5}}{5} \approx 0.89 \qquad \sin L = \frac{2\sqrt{3}}{2\sqrt{15}} = \frac{\sqrt{5}}{5} \approx 0.45$$
$$\cos J = \frac{2\sqrt{3}}{2\sqrt{15}} = \frac{\sqrt{5}}{5} \approx 0.45 \qquad \cos L = \frac{4\sqrt{3}}{2\sqrt{15}} = \frac{2\sqrt{5}}{5} \approx 0.89$$
$$\tan J = \frac{4\sqrt{3}}{2\sqrt{3}} = 2 \qquad \tan L = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2} = 0.50$$

21. 

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse.
The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse.
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

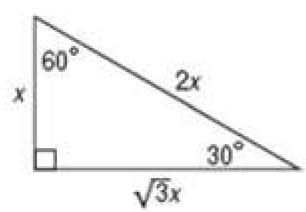
$$\sin J = \frac{2\sqrt{6}}{4\sqrt{2}} = \frac{\sqrt{3}}{2} \approx 0.87 \qquad \sin L = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2} 0.50$$
$$\cos J = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2} = 0.50 \qquad \cos L = \frac{2\sqrt{6}}{4\sqrt{2}} = \frac{\sqrt{3}}{2} \approx 0.87$$
$$\tan J = \frac{2\sqrt{6}}{2\sqrt{2}} = \sqrt{3} \approx 1.73 \qquad \tan L = \frac{2\sqrt{2}}{2\sqrt{6}} = \frac{\sqrt{3}}{3} \approx 0.58$$

Use a special right triangle to express each trigonometric ratio as a fraction and as a decimal to the nearest hundredth.

22. $\tan 60°$

SOLUTION:

Draw and label the side lengths of a 30°-60°-90° right triangle, with *x* as the length of the shorter leg. Then the longer leg measures $x\sqrt{3}$ and the hypotenuse has a measure 2*x*.



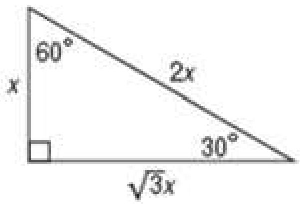
The side opposite to the 60° angle has a measure of $x\sqrt{3}$ and the adjacent side is *x* units long.

$$\tan 60° = \frac{\text{Opp.}}{\text{Adj.}}$$
$$= \frac{\sqrt{3}x}{x}$$
$$= \sqrt{3}$$
$$\approx 1.73$$

23. cos $30°$

SOLUTION:

Draw and label the side lengths of a 30°-60°-90° right triangle, with x as the length of the shorter leg. Then the longer leg measures $x\sqrt{3}$ and the hypotenuse has a measure $2x$.



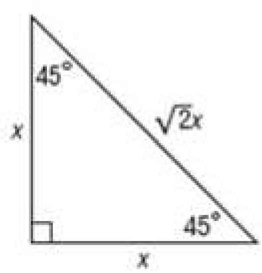
The side adjacent to the 30° angle has a measure of $x\sqrt{3}$ and the hypotenuse is $2x$ units long.

$$\begin{aligned}\cos 30° &= \frac{\text{Adj.}}{\text{Hyp.}}\\ &= \frac{\sqrt{3}x}{2x}\\ &= \frac{\sqrt{3}}{2}\\ &\approx 0.87\end{aligned}$$

24. sin $45°$

SOLUTION:

Draw and label the side lengths of a 45°-45°-90° right triangle, with x as the length of the shorter leg. Then the longer leg measures $x\sqrt{3}$ and the hypotenuse has a measure $2x$.



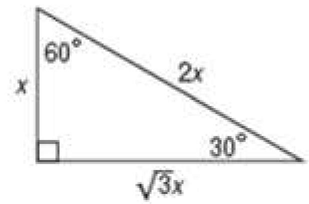
The side opposite to the 45° angle has a measure of x and the hypotenuse is $\sqrt{2}x$ units long.

$$\begin{aligned}\sin 45° &= \frac{\text{Opp.}}{\text{Hyp.}}\\ &= \frac{x}{\sqrt{2}x}\\ &= \frac{\sqrt{2}}{2}\\ &\approx 0.71\end{aligned}$$

25. sin $30°$

SOLUTION:

Draw and label the side lengths of a 30°-60°-90° right triangle, with x as the length of the shorter leg. Then the longer leg measures $x\sqrt{3}$ and the hypotenuse has a measure $2x$.



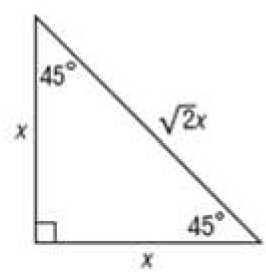
The side opposite to the 30° angle has a measure of x and the hypotenuse is $2x$ units long.

$$\begin{aligned}\sin 30° &= \frac{\text{Opp.}}{\text{Hyp.}}\\ &= \frac{x}{2x}\\ &= \frac{1}{2}\\ &= 0.50\end{aligned}$$

26. tan $45°$

SOLUTION:

Draw and label the side lengths of a 45°-45°-90° right triangle, with x as the length of the shorter leg. Then the longer leg measures $x\sqrt{3}$ and the hypotenuse has a measure $2x$.



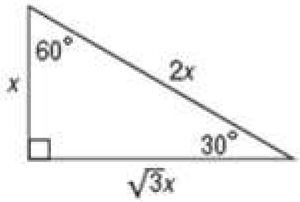
The side opposite to the 45° angle has a measure of x and the adjacent side is also x units long.

$$\begin{aligned}\tan 45° &= \frac{\text{Opp.}}{\text{Adj.}}\\ &= \frac{x}{x}\\ &= 1\end{aligned}$$

27. cos $60°$

SOLUTION:

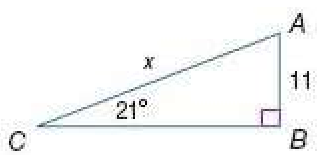
Draw and label the side lengths of a 30°-60°-90° right triangle, with x as the length of the shorter leg. Then the longer leg measures $x\sqrt{3}$ and the hypotenuse has a measure $2x$.



The side adjacent to the 60° angle has a measure of x and the hypotenuse is $2x$ units long.

$$\begin{aligned}\cos 60° &= \frac{\text{Adj.}}{\text{Hyp.}}\\ &= \frac{x}{2x}\\ &= \frac{1}{2}\\ &= 0.50\end{aligned}$$

Find x. Round to the nearest tenth.

28. 

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$\sin 21° = \frac{11}{x}.$

Multiply each side by x.

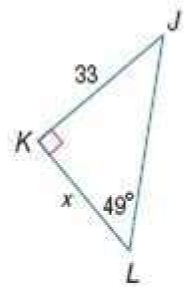
$x \cdot \sin 21° = 11$

Divide each side by sin 21°.

$x = \frac{11}{\sin 21°}$

Use a calculator to find the value of x.

$x \approx 30.7$

29. 

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$\tan 49° = \frac{33}{x}.$

Multiply each side by x.

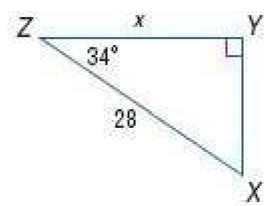
$x \cdot \tan 49° = 33$

Divide each side by tan 49°.

$x = \frac{33}{\tan 49°}$

Use a calculator to find the value of x.

$x \approx 28.7$

30. 

