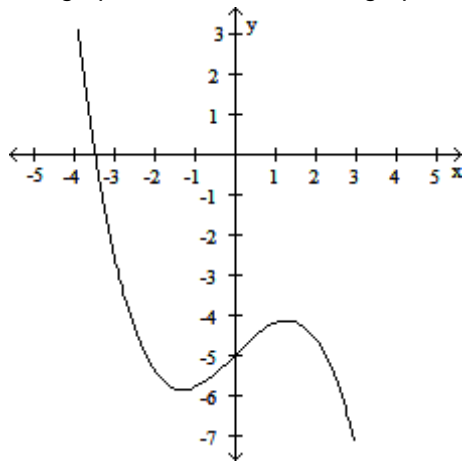Test F Derivatives Part 3

Name_____

1) The graph shown below is the graph of f(x).

a)   Use the graph of f to estimate where f' is 0.

b)   Use the graph of f to estimate where f' is positive.

c)   Use the graph of f to estimate where f' is negative

d)   Use the graph of f to estimate where f'' is 0.

e)   Use the graph of f to estimate where f'' is positive.

f)   Use the graph of f to estimate where f'' is negative

2) The graph shown below is the graph of f′(x), which is the derivative.

a) Determine when f(x) is increasing.

b) Determine when f(x) is decreasing.

c) Determine any local maxima or minima of f(x). Justify your answer.

d) Determine when the function is concave up.

e) Determine when the function is concave down.

f) Determine if there are any inflection points of f(x). Justify your answer.

3) Using the following properties of a twice-differentiable function $y = f(x)$, complete the table by finding the characteristics for each piece of the graph and then sketch a possible graph of f.

x	y	Derivatives	Characteristics
x < -2		$y' < 0, \ y'' > 0$	
-2	1	$y' = 0, y'' > 0$	
-2 < x < 0		$y' > 0, y'' > 0$	
0	3	$y' > 0, y'' = 0$	
0 < x < 2		$y' > 0, y'' < 0$	
2	5	$y' = 0, y'' < 0$	
x > 2		$y' < 0, y'' < 0$	
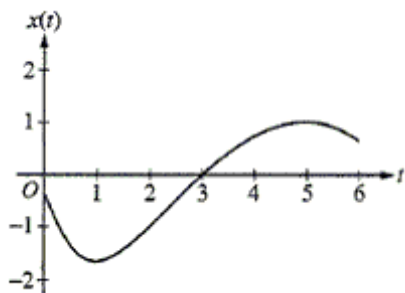
4)

A particle moves along a straight line. The graph of the particles position x(t) is shown above for
$< t < 6$. The graph has horizontal tangents at t = 1 and t =5 and a point of inflection at t = 2.
values of t is the **velocity** of the particle decreasing?

    A) $0 < t < 2$
    B) $2 < t < 5$
    C) $2 < t < 6$
    D) $3 < t < 5$ only
    E) $0 < t < 1$ and $5 < t < 6$

5) Let f be the function with derivative defined by $f'(x) = \sin(x^3)$ on the interval $-1.8 < x < 1.8$.

How many points of inflection does the graph of f have on this interval?
You may want to graph the derivative on your calculator

    A) One   B) Two   C) Three D) Four   E) Five

6) If the velocity of a function is given by $v(t) = -t^3 + 3t^2$, determine when the function achieves its absolute maximu
and absolute minimum acceleration on the interval [0, 4]. Use calculus to justify your answer.

4

7)

Given that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$

a. Find the second derivative at the point $(-2, -2)$

b. Using the fact that the point $(-2,-2)$ is a critical point and using your answer in part a, determine if $(-2, -2)$ is a local maximum or local minimum.

Use your calculus knowledge to find all points of local and absolute extrema, and all points of inflection. In your summary include the intervals of increase, decrease, concave up, and concave down. Make sure you include a reason for each of your answers.

8) $f(x) = 2x^3 + 3x^2 - 12x + 1$ on the interval $[-3, 2]$

9) On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\sin\left(\frac{t^2}{6}\right)$, where t is measured in hours and $0 \le t \le 5$.

a) Evaluate $G'(3)$ and interpret what it means in the context of the problem.

b) What is the maximum rate the unprocessed gravel arrives at the plant during the hours of $0 \le t \le 5$, on this workday? Justify your answer.

10) A rectangle with its base on the x-axis is to be inscribed under the graph

of $y = 10 - x^2$. Find the dimensions of the rectangle if the area is the largest possible area. Justify why the x-value is a maximum.

Solve the problem.

11) A carpenter is building a rectangular room with a fixed perimeter of 120 ft. What are the dimensions of the large room that can be built? What is its area? Justify why the value is a maximum and don't forget to label your answ

12) A man flies a kite at a height of 3 m. The wind carries the kite horizontally away from him at a rate of 10 m/sec. How fast is the distance between the man and the kite changing when the kite is 5 m away from him?

Use l'Hopital's Rule to evaluate the limit.

13) $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{x}$

Give an appropriate answer.

14) Find the value or values of c that satisfy $\dfrac{f(b) - f(a)}{b - a} = f'(c)$ for the function $f(x) = x^2 + 5x + 3$ on the interval $[-1, 2]$.

Find the linearization L(x) of f(x) at x = a. Then find L(a + .1) and f(a + .1) . Use the concavity of the curve to determine if the linearization is above or below the original curve.

15) $f(x) = x + \dfrac{1}{x}$, $a = 3$