NAME DATE

Pre Calc : Identities A

In Exercises 1–40, verify the identity.

1. $\sin t \csc t = 1$
2. $\sec y \cos y = 1$
3. $(1 + \sin \alpha)(1 - \sin \alpha) = \cos^2 \alpha$
4. $\cot^2 y(\sec^2 y - 1) = 1$
5. $\cos^2 \beta - \sin^2 \beta = 1 - 2 \sin^2 \beta$
6. $\cos^2 \beta - \sin^2 \beta = 2 \cos^2 \beta - 1$
7. $\tan^2 \theta + 4 = \sec^2 \theta + 3$
8. $2 - \sec^2 z = 1 - \tan^2 z$
9. $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$
10. $\cos x + \sin x \tan x = \sec x$
11. $\dfrac{\csc^2 \theta}{\cot \theta} = \csc \theta \sec \theta$
12. $\dfrac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$
13. $\dfrac{\cot^2 t}{\csc t} = \csc t - \sin t$
14. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$
15. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x \sqrt{\sin x}$
16. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$
17. $\dfrac{1}{\sec x \tan x} = \csc x - \sin x$
18. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$
19. $\cot \alpha + \tan \alpha = \csc \alpha \sec \alpha$
20. $\sec x - \cos x = \sin x \tan x$
21. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \tan x + \cot x$
22. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$
23. $\dfrac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$
24. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$
25. $\dfrac{1}{\sin x + 1} + \dfrac{1}{\csc x + 1} = 1$
26. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$
27. $\tan\left(\dfrac{\pi}{2} - \theta\right) \tan \theta = 1$
28. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan x$
29. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$
30. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$
31. $\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
32. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$
33. $\dfrac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$
34. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$
35. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \dfrac{1 + \sin \theta}{|\cos \theta|}$
36. $\sqrt{\dfrac{1 - \cos \theta}{1 + \cos \theta}} = \dfrac{1 - \cos \theta}{|\sin \theta|}$
37. $\cos^2 \beta + \cos^2\left(\dfrac{\pi}{2} - \beta\right) = 1$
38. $\sec^2 y - \cot^2\left(\dfrac{\pi}{2} - y\right) = 1$
39. $\sin t \csc\left(\dfrac{\pi}{2} - t\right) = \tan t$
40. $\sec^2\left(\dfrac{\pi}{2} - x\right) - 1 = \cot^2 x$

Example: #7 $\tan^2\theta + 4 = \sec^2\theta + 3$

is it an identity? plugging in a weird angle (2 radians) to test:

$\tan^2(2) + 4 = [\tan(2)]^2 + 4 \approx 8.774$

$\sec^2(2) + 3 = \left[\frac{1}{\cos(2)}\right]^2 + 3 \approx 8.774$

these match, so this is very likely an identity

scribbling:

$\tan^2\theta + 4$

$= \tan^2\theta + 1 + 3$ ⟵ ⟶ $\sec^2\theta + 3$

if I can prove that $\tan^2\theta + 1 = \sec^2\theta$, I am good

$\tan^2\theta + 1$

exponent rule → $\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1$ → clever form of 1 → $\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$ → Pythagorean identity → $= \frac{1}{\cos^2\theta}$

$\sec^2\theta$

$\left(\frac{1}{\cos\theta}\right)^2$

same! yay!

Proof:

$\tan^2\theta + 4$

$= \tan^2\theta + 1 + 3$

$= \left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 + 3$

$= \frac{\sin^2\theta}{\cos^2\theta} + 1 + 3$

$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} + 3$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} + 3$

$= \frac{1}{\cos^2\theta} + 3$

$= \left(\frac{1}{\cos\theta}\right)^2 + 3$

$= \sec^2\theta + 3$

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