SOLUTION:

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

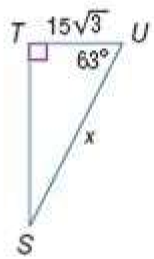
$\cos 34° = \frac{x}{28}.$

Multiply both sides by 28.

$28 \cdot \cos 34° = x$

Use a calculator to find the value of x.

$x \approx 23.2$

31. 

SOLUTION:

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

$\cos 63° = \frac{15\sqrt{3}}{x}$.

Multiply both sides by *x*.

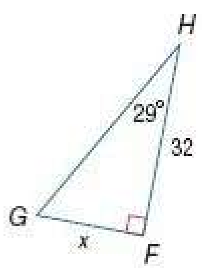
$x \cdot \cos 63° = 15\sqrt{3}$

Divide each side by cos 63°.

$x = \frac{15\sqrt{3}}{\cos 63°}$

Use a calculator to find the value of *x*.

$x \approx 57.2$

32. 

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

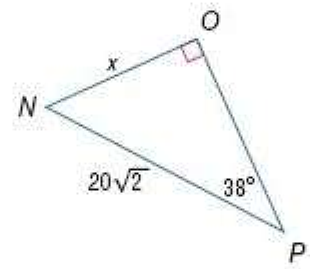
$\tan 29° = \frac{x}{32}$.

Multiply each side by 32.

$32 \cdot \tan 29° = x$

Use a calculator to find the value of *x*.

$x \approx 17.7$

33. 

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$\sin 38° = \frac{x}{20\sqrt{2}}$.

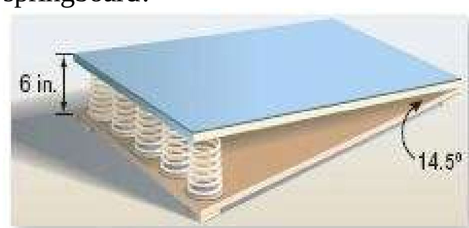
Multiply each side by $20\sqrt{2}$.

$20\sqrt{2} \cdot \sin 38° = x$

Use a calculator to find the value of *x*.

$x \approx 17.4$

34. **GYMNASTICS** The springboard that Eric uses in his gymnastics class has 6-inch coils and forms an angle of $14.5°$ with the base. About how long is the springboard?



SOLUTION:

The base, the coil and the springboard form a right triangle.

Let *x* be the length of the springboard. The length of the coil is 6 inches. Then,

$\sin 14.5° = \frac{6}{x}$.

Multiply each side by *x*.

$x \cdot \sin 14.5° = 6$

Divide each side by sin 14.5°.

$x = \frac{6}{\sin 14.5°}$

Use a calculator to find the value of *x*.

$x \approx 24.0$

Therefore, the springboard is about 24 ft long.

35. **ROLLER COASTERS** The angle of ascent of the first hill of a roller coaster is $55°$. If the length of the track from the beginning of the ascent to the highest point is 98 feet, what is the height of the roller coaster when it reaches the top of the first hill?

SOLUTION:

Let *x* be the height of the roller coaster when it reaches the top, the side opposite the 55 degree angle.

The length of the track from the beginning of the ascent to the highest point is 98 feet, the hypotenuse

Therefore,

$$\sin 55° = \frac{x}{98}.$$

Multiply each side by 98.

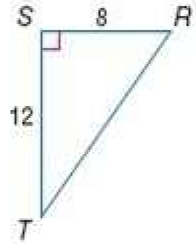
$$98 \cdot \sin 55° = x$$

Use a calculator to find the value of *x*, in degree mode.

$x \approx 80.3$

Therefore, the roller coaster is about 80 ft high when it reaches the top.

CCSS TOOLS Use a calculator to find the measure of $\angle T$ to the nearest tenth.

36. 

SOLUTION:

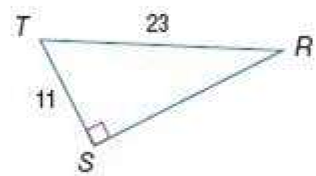
The measures given are those of the legs opposite and adjacent to $\angle T$, so write an equation using the tangent ratio.

$$\tan T = \frac{SR}{ST} = \frac{8}{12} = \frac{2}{3}.$$

If $\tan T = \frac{2}{3}$, then $T = \tan^{-1}\left(\frac{2}{3}\right)$.

Use a calculator.

$m\angle T \approx 33.7°$

37. 

SOLUTION:

The measures given are those of the leg adjacent to $\angle T$ and the hypotenuse, so write an equation using the cosine ratio.

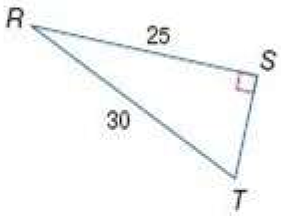
$$\cos T = \frac{TS}{TR} = \frac{11}{23}$$

If $\cos T = \frac{11}{23}$, then $T = \cos^{-1}\left(\frac{11}{23}\right)$.

Use a calculator.

$m\angle T \approx 61.4°$

10-4 Trigonometry

38. 

SOLUTION:

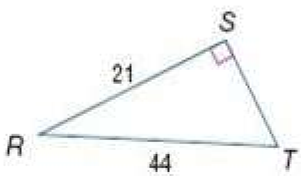
The measures given are those of the leg opposite to $\angle T$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin T = \frac{SR}{TR} = \frac{25}{30} = \frac{5}{6}$$

If $\sin T = \frac{5}{6}$, then $T = \sin^{-1}\left(\frac{5}{6}\right)$.

Use a calculator.

$m\angle T \approx 56.4°$

39. 

SOLUTION:

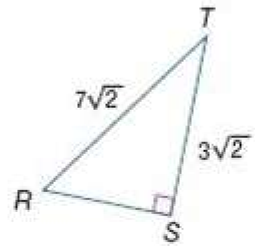
The measures given are those of the leg opposite to $\angle T$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin T = \frac{SR}{TR} = \frac{21}{44}$$

If $\sin T = \frac{21}{44}$, then $T = \sin^{-1}\left(\frac{21}{44}\right)$.

Use a calculator.

$m\angle T \approx 28.5°$

40. 

SOLUTION:

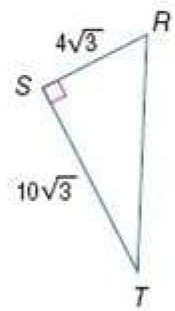
The measures given are those of the leg adjacent to $\angle T$ and the hypotenuse, so write an equation using the cosine ratio.

$$\cos T = \frac{TS}{TR} = \frac{3\sqrt{2}}{7\sqrt{2}} = \frac{3}{7}$$

If $\cos T = \frac{3}{7}$, then $T = \cos^{-1}\left(\frac{3}{7}\right)$.

Use a calculator.

$m\angle T \approx 64.6°$

41. 

SOLUTION:

The measures given are those of the legs opposite and adjacent to $\angle T$, so write an equation using the tangent ratio.

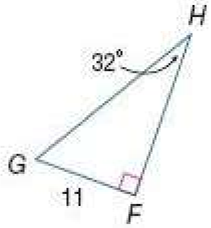
$$\tan T = \frac{SR}{ST} = \frac{4\sqrt{3}}{10\sqrt{3}} = \frac{2}{5}.$$

If $\tan T = \frac{2}{5}$, then $T = \tan^{-1}\left(\frac{2}{5}\right)$.

Use a calculator.

$m\angle T \approx 21.8°$

Solve each right triangle. Round side measures to the nearest tenth and angle measures to the nearest degree.

42. 

