

CALCULUS: Graphical, Numerical, Algebraic by Finney, Demana, Watt and Kennedy  
 Chapter 6: Differential Equations      6.2: Rewriting integrals using u-substitution

Rewrite each definite integral in terms of  $u$  and  $du$

$$1. \int_{x=0}^{x=1} (5x+4)^5 dx \quad \text{Let } u = 5x+4$$

$$u(0) = 4 \quad u(1) = 9$$

$$\frac{du}{dx} = 5 \quad du = 5 dx$$

$$\frac{du}{5} = dx$$

$$\int_4^9 u^5 \frac{du}{5} = \left[ \frac{1}{5} u^6 \right]_4^9$$

$$2. \int_0^2 3x^2(x^3 + 4)^5 dx \quad \text{Let } u = x^3 + 4$$

$$u(0) = 4 \quad u(2) = 12$$

$$\frac{du}{dx} = 3x^2 \quad du = 3x^2 dx$$

$$\int_4^{12} 3x^2(u^5) dx$$

$$\int_4^{12} \frac{du}{dx} (u^5) dx$$

$$\int_4^{12} u^5 du = \left[ \frac{1}{6} u^6 \right]_4^{12}$$

3.  $\int_1^3 \cos(2x+1) dx$  Let  $u = 2x + 1 \rightarrow \frac{du}{dx} = 2$   
 $\frac{du}{2} = dx$

$$\int_3^7 \cos(u) du$$

$$\int_3^7 \cos(u) \frac{du}{2} = \frac{1}{2} \int_3^7 \cos(u) du$$

(1) Change upper/lower limit

(2) Substitute what you know

(3) Take the derivative of u

(4) solve  $\frac{du}{dx}$  for  $dx$

(5) Substitute in for  $dx$

4.  $\int_0^{\pi/4} \frac{\sin x}{(\cos x)^5} dx$  Let  $u = \cos x \quad \frac{du}{dx} = -\sin x$   
 $\frac{du}{-\sin x} = dx$

$$\int_1^{\sqrt{2}/2} \frac{\sin x}{u^5} dx$$

$$\int_1^{\sqrt{2}/2} \frac{\sin x}{u^5} \cdot \frac{du}{-\sin x} = \int_1^{\sqrt{2}/2} \frac{-1}{u^5} du = \boxed{\int_1^{\sqrt{2}/2} -u^{-5} du}$$