AP-CALC-SUMMER WORK / MRS. KOLLCHAKU

Kuta Software - Infinite Calculus

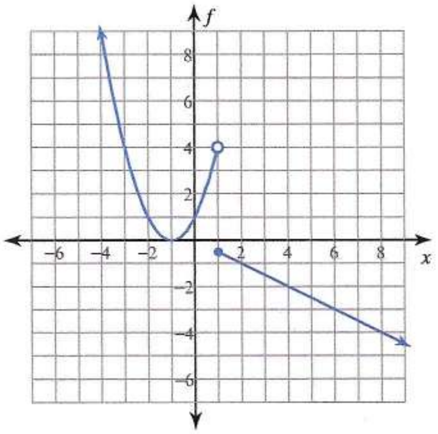
Name____________________________________

Continuity (1)

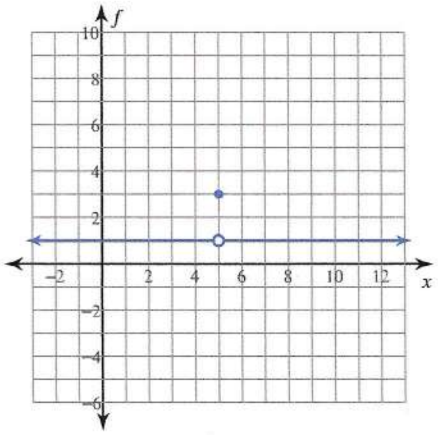
Date________________ Period____

Find the intervals on which each function is continuous.

1) $f(x) = \begin{cases} x^2 + 2x + 1, & x < 1 \\ -\dfrac{x}{2}, & x \geq 1 \end{cases}$

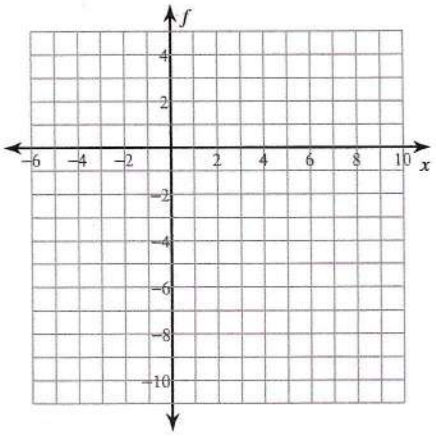


2) $f(x) = \begin{cases} 1, & x \neq 5 \\ 3, & x = 5 \end{cases}$

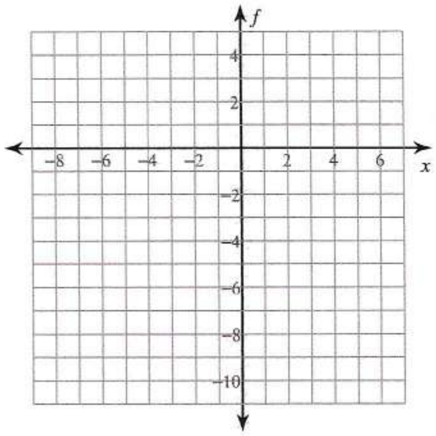


Find the intervals on which each function is continuous. You may use the provided graph to sketch the function.

3) $f(x) = \begin{cases} 2x - 10, & x < 2 \\ 0, & x \geq 2 \end{cases}$



4) $f(x) = \dfrac{x^2 - x - 2}{x + 1}$



Continuity (2)

Find the intervals on which each function is continuous.

5) $f(x) = \dfrac{x^2}{2x + 4}$

6) $f(x) = \begin{cases} -\dfrac{x}{2} - \dfrac{7}{2}, & x \le 0 \\ -x^2 + 2x - 2, & x > 0 \end{cases}$

7) $f(x) = -\dfrac{x^2 - x - 12}{x + 3}$

8) $f(x) = \dfrac{x^2 - x - 6}{x + 2}$

Determine if each function is continuous. If the function is not continuous, find the *x*-axis location of and classify each discontinuity.