SOLUTION:

The sum of the measures of the angles of a triangle is 180. Therefore,

$m\angle G = 180 - (90 + 32) = 58.$

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. Let *x* be the length of the hypotenuse. Then,

$\sin 32° = \frac{11}{x}.$

Multiply each side by *x*.

$x \cdot \sin 32° = 11$

Divide each side by sin 32°.

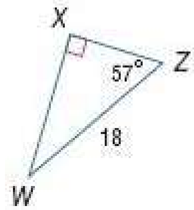
$x = \frac{11}{\sin 32°}$

Use a calculator to find the value of *x*.

$x \approx 20.8$

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So,

$$\begin{aligned} HF &= \sqrt{(20.8)^2 - 11^2} \\ &= \sqrt{311.64} \\ &\approx 17.7. \end{aligned}$$

43. 

SOLUTION:

The sum of the measures of the angles of a triangle is 180. Therefore,

$m\angle W = 180 - (90 + 57) = 33.$

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. Let *WX* = *x*. Then,

$\sin 57° = \frac{x}{18}.$

Multiply each side by 18.

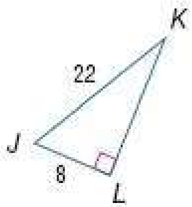
$18 \cdot \sin 57° = x$

Use a calculator to find the value of *x*.

$x \approx 15.1$

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So,

$$\begin{aligned} XZ &= \sqrt{18^2 - (15.1)^2} \\ &= \sqrt{95.99} \\ &\approx 9.8. \end{aligned}$$

44. 

SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So,

$$KL = \sqrt{22^2 - 8^2} = \sqrt{420} \approx 20.5.$$

The measures given are those of the leg opposite to $\angle K$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin K = \frac{JL}{JK} = \frac{8}{22} = \frac{4}{11}$$

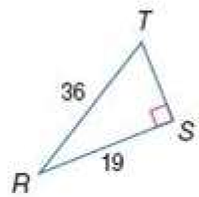
If $\sin K = \frac{4}{11}$, then $K = \sin^{-1}\left(\frac{4}{11}\right)$.

Use a calculator.

$m\angle K \approx 21°$

The sum of the measures of the angles of a triangle is 180. Therefore,

$m\angle J = 180 - (90 + 21) = 69.$

45. 

SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So,

$$TS = \sqrt{36^2 - 19^2} = \sqrt{935} \approx 30.6.$$

The measures given are those of the leg adjacent to $\angle R$ and the hypotenuse, so write an equation using the cosine ratio.

$$\cos R = \frac{SR}{TR} = \frac{19}{36}$$

If $\cos R = \frac{19}{36}$, then $R = \cos^{-1}\left(\frac{19}{36}\right)$.

Use a calculator.

$m\angle R \approx 58°$

The sum of the measures of the angles of a triangle is 180. Therefore,

$m\angle J = 180 - (90 + 58) = 32.$

46. **BACKPACKS** Ramón has a rolling backpack that is $3\frac{3}{4}$ feet tall when the handle is extended. When he is pulling the backpack, Ramon's hand is 3 feet from the ground. What angle does his backpack make with the floor? Round to the nearest degree.

SOLUTION:

Let x be the measure of the angle that his backpack make with the floor. The length of the backpack is $3\frac{3}{4}$ ft or 45 in. and it is 3 ft or 36 in. above the ground. The measures given are those of the leg opposite to x and the hypotenuse, so write an equation using the sine ratio.

$$\sin x = \frac{36}{45} = \frac{4}{5}$$

If $\sin x = \frac{4}{5}$, then $x = \sin^{-1}\left(\frac{4}{5}\right)$.

Use a calculator.

$x \approx 53°$

Therefore, the backpack makes an angle about $53°$ with the floor.

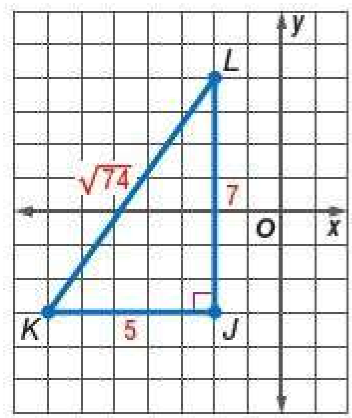
COORDINATE GEOMETRY Find the measure of each angle to the nearest tenth of a degree using the Distance Formula and an inverse trigonometric ratio.

47. $\angle K$ in right triangle JKL with vertices $J(-2, -3)$, $K(-7, -3)$, and $L(-2, 4)$

SOLUTION:

Use the Distance Formula to find the length of each side.

$$JK = \sqrt{(-7-(-2))^2 + (-3-(-3))^2} = \sqrt{(-5)^2 + (0)^2} = \sqrt{25} = 5$$
$$KL = \sqrt{(-2-(-7))^2 + (4-(-3))^2} = \sqrt{(5)^2 + (7)^2} = \sqrt{25+49} = \sqrt{74}$$
$$JL = \sqrt{(-2-(-2))^2 + (4-(-3))^2} = \sqrt{(0)^2 + (7)^2} = \sqrt{49} = 7$$

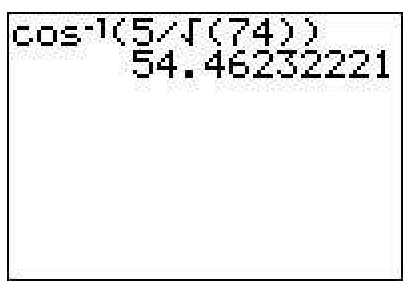


Since KL is the hypotenuse and JK is the leg adjacent to $\angle K$, then write an equation using the cosine ratio.

$$\cos K = \frac{JK}{KL} = \frac{5}{\sqrt{74}}$$

If $\cos K = \frac{5}{\sqrt{74}}$, then $K = \cos^{-1}\left(\frac{5}{\sqrt{74}}\right)$.

Use a calculator, in degree mode, to find the measure of angle K.



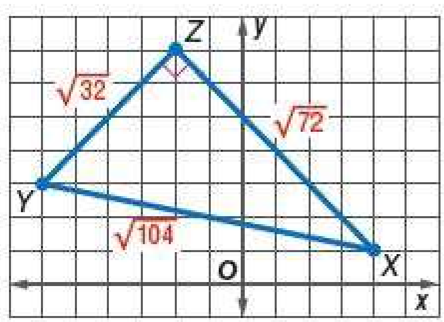
$m\angle K \approx 54.5°$

48. $\angle Y$ in right triangle XYZ with vertices $X(4, 1)$, $Y(-6, 3)$, and $Z(-2, 7)$

SOLUTION:

Use the Distance Formula to find the length of each side.

$$XY = \sqrt{(-6-4)^2 + (3-1)^2} = \sqrt{(-10)^2 + (2)^2} = \sqrt{100+4} = \sqrt{104}$$
$$YZ = \sqrt{(-2-(-6))^2 + (7-3)^2} = \sqrt{(4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$$
$$XZ = \sqrt{(-2-4)^2 + (7-1)^2} = \sqrt{(-6)^2 + (6)^2} = \sqrt{36+36} = \sqrt{72}$$



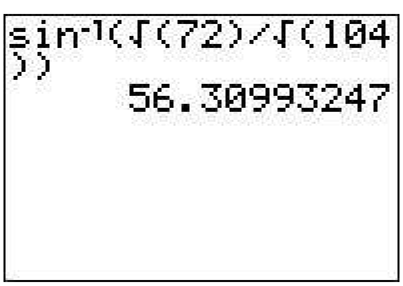
Since $\overline{XY}$ is the hypotenuse and $\overline{XZ}$ is the leg opposite to $\angle Y$, then write an equation using the sine ratio.

$$\sin Y = \frac{XZ}{XY} = \frac{\sqrt{72}}{\sqrt{104}}$$

Solve for *Y*:

$$\text{If } \sin Y = \frac{\sqrt{72}}{\sqrt{104}}, \text{ then } Y = \sin^{-1}\left(\frac{\sqrt{72}}{\sqrt{104}}\right).$$

Use a calculator, in degree mode, to find the measure of angle Y.



