MATHEMATICS TASKS for the THINKING CLASSROOM

GRADES K–5

NOTES

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ILLUSTRATIONS BY LAURA WHEELER
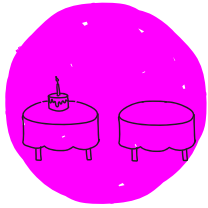
CORWIN Mathematics

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Please enjoy this complimentary excerpt from Mathematics Tasks for the Thinking Classroom, Grades K-5.

LEARN MORE about this title!

CORWIN

TASK 12: TABLES AT A PARTY

What size tables can you use to seat 12 guests, if every table is the same size and there must be the same number of guests at each table?

Grades: K–5

Content Potential: skip counting, repeated addition, multiplication/division, commutative property, perimeter

Perseverance Scale:

Grade Level:	K	1	2	3	4	5
Perseverance Level:	3	3	2	2	1	1

Macro-Moves: using hints and extensions to maintain flow

Competencies: risk taking, trial and error, organization

LAUNCH SCRIPT

Teacher: So, I decided that I want to have a party. Can you be my party planners?

Students: Yes!

Teacher: Great! Here are some details. I am going to rent tables for the party. The tables come in all different sizes. I can get tables that seat 1 person, 2 people, 3 people, all the way up to 100 people. But, even though the tables come in all different sizes, I want all the tables that I rent to be the same size. I just don't know what size tables to get.

Teacher: I also do not want any empty seats at any of the tables. So, if I order tables that each seat 4 people, I want there to be 4 people at every table.

Teacher: I have invited 12 people to my party. What size tables should I get? What are my options?

CREATING ACCESS

- For students who have difficulty representing numbers, give them round, square, or rectangular pieces of paper with numbers on them, each of which represents a table and how many people can sit at it.

- For students who have difficulty with skip counting, repeated addition, or multiplication, offer them a multiplication grid, a calculator, Unifix cubes, or the pieces of paper recommended above.

- This task can start at any number to increase or decrease the difficulty for different grades or groups with different abilities. For instance, you can start at 6 or 8, which will have fewer factors. Likewise, you can start at 20, 24, or 28 for more challenge. You can start with any number you wish as long as it is not a prime number or a square number. Regardless of what number you start with, the sequence of extensions (as seen in the extension scripts) should follow the following pattern before you jump to the noticing questions beginning at extension script 9:

  0. Even composite number [this is the launch script]

  1. Even composite number

  2. Odd composite number

  3. Composite number (odd or even)

  4. Composite number (odd or even)

  5. Square number

  6. Square number

  7. Prime number

  8. Prime number

EXTENSION SCRIPTS

1. Oh no! I just got an e-mail from some more friends saying they want to come to the party. I now have 20 people coming. What are my options now?

2. Oh no! Five of my friends just cancelled and I now only have 15 people coming. What are my options now?

3. What if there were 18 people at my party? What are my options now?

4. What if there were 24 people at my party? What are my options now?

5. What if there were 16 people at my party? What are my options now?

6. What if there were 25 people at my party? What are my options now?

7. What if there were 11 people at my party? What are my options now?

8. What if there were 17 people at my party? What are my options now?

9. OK. We have now tried a whole bunch of different numbers of guests, and I notice that sometimes you give me an even number of options and sometimes you give me an odd number of options. What kinds of numbers give me an even number of options and what kinds of numbers give me an odd number of options?

10. I notice that for some numbers of guests there are exactly two options and for others there are more than two options. What kinds of numbers produce exactly two options?

AUTHOR SOLUTION(S)

This task is fundamentally about factors and factor pairs. For example, with 12 guests, we can have 1 table of 12 guests (1 × 12), 2 tables of 6 guests (2 × 6), 3 tables of 4 guests (3 × 4), 4 tables of 3 guests (4 × 3), 6 tables of 2 guests (6 × 2), or 12 tables of 1 guest (12 × 1). These are all the factor pairs of 12. There are some things to notice just in these pairs. First, for every pair of factors there is an opposite pair. For example, we can have 3 tables of 4 guests (3 × 4) or 4 tables of 3 guests (4 × 3). For every $m \times n$ we can also have $n \times m$. This is the commutative property of multiplication: 4 × 3 = 3 × 4. And because of this pairing, there is an even number of options for 12 guests. This is true for almost any number of guests.

But not all. If we start with a number of guests that is a square number, for example 16, we will have an odd number of options. We can see this if we arrange the options for 16 in a table.