9) $f(x) = -\dfrac{x^2}{2x + 4}$

10) $f(x) = \dfrac{x + 1}{x^2 - x - 2}$

11) $f(x) = \dfrac{x + 1}{x^2 + x + 1}$

12) $f(x) = -\dfrac{x^2}{x - 1}$

13) $f(x) = \begin{cases} x^2 - 4x + 3, & x \ne 0 \\ 3, & x = 0 \end{cases}$

14) $f(x) = \begin{cases} -x^2, & x \ne 1 \\ 0, & x = 1 \end{cases}$

Critical thinking questions:

15) Give an example of a function with discontinuities at $x = 1$, 2, and 3.

16) Of the six basic trigonometric functions, which are continuous over all real numbers? Which are not? What types of discontinuities are there?

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Name________________________

Evaluating Limits

Date________________ Period____

Evaluate each limit.

1) $\lim_{x \to -1} 5$

2) $\lim_{x \to -\frac{5}{2}} (-x + 2)$

3) $\lim_{x \to 2} (x^3 - x^2 - 4)$

4) $\lim_{x \to 1} \left(-\frac{x^2}{2} + 2x + 4\right)$

5) $\lim_{x \to 3} -\sqrt{x + 3}$

6) $\lim_{x \to \frac{3}{2}} -\sqrt{2x + 4}$

7) $\lim_{x \to 1} -\frac{x - 4}{x^2 - 6x + 8}$

8) $\lim_{x \to \frac{3}{2}} \frac{-x - 3}{x^2 + x + 1}$

9) $\lim_{x \to \pi} \sin(x)$

10) $\lim_{x \to \frac{3\pi}{4}} 2\cos(x)$

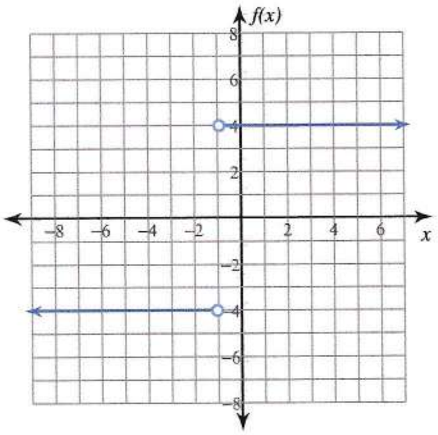
Critical thinking questions:

11) Give an example of a limit that evaluates to 4.

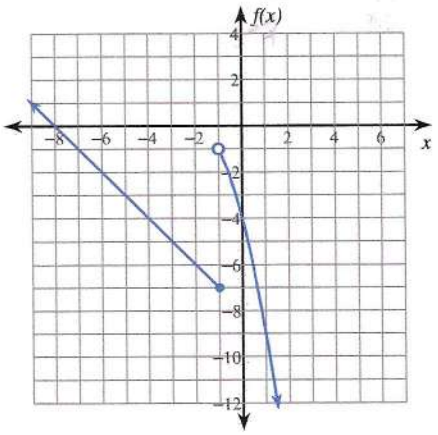
12) Give an example of a limit of a quadratic function where the limit evaluates to 9.

Evaluate each limit.

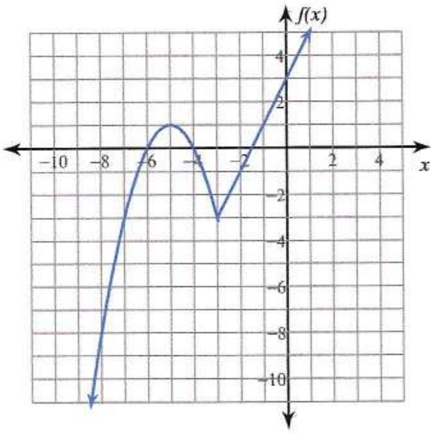
1) $\lim_{x \to -1^+} \frac{4x + 4}{|x + 1|}$