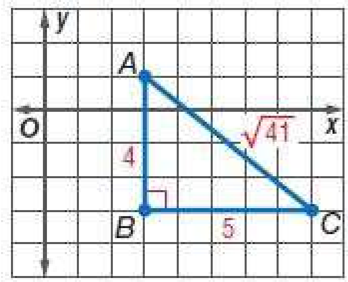
$m\angle Y \approx 56.3°$

49. $\angle A$ in in right triangle ABC with vertices $A(3, 1)$, $B(3, -3)$, and $C(8, -3)$

SOLUTION:

Use the Distance Formula to find the length of each side.

$$AB = \sqrt{(3-3)^2 + (-3-1)^2} = \sqrt{(0)^2 + (-4)^2} = \sqrt{16} = 4$$
$$BC = \sqrt{(8-3)^2 + (-3-(-3))^2} = \sqrt{(5)^2 + (0)^2} = \sqrt{25} = 5$$
$$AC = \sqrt{(8-3)^2 + (-3-1)^2} = \sqrt{(5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

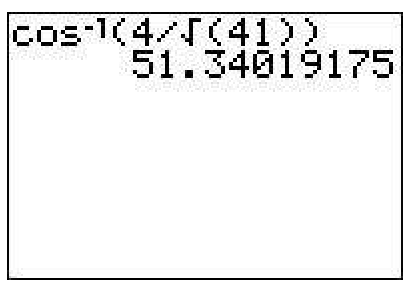


Since $\overline{AC}$ is the hypotenuse and $\overline{AB}$ is the leg adjacent to $\angle A$, then write an equation using the cosine ratio.

$$\cos A = \frac{AB}{AC} = \frac{4}{\sqrt{41}}$$

$$\text{If } \cos A = \frac{4}{\sqrt{41}}, \text{ then } A = \cos^{-1}\left(\frac{4}{\sqrt{41}}\right).$$

Use a calculator, in degree mode, to find the measure of angle A.



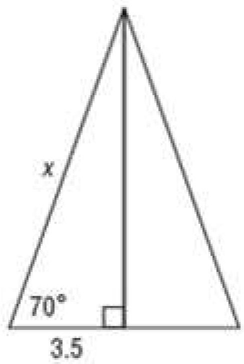
$m\angle A \approx 51.3°$

50. **SCHOOL SPIRIT** Hana is making a pennant for each of the 18 girls on her basketball team. She will use 0.5-inch seam binding to finish the edges of the pennants.

a. What is the total length of the seam binding needed to finish all of the pennants?

b. If the seam binding is sold in 3-yard packages at a cost of $1.79, how much will it cost?

SOLUTION:
The altitude to the base of an isosceles triangle bisects the base, which gives us the following diagram:



Write an equation using the cosine ratio.

$$\cos 70° = \frac{3.5}{x}$$

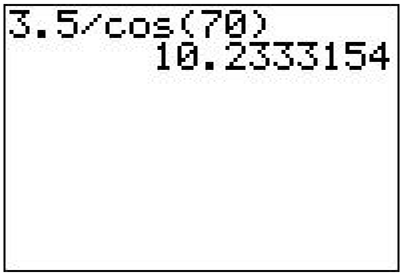
Multiply each side by *x*.

$$x \cdot \cos 70° = 3.5$$

Divide each side by cos 70°.

$$x = \frac{3.5}{\cos 70°}$$

Use a calculator, in degree mode, to find the value of *x*.



$$x \approx 10.23$$

The perimeter of one pennant is about 7 + 10.23 + 10.23 = 27.46 in .

Therefore, for 18 pennants, the total perimeter would be 27.46 (18)= 494.28 or 494 in.

b. To determine the cost of the binding, there are three things to consider: 1) how many yards of binding does Hana need? , 2) How many packages of binding would be sufficient?, and 3) What is the cost per package?

1) To determine the yards of binding Hana needs to buy, we need to convert 494 inches to yards. Since there are 36 inches in one yard, then $\frac{494}{36} = 13.72 yards.$

2) Since the binding comes in packages of 3 yards, we will need to buy 5 packages. (4 packages would only allow for 12 yards, which is not enough.)

3) The cost of one package of binding is $1.79, so 5(1.79) = $8.95

CCSS SENSE-MAKING Find the perimeter and area of each triangle. Round to the nearest hundredth.

51. 

SOLUTION:
Let *x* be the length of the side adjacent to the angle of measure 59°. Since *x* is the side adjacent to the given acute angle and 5 is opposite to it, write an equation using the tangent ratio.

$$\tan 59° = \frac{5}{x}$$

Multiply each side by *x*.

$$x \cdot \tan 59° = 5$$

Divide each side by tan 59°.

$$x = \frac{5}{\tan 59°}$$

Use a calculator to find the value of x. Make sure your calculator is in degree mode.

$$x \approx 3$$

Therefore, the area A of the triangle is

$$A = \frac{1}{2}bh = \frac{1}{2}(5)(3) = 7.5 \text{ in}^2.$$

Use the Pythagorean Theorem to find the length of the hypotenuse.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So,

$$3^2 + 5^2 = hypotenuse^2$$
$$9 + 25 = hypotenuse^2$$
$$34 = hypotenuse^2$$
$$hypotenuse = \sqrt{34} \approx 5.83$$

Therefore, the perimeter of the triangle is about 3 + 5 + 5.83 = 13.83 ft.

52. 

SOLUTION:

Let x be the length of the hypotenuse of the right triangle. Since x is the hypotenuse and the side adjacent to the acute angle is given, write an equation using the cosine ratio.

$$\cos 18° = \frac{12}{x}$$

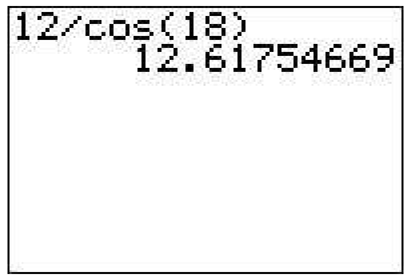
Multiply each side by x.

$$x \cdot \cos 18° = 12$$

Divide each side by cos 18°.

$$x = \frac{12}{\cos 18°}$$

Use a calculator to find the value of x. make sure your calculator is in degree mode.



$$x \approx 12.62$$

Use the Pythagorean Theorem to find the length of the hypotenuse.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So,

$$leg^2 + 12^2 = 12.62^2$$
$$leg^2 + 144 = 159.26$$
$$leg^2 = 15.26$$
$$leg = \sqrt{15.26} \approx 3.91$$

Therefore, the perimeter of the triangle is about 12 + 12.62 + 3.91 = 28.53 cm.

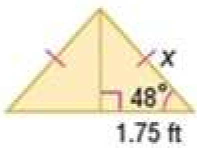
The area A of the triangle is

$$A = \frac{1}{2}bh = \frac{1}{2}(12)(3.91) = 23.46 \text{ cm}^2.$$

53. 

SOLUTION:

The altitude to the base of an isosceles triangle bisects the base. Then we have the following diagram.



