

# CHAPTER 4 • STUDY GUIDE AND ASSESSMENT

## SKILLS AND CONCEPTS

### OBJECTIVES AND EXAMPLES

**Lesson 4-1** Determine roots of polynomial equations.

Determine whether 2 is a root of  $x^4 - 3x^3 - x^2 - x = 0$ . Explain.

$$f(2) = 2^4 - 3(2^3) - 2^2 - 2$$

$$f(2) = 16 - 24 - 4 - 2 \text{ or } -14$$

Since  $f(2) \neq 0$ , 2 is not a root of  $x^4 - 3x^3 - x^2 - x = 0$ .

**Lesson 4-2** Solve quadratic equations.

Find the discriminant of  $3x^2 - 2x - 5 = 0$  and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

The value of the discriminant,  $b^2 - 4ac$ , is  $(-2)^2 - 4(3)(-5)$  or 64. Since the value of the discriminant is greater than zero, there are two distinct real roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{64}}{2(3)}$$

$$x = \frac{2 \pm 8}{6}$$

$$x = -1 \text{ or } \frac{5}{3}$$

**Lesson 4-3** Find the factors of polynomials using the Remainder and Factor Theorems.

Use the Remainder Theorem to find the remainder when  $(x^3 + 2x^2 - 5x - 9)$  is divided by  $(x + 3)$ . State whether the binomial is a factor of the polynomial.

$$f(x) = x^3 + 2x^2 - 5x - 9$$

$$f(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 9 \\ = -27 + 18 + 15 - 9 \text{ or } -3$$

Since  $f(-3) = -3$ , the remainder is  $-3$ . So the binomial  $x + 3$  is not a factor of the polynomial by the Remainder Theorem.

### REVIEW EXERCISES

Determine whether each number is a root of  $a^3 - 3a^2 - 3a - 4 = 0$ . Explain.

11. 0

12. 4

13.  $-2$

14. Is  $-3$  a root of  $t^4 - 2t^2 - 3t + 1 = 0$ ?

15. State the number of complex roots of the equation  $x^3 + 2x^2 - 3x = 0$ . Then find the roots and graph the related function.

Find the discriminant of each equation and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

16.  $2x^2 - 7x - 4 = 0$

17.  $3m^2 - 10m + 5 = 0$

18.  $x^2 - x + 6 = 0$

19.  $-2y^2 + 3y + 8 = 0$

20.  $a^2 + 4a + 4 = 0$

21.  $5r^2 - r + 10 = 0$

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

22.  $(x^3 - x^2 - 10x - 8) \div (x + 2)$

23.  $(2x^3 - 5x^2 + 7x + 1) \div (x - 5)$

24.  $(4x^3 - 7x + 1) \div \left(x + \frac{1}{2}\right)$

25.  $(x^4 - 10x^2 + 9) \div (x - 3)$

## OBJECTIVES AND EXAMPLES

**Lesson 4-4** Identify all possible rational roots of a polynomial equation by using the Rational Root Theorem.

List the possible rational roots of  $4x^3 - x^2 - x - 5 = 0$ . Then determine the rational roots.

If  $\frac{p}{q}$  is a root of the equation, then  $p$  is a factor of 5 and  $q$  is a factor of 4.

possible values of  $p$ :  $\pm 1, \pm 5$

possible values of  $q$ :  $\pm 1, \pm 2, \pm 4$

possible rational roots:  $\pm 1, \pm 5, \pm \frac{1}{2},$

$\pm \frac{1}{4}, \pm \frac{5}{2}, \pm \frac{5}{4}$

Graphing and substitution show a zero at  $\frac{5}{4}$ .

**Lesson 4-4** Determine the number and type of real roots a polynomial function has.

For  $f(x) = 3x^4 - 9x^3 + 4x - 6$ , there are three sign changes. So there are three or one positive real zeros.

For  $f(-x) = 3x^4 + 9x^3 - 4x - 6$ , there is one sign change. So there is one negative real zero.

**Lesson 4-5** Approximate the real zeros of a polynomial function.

Determine between which consecutive integers the real zeros of  $f(x) = x^3 + 4x^2 + x - 2$  are located.

Use synthetic division.

$r$	1	4	1	-2	
-4	1	0	1	-6	← change in signs
-3	1	1	2	4	← change in signs
-2	1	2	-3	4	
-1	1	3	-2	0	← zero
0	1	4	1	-2	← change in signs
1	1	5	6	4	← change in signs

One zero is  $-1$ . Another is located between  $-4$  and  $-3$ . The other is between  $0$  and  $1$ .

## REVIEW EXERCISES

List the possible rational roots of each equation. Then determine the rational roots.

26.  $x^3 - 2x^2 - x + 2 = 0$

27.  $x^4 - x^2 + x - 1 = 0$

28.  $2x^3 - 2x^2 - 2x - 4 = 0$

29.  $2x^4 + 3x^3 - 6x^2 - 11x - 3 = 0$

30.  $x^5 - 7x^3 + x^2 + 12x - 4 = 0$

31.  $3x^3 + 7x^2 - 2x - 8 = 0$

32.  $4x^3 + x^2 + 8x + 2 = 0$

33.  $x^4 + 4x^2 - 5 = 0$

Find the number of possible positive real zeros and the number of possible negative real zeros for each function. Then determine the rational zeros.

34.  $f(x) = x^3 - x^2 - 34x - 56$

35.  $f(x) = 2x^3 - 11x^2 + 12x + 9$

36.  $f(x) = x^4 - 13x^2 + 36$

Determine between which consecutive integers the real zeros of each function are located.