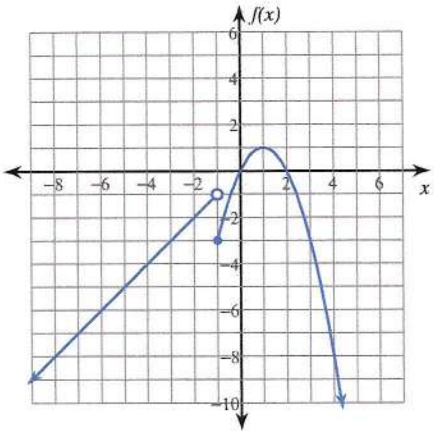
2) $\lim_{x \to -1^-} f(x),\ f(x) = \begin{cases} -x - 8, & x \le -1 \\ -x^2 - 4x - 4, & x > -1 \end{cases}$



3) $\lim_{x \to -3} f(x),\ f(x) = \begin{cases} -x^2 - 10x - 24, & x \le -3 \\ 2x + 3, & x > -3 \end{cases}$



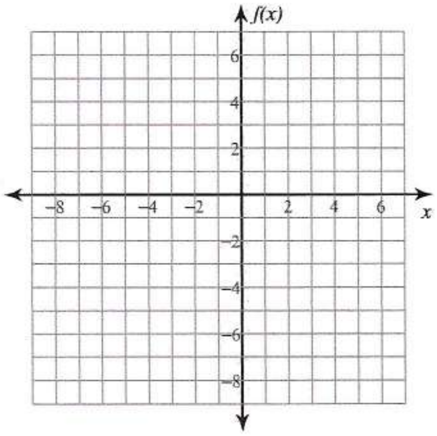
4) $\lim_{x \to -1} f(x),\ f(x) = \begin{cases} x, & x < -1 \\ -x^2 + 2x, & x \ge -1 \end{cases}$



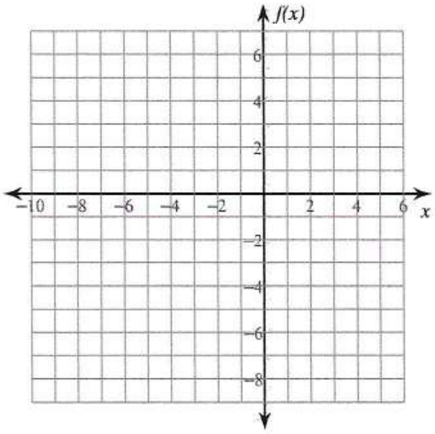
(2)

Evaluate each limit. You may use the provided graph to sketch the function.

5) $\lim\limits_{x\to -1^-} f(x),\ f(x) = \begin{cases} -x-3, & x \le -1 \\ x+1, & x > -1 \end{cases}$



6) $\lim\limits_{x\to -2} f(x),\ f(x) = \begin{cases} -x^2-4x-5, & x \le -2 \\ -1, & x > -2 \end{cases}$



Evaluate each limit.

7) $\lim\limits_{x\to 0^+} f(x),\ f(x) = \begin{cases} 1, & x \le 0 \\ -x^2+4x-3, & x > 0 \end{cases}$

8) $\lim\limits_{x\to 0^-} \dfrac{|x|}{x}$

9) $\lim\limits_{x\to 0^+} \lfloor -2x+1 \rfloor$

10) $\lim\limits_{x\to 1} f(x),\ f(x) = \begin{cases} \dfrac{x}{2}+\dfrac{9}{2}, & x < 1 \\ x^2-6x+10, & x \ge 1 \end{cases}$

11) $\lim\limits_{x\to -1} \dfrac{3|x+1|}{x+1}$

12) $\lim\limits_{x\to -2} f(x),\ f(x) = \begin{cases} x^2, & x \le -2 \\ -\dfrac{x}{2}+3, & x > -2 \end{cases}$

Critical thinking questions:

13) Give an example of a two-sided limit of a piecewise function where the limit does not exist.

14) Given an example of a two-sided limit of a function with an absolute value where the limit does not exist.