Let x represent the hypotenuse of the right triangle formed. Since we are solving for *x* and have the length of the side adjacent to the given acute angle, write an equation using the cosine ratio.

$$\cos 48° = \frac{1.75}{x}$$

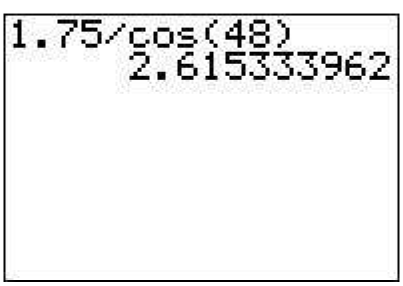
Multiply each side by *x*.

$$x \cdot \cos 48° = 1.75$$

Divide each side by cos 48°.

$$x = \frac{1.75}{\cos 48°}$$

Use a calculator to find the value of *x*.Make sure your calculator is in degree mode.



$$x \approx 2.62$$

The perimeter of the triangle is about 3.5 + 2.62 + 2.62 = 8.74 ft.

Use the Pythagorean Theorem to find the altitude.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

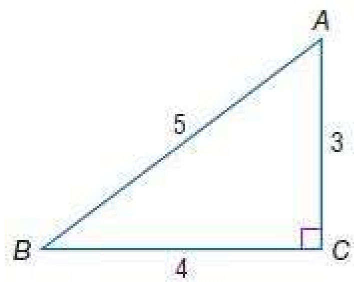
$$altitude^2 + 1.75^2 = 2.62^2$$
$$altitude^2 + 3.06 = 6.86$$
$$altitude^2 = 3.8$$
$$altitude = \sqrt{3.8} \approx 1.95$$

Therefore, the area *A* of the triangle is

$$A = \frac{1}{2}bh = \frac{1}{2}(3.5)(1.95) = 3.41\text{ft}^2.$$

54. Find the tangent of the greater acute angle in a triangle with side lengths of 3, 4, and 5 centimeters.

SOLUTION:



The lengths of the sides 3, 4, and 5 form a Pythagorean triple, so the triangle is right triangle.

The length of the hypotenuse is 5 units, the longer leg is 4 units and the shorter leg is 3 units.

The greater acute angle is the one opposite to the longer leg. Let *A* be the angle.

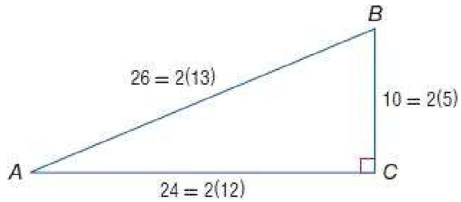
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

The side opposite to *A* measures 4 units and that adjacent to *A* measures 3 units.

$$\tan A = \frac{4}{3} \approx 1.33$$

55. Find the cosine of the smaller acute angle in a triangle with side lengths of 10, 24, and 26 inches.

SOLUTION:



The lengths of the sides, 10, 24, and 26 can be written as 2(5), 2(12), and 2(13).

We know that the numbers 5, 12, and 13 form a Pythagorean triple, so the triangle is right triangle.

The length of the hypotenuse is 26 units, the longer leg is 24 units and the shorter leg is 10 units long.

The smaller acute angle is the one opposite to the shorter leg. Let A be the angle.

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse.

The side adjacent to A measures 24 units and the hypotenuse is 26 units.

$$\cos A = \frac{24}{26} = 0.92$$

56. **ESTIMATION** Ethan and Tariq want to estimate the area of the field that their team will use for soccer practice. They know that the field is rectangular, and they have paced off the width of the field as shown. They used the fence posts at the corners of the field to estimate that the angle between the length of the field and the diagonal is about $40°$. If they assume that each of their steps is about 18 inches, what is the area of the practice field in square feet? Round to the nearest square foot.

SOLUTION:

Use the trigonometric ratios to find the length of the play ground. The width of the ground is 280 steps which is about 5040 inches = 420 feet. Let x be the length of the play ground.

Since x represents the side opposite the acute angle given and we are solving for the side adjacent to the acute angle, write an equation using the tangent ratio.

$$\tan 40° = \frac{420}{x}$$

Multiply each side by x.

$$x \cdot \tan 40° = 420$$

Divide each side by tan 40°.

$$x = \frac{420}{\tan 40°}$$

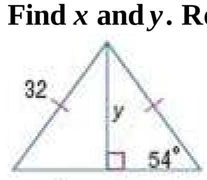
Use a calculator to find the value of x. Make sure your calculator is in degree mode.

$$x \approx 500.54$$

Therefore, the area of the play ground is about

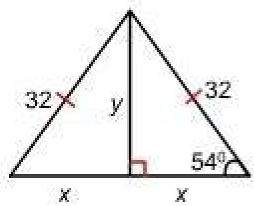
$$420(500.54) \approx 210227 \text{ ft}^2.$$

Find x and y. Round to the nearest tenth.

57. 

SOLUTION:

The altitude to the base of an isosceles triangle bisects the base. Then we have the following diagram:



Since x is the side adjacent to the given acute angle and the hypotenuse is given, write an equation using the cosine ratio.

$$\cos 54° = \frac{x}{32}$$

Multiply each side by 32.

$$32 \cdot \cos 54° = x$$

Use a calculator to find the value of x. Make sure your calculator is in degree mode.

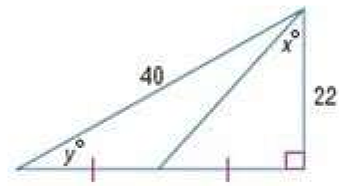
$$x \approx 18.8$$

Use the Pythagorean Theorem to find the altitude.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$y^2 + 18.8^2 = 32^2$$
$$y^2 + 353.44 = 1024$$
$$y^2 = 670.56$$
$$y = \sqrt{670.56} \approx 25.9$$

58.



SOLUTION:

Use the Pythagorean Theorem to find the length of the third side of the right triangle.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$leg^2 + 22^2 = 40^2$$
$$leg^2 + 484 = 1600$$
$$leg^2 = 1116$$
$$leg = \sqrt{1116} \approx 33.4$$

The side of length 33.4 units is divided into two congruent segments. So, each segment is about 16.7 units long.

Use the trigonometric ratios to find the values of x and y.

Since x is opposite 16.7 and adjacent to 22, write an equation for x using the tangent ratio.

$$\tan x = \frac{16.7}{22} \Rightarrow x = \tan^{-1}\left(\frac{16.7}{22}\right)$$

Use a calculator to solve for x. Make sure your calculator is in degree mode.

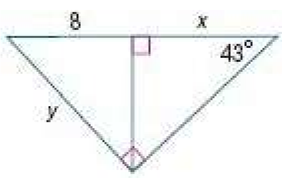
$$x \approx 37.2$$

Since y is opposite 22 and the hypotenuse is 40, write an equation for y using the sine ratio.

$$\sin y° = \frac{22}{40} \Rightarrow y° = \sin^{-1}\left(\frac{22}{40}\right)$$

```
sin⁻¹(22/40)
     33.36701297
```

$y \approx 33.4$

59. 

