Precalc notes **Name:**

Math 3 Unit 7: Trigonometric Functions, Equations, and Identities

Standards

M3 7.1 I can transform sine and cosine functions to fit contexts.
M3 7.2 I can calculate the tangent, secant, cosecant, and cotangent of an angle.
M3 7.3 I can prove trigonometric identities.
M3 7.4 I can use trigonometric identities to evaluate trigonometric expressions.
M3 7.5 I can find all solutions to a trigonometric equation.

$f(x) = a\sin(b(x-c)) + d$

a = amplitude of wave / radius of circle
$b = 2\pi \div$ period (or $360° \div$ period)
c = phase shift, translates wave c units to the right
d = midline translated up d units

phase shift

translating a periodic function right or left
↳ repeating

solving trig equations with graphs

★ click the wrench in Desmos to change radians vs. degrees ★

graph the trig function
graph the desired y-value as $y = \#$
find the points where the wave and the line intersect

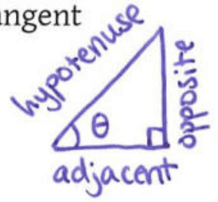
solving trig equations with algebra

do algebra until you get just the trig function on one side
(like sin(something) = something)
do the inverse trig function to both sides ★ check DEG/RAD mode ★
this gives you one angle. find the others. (there are ∞ more.)
do more algebra (on all the answers) to finish solving

frequency — how often something happens

$$\frac{\#\text{ repeats or }\#\text{ rotations}}{\text{time}} \qquad \text{frequency} = \frac{1}{\text{period}}$$

tangent



$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} \qquad \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

secant

$$\sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \sec(\theta) = \frac{1}{\cos(\theta)}$$

cosecant

$$\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

cotangent

$$\cot(\theta) = \frac{\text{adjacent}}{\text{opposite}} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

notation convention: $\sin^2\theta$ $= (\sin\theta)^2$

$\cos^2\theta = (\cos\theta)^2$ etc.

equation vs. identity

an identity is an equation that is true no matter what the variables are.

example: $x+1=3$ is an equation that is only true if $x=2$ (<u>not</u> an identity)

example: $x+0=x$ is an <u>identity</u> because it is true no matter what x is

trigonometric identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin\theta = \cos(90^\circ - \theta) \quad \text{or} \quad \sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\sin^2\theta + \cos^2\theta = 1 \quad \text{(Pythagorean identity)}$$

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\sin(\pi - \theta) = \sin\theta \qquad \cos(\pi - \theta) = -\cos\theta$$

$$\sin(\pi + \theta) = -\sin\theta \qquad \cos(\pi + \theta) = -\cos\theta$$

$$\sin(2\pi - \theta) = -\sin\theta \qquad \cos(2\pi - \theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

for all angles θ, α, and β

Is it an identity? If so, then it is true for all possible angles. Try plugging in an angle to see if it's true.

Protip: Avoid special angles like 90°, 45°, etc.

Protip: If you're not sure, try another angle!

If it's <u>not</u> an identity, disprove it by showing that there is an angle it doesn't work for. Plug in an angle that doesn't work and show that the equation is false.

Example: Prove that $\sin(2\theta) = 2\sin(\theta)$ is not an identity.

if $\theta = 78°$, then $\sin(2\theta) = \sin(2 \cdot 78°) \approx 0.4067$

but $2\sin(\theta) = 2\sin(78°) \approx 1.956$

the equation does not hold for $\theta = 78°$.

If it <u>is</u> an identity, prove it like this:

This is not the only way to prove things!

① Start with one side of the equation.

Protip: It's usually easier to start with the more complicated side.