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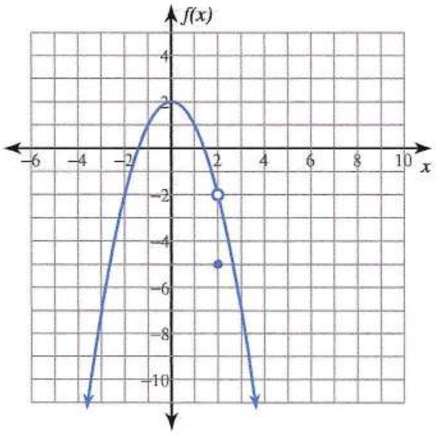
Name________________________

Evaluating Limits (1)

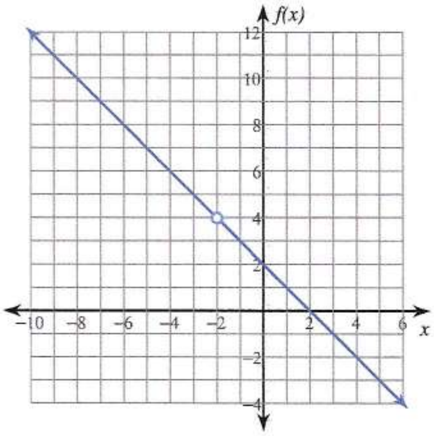
Date________________ Period____

Evaluate each limit.

1) $\lim_{x\to 2} f(x),\ f(x) = \begin{cases} -x^2+2, & x\neq 2 \\ -5, & x=2 \end{cases}$

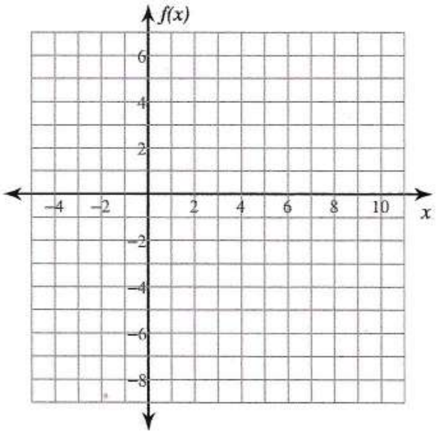


2) $\lim_{x\to -2} -\frac{x^2-4}{x+2}$

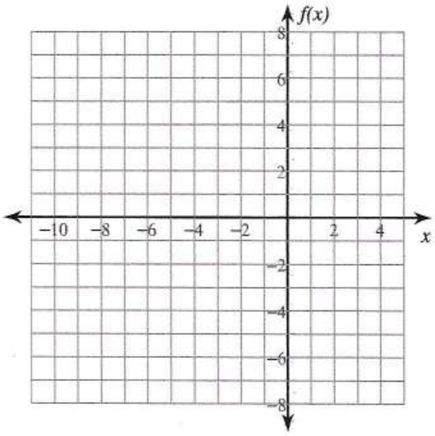


Evaluate each limit. You may use the provided graph to sketch the function.

3) $\lim_{x\to 3} \frac{x^2-7x+12}{x-3}$



4) $\lim_{x\to -3} \frac{x+3}{x^2+2x-3}$



Evaluate each limit.

5) $\lim_{x\to 0} f(x),\ f(x) = \begin{cases} x+1, & x\neq 0 \\ 2, & x=0 \end{cases}$

6) $\lim_{x\to 3} f(x),\ f(x) = \begin{cases} 2+\frac{x}{2}, & x\neq 3 \\ 2, & x=3 \end{cases}$

7) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ (2)

8) $\lim_{x \to 5} \frac{x^2 - 5x}{x - 5}$

9) $\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}$

10) $\lim_{x \to -5} \frac{x^2 + 3x - 10}{x + 5}$

11) $\lim_{x \to 0} \frac{\frac{1}{-4 + x} + \frac{1}{4}}{x}$

12) $\lim_{x \to -3} \frac{x}{\frac{1}{3 + x} - \frac{1}{3}}$

13) $\lim_{x \to 5} \frac{x - 5}{\sqrt{x + 4} - 3}$

14) $\lim_{x \to 3} \frac{\sqrt{x + 6} - 3}{x - 3}$

Critical thinking questions:

15) Give an example of a limit of a rational function where the limit at -1 exists, but the rational function is undefined at -1.