SOLUTION:

The sum of the measures of the angles of a triangle is 180. Therefore,

$$\text{measure of the third angle} = 180 - (90 + 43) = 47.$$

Since y is the hypotenuse and 8 is adjacent to the given angle, write an equation using the cosine ratio.

$$\cos 47° = \frac{8}{y}$$

Solve the equation for y.

$$y = \frac{8}{\cos 47°}$$

Use your calculator to solve for y. Make sure your calculator is in degree mode.

```
8/cos(47)
     11.73023349
```

$y \approx 11.7$

Use the Pythagorean Theorem to find the altitude of the triangle.

$$altitude^2 + 8^2 = 11.7^2$$
$$altitude^2 + 64 = 136.89$$
$$altitude^2 = 72.89$$
$$altitude = \sqrt{72.89} \approx 8.5$$

Since x is the side adjacent the given acute angle and 8.5 is side opposite the given angle, write an equation using the tangent ratio of the angle of measure 43°.

$$\tan 43° = \frac{8.5}{x}$$

Solve the equation for x.

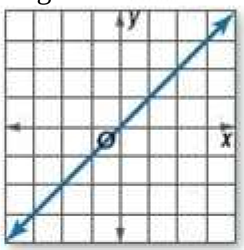
$$x = \frac{8.5}{\tan 43°}$$

Use your calculator to solve for x. Make sure your calculator is in degree mode.

```
8.5/tan(43)
     9.115134035
```

$x \approx 9.1$

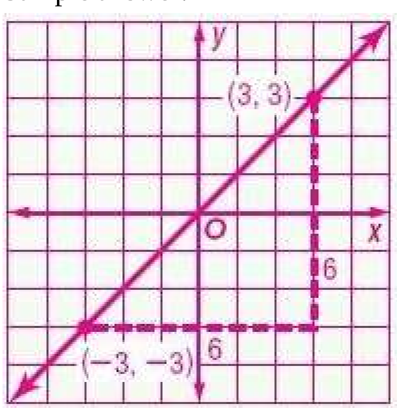
60. **COORDINATE GEOMETRY** Show that the slope of a line at $225°$ from the *x*-axis is equal to the tangent of $225°$.



SOLUTION:

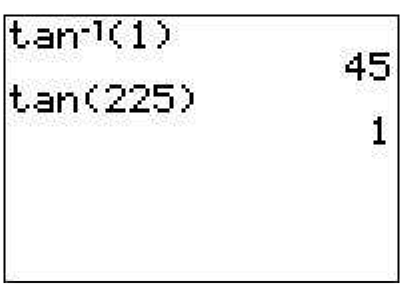
Plot two points that lie on the line and find their slope, sketching a right triangle as shown below. Slope is determined by the tangent ratio of the triangle. It is the rise (opposite) over run (adjacent side) of the line.

Sample answer:



$$\text{Slope} = \frac{-3-3}{-3-3} = \frac{6}{6} = 1$$

Since the slope is 1 and slope is rise over run, which is tan $45°$ or tan $225°$.



61. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate an algebraic relationship between the sine and cosine ratios.

a. GEOMETRIC Draw three nonsimilar right triangles that are not similar to each other. Label the triangles *ABC*, *MNP*, and *XYZ*, with the right angles located at vertices *B*, *N*, and *Y*, respectively. Measure and label each side of the three triangles.

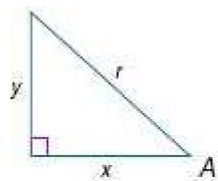
b. TABULAR Copy and complete the table below.

Triangle	Trigonometric Ratios				Sum of Ratios Squared	
ABC	cos *A*		sin *A*		$(\cos A)^2 + (\sin A)^2 =$	
	cos *C*		sin *C*		$(\cos C)^2 + (\sin C)^2 =$	
MNP	cos *M*		sin *M*		$(\cos M)^2 + (\sin M)^2 =$	
	cos *P*		sin *P*		$(\cos P)^2 + (\sin P)^2 =$	
XYZ	cos *X*		sin *X*		$(\cos X)^2 + (\sin X)^2 =$	
	cos *Z*		sin *Z*		$(\cos Z)^2 + (\sin Z)^2 =$	

c. VERBAL Make a conjecture about the sum of the squares of the cosine and sine of an acute angle of a right triangle.

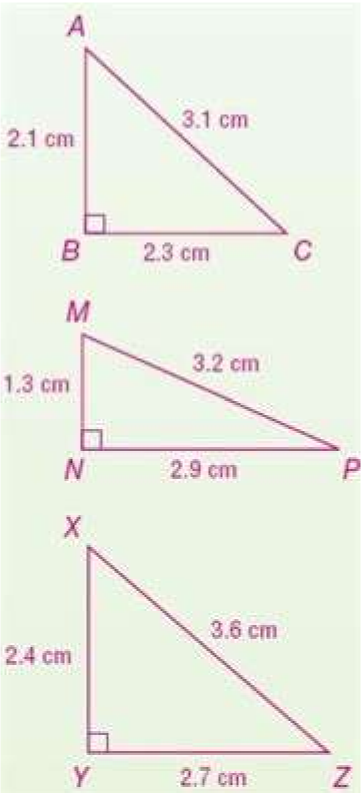
d. ALGEBRAIC Express your conjecture algebraically for an angle X.

e. ANALYTICAL Show that your conjecture is valid for angle A in the figure using the trigonometric functions and the Pythagorean Theorem.



SOLUTION:

a. Create three right triangles, using a protractor and straight edge. Measure the side lengths carefully, in centimeters.



b. Calculate the sine, cosine, and tangent of each acute angle, in triangle you created. Then, write their ratios in the table provided. Round your ratios to the nearest thousandth. Then, substitute those values into the formula $(\sin X)^2 + (\cos X)^2$ for each angle requested.

Triangle	Trigonometric Ratios				Sum of Ratios Squared	
ABC	cos A	0.677	sin A	0.742	$(\cos A)^2 + (\sin A)^2 =$	1
	cos C	0.742	sin C	0.677	$(\cos C)^2 + (\sin C)^2 =$	1
MNP	cos M	0.406	sin M	0.906	$(\cos M)^2 + (\sin M)^2 =$	1
	cos P	0.906	sin P	0.406	$(\cos P)^2 + (\sin P)^2 =$	1
XYZ	cos X	0.667	sin X	0.75	$(\cos X)^2 + (\sin X)^2 =$	1
	cos Z	0.75	sin Z	0.667	$(\cos Z)^2 + (\sin Z)^2 =$	1

c. Observe any pattern you see on last column of the table. Sample answer: The sum of the cosine squared and the sine squared of an acute angle of a right triangle is 1.

d. $(\sin X)^2 + (\cos X)^2 = 1$

e. Conjecture that $(\sin A)^2 + (\cos A)^2$ might not equal one, then prove that it must be, using the trigonometric ratios from the provided triangle and the Pythagorean theorem.

Sample answer:

$(\sin A)^2 + (\cos A)^2 \stackrel{?}{=} 1$ (Conjecture)

$\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 \stackrel{?}{=} 1$ $\left(\sin A = \frac{y}{r}, \cos A = \frac{x}{r}\right)$

$\frac{y^2}{r^2} + \frac{x^2}{r^2} \stackrel{?}{=} 1$ (Simplify)

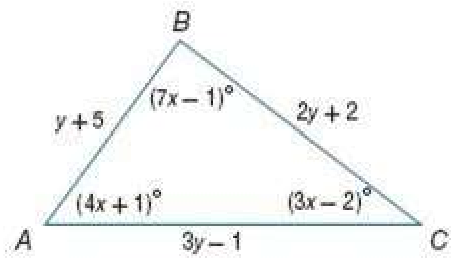
$\frac{y^2 + x^2}{r^2} \stackrel{?}{=} 1$ (Combine fractions)

Since $y^2 + x^2 = r^2$, then

$\frac{r^2}{r^2} \stackrel{?}{=} 1$ (Pythagorean Theorem)

$1 = 1$ (Simplify)

62. **CHALLENGE** Solve ΔABC. Round to the nearest whole number.