② Do legal algebra moves to change your expression into equivalent expressions, ending up at the other side of the equation. Make sure you can explain why each step works! Some options:

- start by converting everything to sin & cos ← highly recommended!
- algebra / simplifying moves
- use trig identities that were proven previously
- you can replace a 1 with $\sin^2\theta + \cos^2\theta$
- multiply by a clever form of 1
- if there is a $\sin(2\theta)$ or similar, use double angle or angle sums identity

solving trig equations

with algebra

- isolate a trig function on one side
  (so like $\sin(\sim) = \sim$ or $\cos(\sim) = \sim$)
  - by using algebra
  - by using trig identities
- use the inverse trig function to solve for the angle
  → remember, there are multiple solutions! consult the unit circle, usually there are 2 there. when you include additional rotations, there are ∞ more.
- finish solving with algebra

with graphing

graph each side of the equation, look for intersections

restricted domain

maybe we only want the solutions between 0 and 2π. then there would only be a few solutions.
(usually 2 solutions, but 4 if there was a 2θ, 6 if there was a 3θ, etc.)

writing all solutions to a trig equation

when you use your inverse trig function to get an angle, your calculator will only give you one angle, but there are ∞. find all of them!

for inverse sine: $\# \pm 2\pi n$ or $\pi - \# \pm 2\pi n$
for inverse cosine: $\# \pm 2\pi n$ or $-\# \pm 2\pi n$
for inverse tangent: $\# \pm \pi n$

n is any whole number

remember that this happens when you do the inverse trig function! any additional algebra steps happen to all of the solutions!

Example of trig proof

Proof of: $\sec\theta(\sec\theta - \cos\theta) = \tan^2\theta$

Proof: $\sec\theta(\sec\theta - \cos\theta)$ start with complex side

replace with sin & cos

$$= \frac{1}{\cos\theta}\left(\frac{1}{\cos\theta} - \cos\theta\right)$$

$$= \frac{1}{\cos\theta}\left(\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\cos\theta}\right)$$

multiply by clever form of 1 to get a common denominator

$$= \frac{1}{\cos\theta}\left(\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}\right)$$

subtract fractions

$$= \frac{1}{\cos\theta}\left(\frac{1-\cos^2\theta}{\cos\theta}\right)$$

multiply fractions

$$= \frac{1-\cos^2\theta}{\cos^2\theta}$$

replace 1 with $\sin^2+\cos^2$ because of Pythagorean identity

$$= \frac{\sin^2\theta+\cos^2\theta-\cos^2\theta}{\cos^2\theta}$$

combine like terms

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

exponent property

$$= \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

because $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$= \tan^2\theta$$

□

Example of solving trig equation

Solve $\cos(3\theta)\cdot\tan(3\theta) = \frac{\sqrt{3}}{2}$.

$$\cos(3\theta)\cdot\tan(3\theta) = \frac{\sqrt{3}}{2}$$

trig identity

$$\cos(3\theta)\cdot\frac{\sin(3\theta)}{\cos(3\theta)} = \frac{\sqrt{3}}{2}$$

multiply fractions

$$\sin(3\theta) = \frac{\sqrt{3}}{2}$$

inverse trig function

$$3\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

make sure you get all the angles! there are usually 2 on the unit circle, but then ∞ more when you go around multiple times.

$$\frac{3\theta}{3} = \frac{\frac{\pi}{3} \pm 2\pi n}{3} \quad \text{or} \quad \frac{3\theta}{3} = \frac{\frac{2\pi}{3} \pm 2\pi n}{3}$$

(divide both sides by 3)

$$\theta = \frac{\pi}{9} \pm \frac{2\pi}{3}n \quad \text{or} \quad \theta = \frac{2\pi}{9} \pm \frac{2\pi}{3}n$$

what if you needed all the solutions between 0 and 2π? usually there are 2, but since there was a 3θ, there are 3 times as many (total of 6.)

$$\frac{\pi}{9},\ \frac{\pi}{9}+\frac{2\pi}{3},\ \frac{\pi}{9}+\frac{2\pi}{3}+\frac{2\pi}{3} = \frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}$$

$$\frac{2\pi}{9},\ \frac{2\pi}{9}+\frac{2\pi}{3},\ \frac{2\pi}{9}+\frac{2\pi}{3}+\frac{2\pi}{3} = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}$$