Option	Tables	Guests per table
1	1	16
2	2	8
3	4	4
4	8	2
5	16	1

We can see that for every option there is an opposite pair—option 1 pairs with option 5 and option 2 pairs with option 4—except for option 3. Option 3 has no pair because 4 tables of 4 guests is the same as 4 tables of guests. This is true of all square number of guests. If we have 9 guests, we will have an option that is 3 tables of 3 guests. If we have 25 guests, we will have an option that is 5 tables of 5 guests. Square numbers, and only square numbers, have an odd number of factor pairs. And, the option that has no pair is the square root of the number of quests. That is:

- $\sqrt{9} = 3$

- $\sqrt{16} = 4$

- $\sqrt{25} = 5$

There is also something interesting that happens if the number of guests is a prime number. For example, if we have 13 guests, the only options are 1 table of 13 guests (1 × 13) or 13 tables of 1 guest (13 × 1). There are only two options. In fact, there are exactly two options. This is true of any prime number and occurs because a prime number has no factors other than 1 and itself.
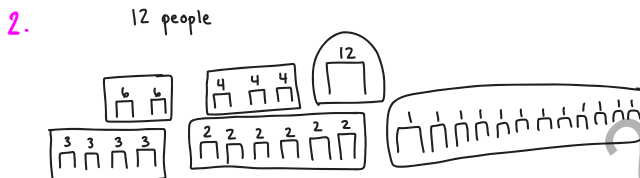
STUDENT SOLUTIONS

1.
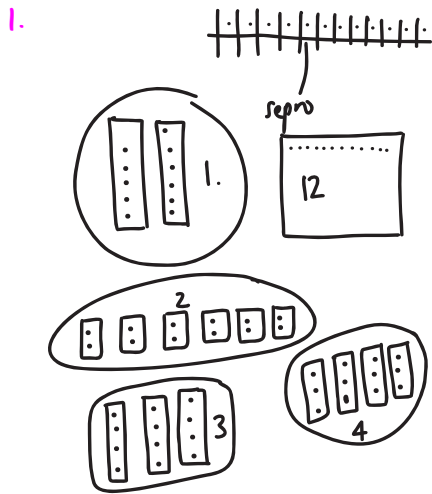
Commentary:

This solution is the first level of understanding that students will show. It shows they know the context and what is being asked but may not be connecting it to any specific mathematical concept.

Suggested hints:

This is interesting. Is there an easier way to represent, for example, 4 tables of 3 guests [teacher points at correct image] other than with dots?


2.

Commentary:

This solution shows the students beginning to bridge the pictorial to the abstract as they are aligning the tables as if to add them. These students may understand that they are creating groups, but they may not yet see connection to addition or multiplication.

Suggested hints:

Is there a way we can represent each of these images using multiplication (×)?

3.
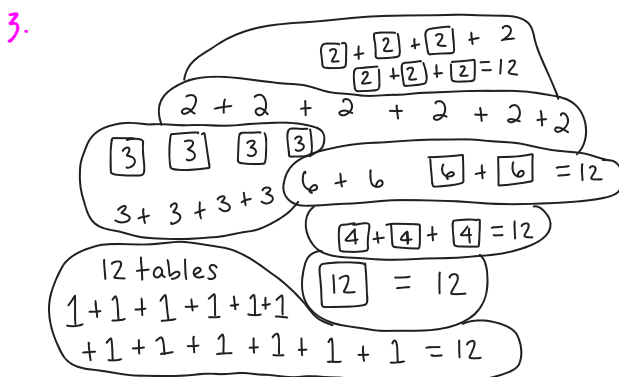
Commentary:

These students understand that these solutions can be found through repeated addition and are beginning to understand that the solutions can be paired up to represent the commutative property. You want them to connect their understanding to multiplication for cleaner notation and organization.

Suggested hints:

Is there a way we can represent each of these images using multiplication (×)?

(Continued)

4.

2
2
2   2×6=12
2   2 tables with 6 people
2
2   = 12
6   6 tables with 2 people
6 =12   6x2=12

3 × 4 = 12
4 × 3 = 12

3 tables with 4 People
4 tables with 3 people

1 × 12 = 12

12×1=12
12 tables with one person

1 table with 12 people

Commentary:

These students have grasped the concept that 2 tables with 6 guests is synonymous with 2 × 6. They also have clearly connected the solutions to commutative property and repeated addition. The next step for students would be to name and recognize each number as a factor of 12.

Suggested hints:

Can you put all your options into a list? How can you be sure you didn't miss any options?

CONSOLIDATION

The purpose of consolidation for this task depends on the grade you are doing it with. For all grades, you want to use consolidation to move them from the pictorial literal representation of tables at a party to the symbolic representation of multiplication and the commutative property. For older students, you also want to use consolidation to showcase the patterns that emerge in the relationship between the type of number (composite, square, prime) and the number of options that these types of numbers produce.

CHECK-YOUR-UNDERSTANDING QUESTIONS

MILD

A. 2 tables of 7 guests are ____ guests in total.

B. 4 tables of 6 guests are ___ guests in total.

C. 5 tables of 8 guests are ___ guests in total.

MEDIUM

A. How many options are there to seat 32 guests?

B. How many people sit at each table if there are 6 tables and 42 guests?

C. How many tables do I need if I have 56 guests total and 8 guests per table?

SPICY

A. What are all the options for 64 guests?

B. What number of guests can I invite if I want exactly 5 tables?

C. What numbers of guests have exactly 6 options?

This is a great task for almost any grade level. The context of tables and guests is one that every student can access. At the same time, it offers a very high ceiling with exploration of the types of numbers (composite, square, prime) and the effect that this has on the number of options. If you are moving toward having students notice the relationship between the number of options and the type of number, we recommend introducing the bank (see Task 8, The Carnival Conundrum) as a way for students to track their data.

✄ Notes to My Future Forgetful Self

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