SOLUTION:

The sum of the measures of the angles of a triangle is 180, therefore $7x - 1 + 4x + 1 + 3x - 2 = 180$.

Solve for *x*.

$$\begin{aligned} 7x - 1 + 4x + 1 + 3x - 2 &= 180 \\ 14x - 2 &= 180 \\ 14x &= 182 \\ x &= 13 \end{aligned}$$

Use the value of *x* to find the measures of the angles.

$$m\angle A = 4(13) + 1 = 53$$
$$m\angle B = 7(13) - 1 = 90$$
$$m\angle C = 3(13) - 2 = 37$$

So, ΔABC is a right triangle with the right angle *B*. In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. Therefore, $(3y-1)^2 = (y+5)^2 + (2y+2)^2$.

Simplify.

$$(3y-1)^2 = (y+5)^2 + (2y+2)^2$$
$$(3y-1)(3y-1) = (y+5)(y+5) + (2y+2)(2y+2)$$
$$9y^2 - 3y - 3y + 1 = y^2 + 5y + 5y + 25 + 4y^2 + 4y + 4y + 4$$
$$9y^2 - 6y + 1 = y^2 + 10y + 25 + 4y^2 + 8y + 4$$
$$4y^2 - 24y - 28 = 0$$
$$4\left(y^2 - 6y - 7\right) = 0$$
$$4 \neq 0 \Rightarrow y^2 - 6y - 7 = 0$$

Use the Quadratic formula to find the roots of the equation.

$$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-7)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{36+28}}{2}$$
$$= \frac{6 \pm \sqrt{64}}{2}$$
$$= \frac{6 \pm 8}{2}$$
$$= 7 \text{ or } -1$$

The root $y = -1$ will make the length $AC = 3(-1) - 1 = -4$. So, y cannot be –1. Then, $y = 7$.

Use the value of y to find the lengths of the sides.

$$AB = 7 + 5 = 12$$
$$BC = 2(7) + 2 = 16$$
$$AC = 3(7) - 1 = 20$$

63. **REASONING** Are the values of sine and cosine for an acute angle of a right triangle always less than 1? Explain.

SOLUTION:

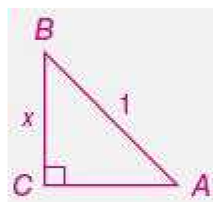
Sample answer: Yes; since the values of sine and cosine are both calculated by dividing one of the legs of a right triangle by the hypotenuse, and the hypotenuse is always the longest side of a right triangle, the values will always be less than 1. You will always be dividing the smaller number by the larger number.

64. **CCSS REASONING** What is the relationship between the sine and cosine of complementary angles? Explain your reasoning and use the relationship to find cos 50 if sin 40 ≈ 0.64.

SOLUTION:

The trigonometric functions that we have discussed in this lesson describe relationships in right triangles. In a right triangle, the non-right angles sum to 90 degrees, so they will always be complementary. *Sine* relates the opposite side to the hypotenuse and *cosine* relates the adjacent side to the hypotenuse. Set the hypotenuse equal to 1 to make the calculations easier.

In the diagram below, angles A and B are the complementary angles for the right triangle.



$\sin A = x$ and $\cos B = x$; therefore $\sin A = \cos B$.

Since the acute angles of a right triangle are complementary, $m\angle B = 90 - m\angle A$. By substitution, $\sin A = \cos(90 - A)$. Since $\sin A = x$, $\cos(90 - A) = x$ by substitution.

Applying this relationship, if sin 40 ≈ 0.64, then cos (90 – 40) ≈ 0.64. Since 90 – 40 = 50, cos 50 ≈ 0.64.

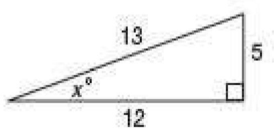
65. **WRITING IN MATH** Explain how you can use ratios of the side lengths to find the angle measures of the acute angles in a right triangle.

SOLUTION:

Describe the process for how you find the acute angle measure of a right triangle, when you know the side lengths. Include how you use the calculator to solve for the angle measure, once you know the correct ratio of the sides.

Sample answer: To find the measure of an acute angle of a right triangle, you can find the ratio of the leg opposite the angle to the hypotenuse and use a calculator to find the inverse sine of the ratio, you can find the ratio of the leg adjacent to the angle to the hypotenuse and use a calculator to find the inverse cosine of the ratio, or you can find the ratio of the leg opposite the angle to the leg adjacent to the angle and use a calculator to find the inverse tangent of the ratio.

66. What is the value of tan x?



A $\tan x = \frac{13}{5}$

B $\tan x = \frac{12}{5}$

C $\tan x = \frac{5}{13}$

D $\tan x = \frac{5}{12}$

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side.

Therefore, $\tan x = \frac{5}{12}$ and the correct choice is D.

67. **ALGEBRA** Which of the following has the same value as $2^{-12} \times 2^3$?

F 2^{-36}

G 4^{-9}

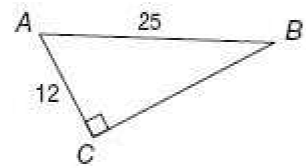
H 2^{-9}

J 2^{-4}

SOLUTION:

We have $a^m \times a^n = a^{m+n}$. So, $2^{-12} \times 2^3 = 2^{-12+3} = 2^{-9}$. Therefore, the correct choice is H.

68. **GRIDDED RESPONSE** If AC = 12 and AB = 25, what is the measure of $\angle B$ to the nearest tenth?



SOLUTION:

The measures given are those of the leg opposite to $\angle B$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin B = \frac{AC}{AB} = \frac{12}{25}$$

If $\sin B = \frac{12}{25}$, then $B = \sin^{-1}\left(\frac{12}{25}\right)$.

Use a calculator to solve for the measure of angle B. Make sure it is in degree mode.

$m\angle B \approx 28.7°$

69. **SAT/ACT** The area of a right triangle is 240 square inches. If the base is 30 inches long, how many inches long is the hypotenuse?

A 5
B 8
C 16
D $2\sqrt{241}$
E 34

SOLUTION:

The area of a triangle is given by the formula $A = \frac{1}{2}bh.$

A = 240 and b = 30. Substitute into the area formula and solve for h:

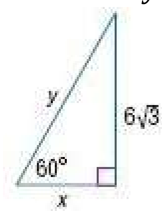
$$A = \frac{1}{2}bh$$
$$240 = \frac{1}{2}(30)h$$
$$240 = 15h$$
$$16 in. = h$$

Use the Pythagorean Theorem to find the length of the third side of the right triangle.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$(16)^2 + (30)^2 = (hyp)^2$$
$$256 + 900 = (hyp)^2$$
$$\sqrt{1156} = hypotenuse$$
$$hypotenuse = 34 \text{ inches}$$

Find x and y.

70. 

SOLUTION:

In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s ($h=2s$) , and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg ($l = s\sqrt{3}$).

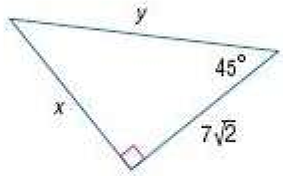
The length of the hypotenuse is y, the shorter leg is x, and the longer leg is $6\sqrt{3}$.

Solve for x:

$$x\sqrt{3} = 6\sqrt{3}$$
$$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3}}$$
$$x = 6$$

Solve for y:

$$y = 2(6) = 12$$

71. 

