What you'll Learn About

- How to integrate a product by that cannot be done by recognition

Proof of Integration by Parts

1. *Find* $\frac{d}{dx}(uv) =$

$\frac{d}{dx}(uv) = u\,dv + v\,du$

2. Integrate both sides

$\int \frac{d}{dx}(uv) = \int u\,dv + v\,du$

$\int \frac{d}{dx}(uv) = \int u\,dv + \int v\,du$

$uv = \int u\,dv + \int v\,du$

3. Solve for $\int u\,dv$

$\int u\,dv = uv - \int v\,du$

Left - Derivative

Right - Antiderivative

Use ultra violet minus super vdu to integrate the following

2. $\int xe^x = xe^x - \int e^x\,dx = \boxed{xe^x - e^x + C}$

$\int u\,dv = uv - \int v\,du$

$\int e^x \qquad \frac{d}{dx}(x)$

$\checkmark\ xe^x + e^x - e^x$

Use tabular integration to integrate the following

2. $\int xe^x =$

$\int x \mid e^x\,dx = xe^x - \int e^x(1)\,dx =$

$1 \rightarrow e^x$

$= \boxed{xe^x - e^x + C}$

Use tabular integration to integrate the following

6. $\int x^2 e^{-x} = -x^2e^{-x} + \int 2x e^{-x}\,dx$

$2x \quad -e^{-x}$

$2 \quad -e^{-x}$

$= -x^2e^{-x} - 2xe^{-x} + \int 2e^{-x}\,dx$

$= \boxed{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C}$

8. $\int x^2 \cos\left(\frac{x}{2}\right) = 2x^2 \sin\left(\tfrac{1}{2}x\right) - \int 4x \sin\left(\tfrac{1}{2}x\right) dx$

$2x \quad 2\sin\left(\tfrac{1}{2}x\right)$

$4 \quad -2\cos\left(\tfrac{1}{2}x\right)$

$2x^2 \sin\left(\tfrac{1}{2}x\right) - \left[-8x\cos\left(\tfrac{1}{2}x\right) + \int 8\cos\left(\tfrac{1}{2}x\right)\right]$

$\boxed{2x^2 \sin\left(\tfrac{1}{2}x\right) + 8x\cos\left(\tfrac{1}{2}x\right) - 16\sin\left(\tfrac{1}{2}x\right) + C}$

Solve the initial value problem using tabular integration

~~Find the general solution~~ Solve the initial value problem

11. $\int \frac{dy}{dx} = \int (x+2)\sin x \quad y = 2 \text{ and } x = 0$

$$\begin{array}{c|c} x+2 & \sin x \\ \hline 1 & -\cos x \end{array}$$

$y = -(x+2)\cos x + \int \cos x$

$y = -(x+2)\cos x + \sin x + C$

$2 = -(2) + C$

$4 = C$

$\boxed{y = -(x+2)\cos x + \sin x + 4}$

16. $\frac{dy}{dx} = 2x\sqrt{x+2} \quad y(-1) = 0$

x y

$\int \frac{dy}{dx} = \begin{array}{c|c} 2x & (x+2)^{1/2} \\ \hline 2 & \frac{2}{3}(x+2)^{3/2} \end{array}$

$-\frac{4(5)}{3(5)} - \frac{8}{15}$

$-\frac{20}{15} - \frac{8}{15}$

$y = \frac{4x}{3}(x+2)^{3/2} - \int \frac{4}{3}(x+2)^{3/2}$

$y = \frac{4x}{3}(x+2)^{3/2} - \frac{4 \cdot 2}{3 \cdot 5}(x+2)^{5/2} + C$

$0 = -\frac{4}{3} - \frac{8}{15} + C$

$\boxed{y = \frac{4x}{3}(x+2)^{3/2} - \frac{8}{15}(x+2)^{5/2} + \frac{28}{15}}$

$0 = -\frac{28}{15} + C$

	Use tabular integration to integrate the following 10. $\int x^2 \ln x dx$ Use ultra violet minus super vdu to integrate the following 10. $\int x^2 \ln x dx$

	Use tabular integration to integrate the following $A.\ \int \arcsin(x)dx$ $19.\ \int e^{x}\cos(2x)dx$

	Top Heavy Integrals $A.\ \int \frac{x^2 + x}{x} dx$ $B.\ \int \frac{\sqrt{x} + 5}{x} dx$ $C.\ \int \frac{x^3 + 2x}{\sqrt{x}} dx$