

## Unit F - Beyond Straight Lines - Quadratic and Absolute Value Functions

### Overview

In this unit students work with quadratic expressions, quadratic equations, radicals and rational expressions to see how changing the form of an expression or equation can give the item a clearer meaning and can make it easier to work with. By the end of the unit students should be able to fluently solve quadratic equations. They should be able to fluently identify transformations made to the parent function so they are able to visualize the graph to make estimations and to check to see if their solution makes sense. The unit ends with students using their new factoring skills to simplify rational expressions and equations into manageable problems.

**21<sup>st</sup> Century Capacities:** Analyzing, Product Creation

### Stage 1 - Desired Results

| ESTABLISHED GOALS/ STANDARDS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  | <b>Transfer:</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
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| <b>MP4</b> Model with Mathematics<br><b>MP5</b> Use appropriate tools strategically<br><b>MP7</b> Look for and make use of structure<br><br><b>A.SSE.1a</b> Interpret parts of an expression, such as terms, factors, and coefficients.<br><b>A.SSE.1b</b> Interpret complicated expressions by viewing one or more of their parts as a single entity.<br><b>A.CED.3</b> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.<br><b>A.REI.10</b> Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).<br><b>8.F.2</b> Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.<br><b>CC.8.F.5</b> Describe qualitatively the functional relationship |  | <i>Students will be able to independently use their learning in new situations to...</i><br><br>1. Explain real world phenomena mathematically for events that are parabolic in nature;<br>2. Draw conclusions about graphs and equations;(Analyzing)<br>3. Manipulate equations/expressions or objects to create order and establish relationships. (Analyzing)(Product Creation)                                                                                                                                                                                                                                                                                                                                                                                                                               |
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| <p>between two quantities by reading a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p> <p><b>F.IF.4</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity</p> <p><b>F.IF.5</b> Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.</p> <p><b>Analyze functions using different representations.</b></p> <p><b>F.IF.7</b> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases</p> <p><b>F.IF.7a</b> Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p><b>F.IF.7b</b> Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p><b>F.IF.8</b> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p><b>F.IF.8a</b> Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p><b>F.IF.9</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</p> <p><b>Build a function that models a relationship between two quantities.</b></p> | <p>written in different but equivalent forms to build meaning or ease of use.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | <p>D. What is another way to represent this?</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
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| <p><b>F.BF.1</b> Write a function that describes a relationship between two quantities</p> <p><b>Build new functions from existing functions.</b></p> <p><b>F.BF.3</b> Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.</p> <p><b>A.SSE.2</b> Use the structure of an expression to identify ways to rewrite it.</p> <p><b>A.SSE.3</b> Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression</p> <p><b>A.SSE.3a</b> Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p><b>A.SSE.3b</b> Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p><b>A.APR.1</b> Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials</p> <p><b>A.REI.4</b> Solve quadratic equations in one variable.</p> <p><b>A.REI.4a</b> Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form. (done in this unit as an extension)</p> <p><b>A.REI.4b</b> Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</p> |  | <p>expressions</p> <p>18. Using factoring to simplify rational equations before solving</p> <p>19. Finding the restrictions on the variable of a rational expression</p> |
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