SOLUTION:
In a 45°-45°-90° triangle, the legs l are congruent and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg $(h = l\sqrt{2})$.

Since x is one of the congruent legs, then $x = 7\sqrt{2}$.

Solve for y:

$$y = 7\sqrt{2}(\sqrt{2})$$
$$y = 7 \cdot 2$$
$$y = 14$$

72. 

SOLUTION:
In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s $(h=2s)$, and the length of the longer leg l is $\sqrt{3}$ times the length of the shorter leg ($l = s\sqrt{3}$).

The length of the hypotenuse is 14 the shorter leg is x, and the longer leg is y.

Solve for y:

$$14 = 2y$$
$$y = 7$$

Solve for x:

$$x = 7\sqrt{3}$$

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute, obtuse,* or *right*. Justify your answer.

73. 8, 15, 17

SOLUTION:
By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$$8 + 15 > 17 \checkmark$$
$$15 + 17 > 8 \checkmark$$
$$8 + 17 > 15 \checkmark$$

Therefore, the set of numbers can be measures of a triangle.

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$17^2 \stackrel{?}{=} 15^2 + 8^2$$
$$289 \stackrel{?}{=} 225 + 64$$
$$289 = 289 \checkmark$$

Therefore, by the converse of Pythagorean Theorem, a triangle with the given measures will be a right triangle.

74. 11, 12, 24

SOLUTION:
By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$$11 + 12 > 24 \times$$
$$12 + 24 > 11 \checkmark$$
$$11 + 24 > 12 \checkmark$$

Since the sum of the lengths of two sides is less than that of the third side, the set of numbers cannot be measures of a triangle.

75. 13, 30, 35

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$13 + 30 > 35$ ✓

$30 + 35 > 13$ ✓

$13 + 35 > 30$ ✓

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$35^2 \stackrel{?}{=} 30^2 + 13^2$$
$$1225 \stackrel{?}{=} 900 + 169$$
$$1225 > 1069$$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an obtuse triangle.

76. 18, 24, 30

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$30 + 18 > 24$ ✓

$18 + 24 > 30$ ✓

$30 + 24 > 18$ ✓

Therefore, the set of numbers can be measures of a triangle.

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$30^2 \stackrel{?}{=} 18^2 + 24^2$$
$$900 \stackrel{?}{=} 324 + 576$$
$$900 = 900 \checkmark$$

Therefore, by the converse of Pythagorean Theorem, a triangle with the given measures will be a right triangle.

77. 3.2, 5.3, 8.6

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$3.2 + 5.3 > 8.6$ ✗

$5.3 + 8.6 > 3.2$ ✓

$3.2 + 8.6 > 5.3$ ✓

Since the sum of the lengths of two sides is less than that of the third side, the set of numbers cannot be measures of a triangle.

78. $6\sqrt{3}, 14, 17$

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$$6\sqrt{3} \approx 10.4$$
$$6\sqrt{3} + 14 > 17 \checkmark$$
$$14 + 17 > 6\sqrt{3} \checkmark$$
$$6\sqrt{3} + 17 > 14 \checkmark$$

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$17^2 \stackrel{?}{=} \left(6\sqrt{3}\right)^2 + 14^2$$
$$289 \stackrel{?}{=} 108 + 196$$
$$289 < 304$$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an acute triangle.

79. **MAPS** The scale on the map of New Mexico is 2 centimeters = 160 miles. The width of New Mexico through Albuquerque on the map is 4.1 centimeters. How long would it take to drive across New Mexico if you drove at an average of 60 miles per hour?

SOLUTION:

First, find the actual distance from one end to the other end through Albuquerque.

Let x be the distance. Then,

$$\frac{2}{160} = \frac{\text{distance on map}}{\text{actual distance}}$$
$$\frac{2}{160} = \frac{4.1}{x}$$

Solve the proportion to find the value of x.

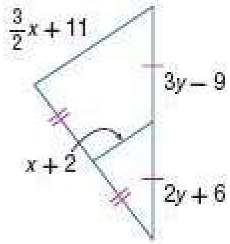
$$2x = 656$$
$$x = 328$$

The distance from one end to the other end through Albuquerque is 328 miles.

The average speed is 60 miles per hour. Therefore, the time taken to travel from one end to the other end through Albuquerque is

$$\frac{328}{60} = 5\frac{7}{15} \text{ or } 5 \text{ hr } 28 \text{ min.}$$

ALGEBRA Find x and y.

80. 

SOLUTION:

The measures of the segments of lengths $3y - 9$ and $2y + 6$ are equal. Solve for y:

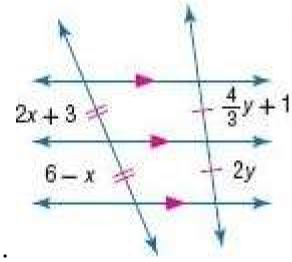
$$3y - 9 = 2y + 6$$
$$y = 15$$

By the Triangle Midsegment Theorem, a midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.
The segment of length $x + 2$ is a midsegment and the length is half of $\frac{3}{2}x + 11$.

Solve for x:

$$\frac{3}{2}x + 11 = 2(x + 2)$$
$$\frac{3}{2}x + 11 = 2x + 4$$
$$3x + 22 = 4x + 8$$
$$-x = -14$$

$$x = 14$$

81. 

SOLUTION:

The measures of the segments of lengths $2x + 3$ and $6 - x$ are equal.

Solve for x:

$$2x + 3 = 6 - x$$
$$3x = 3$$
$$x = 1$$

Similarly, the measures of the segments of lengths $\frac{4}{3}y + 1$ and $2y$ are equal.

Solve for y:

$$\frac{4}{3}y + 1 = 2y$$
$$4y + 3 = 6y$$
$$3 = 2y$$
$$y = \frac{3}{2}$$

Solve each proportion. Round to the nearest tenth if necessary.

82. $2.14 = \frac{x}{12}$

SOLUTION:

Multiply each side by 12.

$$2.14 = \frac{x}{12}$$
$$25.68 = x$$
$$x \approx 25.7$$

83. $0.05x = 13$

SOLUTION:
Divide each side by 0.05.

$$\frac{0.05x}{0.05} = \frac{13}{0.05}$$
$$x = 260$$

84. $0.37 = \frac{32}{x}$

SOLUTION:
First multiply each side by *x*.

$$x(0.37) = x\left(\frac{32}{x}\right)$$
$$0.37x = 32$$
$$\frac{0.37x}{0.37} = \frac{32}{0.37}$$
$$x = 86.4864\ldots$$
$$x \approx 86.5$$

85. $0.74 = \frac{14}{x}$

SOLUTION:
First multiply each side by *x*.

$$x(0.74) = x\left(\frac{14}{x}\right)$$
$$0.74x = 14$$
$$\frac{0.74x}{0.74} = \frac{14}{0.74}$$
$$x = 18.9189\ldots$$
$$x \approx 18.9$$

86. $1.66 = \frac{x}{23}$

SOLUTION:
Multiply each side by 23.

$$23(1.66) = 23\left(\frac{x}{23}\right)$$
$$x = 38.18$$
$$x \approx 38.2$$

87. $0.21 = \frac{33}{x}$

SOLUTION:
First multiply each side by *x*.

$$x(0.21) = x\left(\frac{33}{x}\right)$$
$$0.21x = 33$$
$$\frac{0.21x}{0.21} = \frac{33}{0.21}$$
$$x = 157.1428\ldots$$
$$x \approx 157.1$$