37.  $g(x) = 3x^3 + 1$

38.  $f(x) = x^2 - 4x + 2$

39.  $g(x) = x^2 - 3x - 3$

40.  $f(x) = x^3 - x^2 + 1$

41.  $g(x) = 4x^3 + x^2 - 11x + 3$

42.  $f(x) = -9x^3 + 25x^2 - 24x + 6$

43. Approximate the real zeros of  $f(x) = 2x^3 + 9x^2 - 12x - 40$  to the nearest tenth.

# CHAPTER 4 • STUDY GUIDE AND ASSESSMENT

## OBJECTIVES AND EXAMPLES

**Lesson 4-6** Solve rational equations and inequalities.

Solve  $\frac{1}{9} + \frac{1}{2a} = \frac{1}{a^2}$ .

$$\frac{1}{9} + \frac{1}{2a} = \frac{1}{a^2}$$

$$\left(\frac{1}{9} + \frac{1}{2a}\right)(18a^2) = \left(\frac{1}{a^2}\right)(18a^2)$$

$$2a^2 + 9a = 18$$

$$2a^2 + 9a - 18 = 0$$

$$(2a - 3)(a + 6) = 0$$

$$a = \frac{3}{2} \text{ or } -6$$

**Lesson 4-7** Solve radical equations and inequalities.

Solve  $9 + \sqrt{x-1} = 1$ .

$$9 + \sqrt{x-1} = 1$$

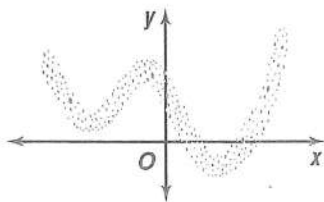
$$\sqrt{x-1} = -8$$

$$x - 1 = 64$$

$$x = 65$$

**Lesson 4-8** Write polynomial functions to model real-world data.

Determine the type of polynomial function that would best fit the data in the scatter plot.



The scatter plot seems to change direction three times. So a quartic function would best fit the scatter plot.

## REVIEW EXERCISES

Solve each equation or inequality.

44.  $n - \frac{6}{n} + 5 = 0$

45.  $\frac{1}{x} = \frac{x+3}{2x^2}$

46.  $\frac{5}{6} = \frac{2m}{2m+2} - \frac{1}{3m-3}$

47.  $\frac{3}{y} - 2 < \frac{5}{y}$

48.  $\frac{2}{x+1} < 1 - \frac{1}{x-1}$

Solve each equation or inequality.

49.  $5 - \sqrt{x+2} = 0$

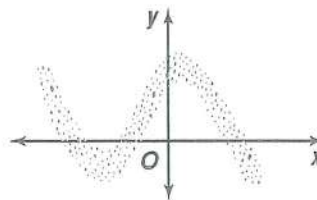
50.  $\sqrt[3]{4a-1} + 8 = 5$

51.  $3 + \sqrt{x+8} = \sqrt{x+35}$

52.  $\sqrt{x-5} < 7$

53.  $4 + \sqrt{2a+7} \geq 6$

54. Determine the type of polynomial function that would best fit the scatter plot.



55. Write a polynomial function to model the data.

$x$	-3	-1	0	2	4	7
$f(x)$	24	6	3	9	31	94

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## APPLICATIONS AND PROBLEM SOLVING

56. **Entertainment** The scenery for a new children's show has a playhouse with a painted window. A special gloss paint covers the area of the window to make them look like glass. If the gloss only covers 315 square inches and the window must be 6 inches taller than it is wide, how large should the scenery painters make the window? (Lesson 4-1)
57. **Gardening** The length of a rectangular flower garden is 6 feet more than its width. A walkway 3 feet wide surrounds the outside of the garden. The total area of the walkway itself is 288 square feet. Find the dimensions of the garden. (Lesson 4-2)
58. **Medicine** Doctors can measure cardiac output in potential heart attack patients by monitoring the concentration of dye after a known amount is injected in a vein near the heart. In a normal heart, the concentration of the dye is given by  $g(x) = -0.006x^4 + 0.140x^3 - 0.053x^2 + 1.79x$ , where  $x$  is the time in seconds. (Lesson 4-4)
- Graph  $g(x)$ .
  - Find all the zeros of this function.
59. **Physics** The formula  $T = 2\pi\sqrt{\frac{\ell}{g}}$  is used to find the period  $T$  of a oscillating pendulum. In this formula,  $\ell$  is the length of the pendulum, and  $g$  is acceleration due to gravity. Acceleration due to gravity is 9.8 meters per second squared. If a pendulum has an oscillation period of 1.6 seconds, determine the length of the pendulum. (Lesson 4-7)

## ALTERNATIVE ASSESSMENT

## OPEN-ENDED ASSESSMENT

- Write a rational equation that has at least two solutions, one which is 2. Solve your equation.
- Write a radical equation that has solutions of 3 and 6, one of which is extraneous.
  - Solve your equation. Identify the extraneous solution and explain why it is extraneous.
- Write a set of data that is best represented by a cubic equation.
  - Write a polynomial function to model the set of data.
  - Approximate the real zeros of the polynomial function to the nearest tenth.

 **PORTFOLIO**

Explain how you can use the leading coefficient and the degree of a polynomial equation to determine the number of possible roots of the equation.

Unit 1 *interNET* Project

## TELECOMMUNICATION

The Pen is Mightier than the Sword!

- Gather all materials obtained from your research for the mini-projects in Chapters 1, 2, and 3. Decide what types of software would help you to prepare a presentation.
- Research websites that offer downloads of software including work processing, graphics, spreadsheet, and presentation software. Determine whether the software is a demonstration version or free shareware. Select at least two different programs for each of the four categories listed above.
- Prepare a presentation of your Unit 1 project using the software that you found. Be sure that you include graphs and maps in the presentation.

**Additional Assessment** See p. A59 for Chapter 4 practice test.