16) Give two values of a where the limit cannot be solved using direct evaluation. Give one value of a where the limit can be solved using direct evaluation.

$$\lim_{x \to a} \frac{x}{\frac{1}{-2 + x} + \frac{1}{2}}$$

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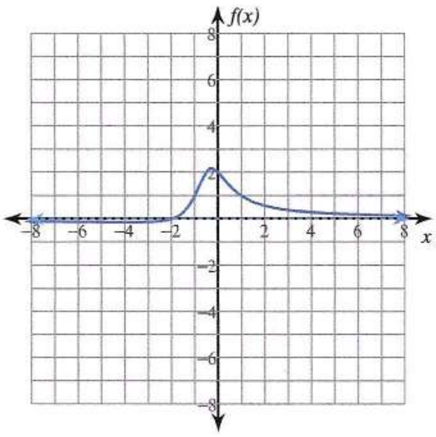
Name______________________________

Evaluating Limits (1)

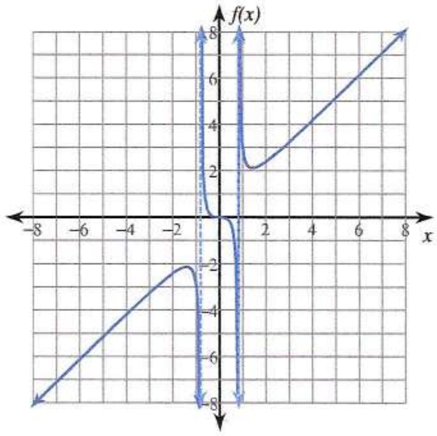
Date________________ Period____

Evaluate each limit.

1) $\lim_{x \to -\infty} \frac{x+2}{x^2+x+1}$

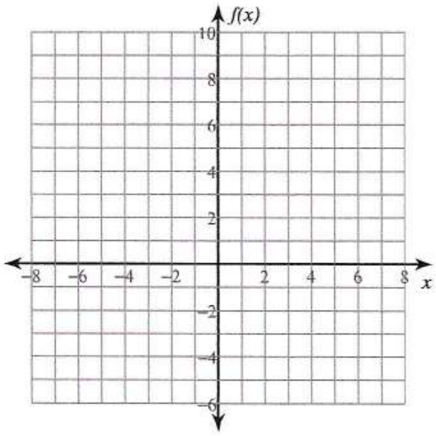


2) $\lim_{x \to -\infty} \frac{3x^3}{3x^2-2}$

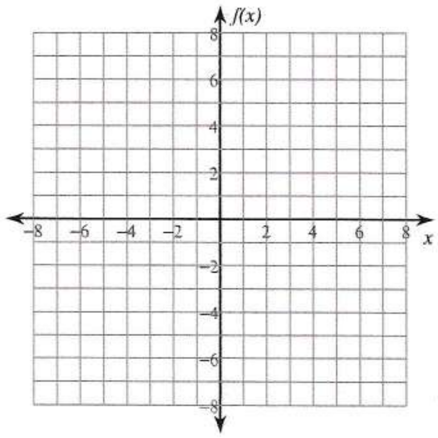


Evaluate each limit. You may use the provided graph to sketch the function.

3) $\lim_{x \to -\infty} \frac{2x^2}{x^2-4}$



4) $\lim_{x \to \infty} -\frac{3x^2}{4x+4}$



Evaluate each limit. (2)

5) $\lim_{x\to-\infty} (x^3 - 4x^2 + 5)$

6) $\lim_{x\to\infty} \frac{2x^3}{3x^2 - 4}$

7) $\lim_{x\to\infty} \frac{x^3}{4x^2 + 3}$

8) $\lim_{x\to\infty} \frac{x+1}{2x^2 + 2x + 1}$

9) $\lim_{x\to-\infty} \frac{\sqrt{2x^2 + 3}}{2x + 3}$

10) $\lim_{x\to-\infty} \frac{\sqrt{2x^2 + 1}}{4x + 2}$

11) $\lim_{x\to\infty} \left(-\frac{\ln x}{x^4} + 1\right)$

12) $\lim_{x\to\infty} \left(-e^{-3x} - 1\right)$

13) $\lim_{x\to\infty} \left(e^{x} - 3\right)$

14) $\lim_{x\to-\infty} -e^{-4x}$

15) $\lim_{x\to\infty} \cos(2x)$

16) $\lim_{x\to-\infty} \frac{x}{\cos(-3x)}$

17) $\lim_{x\to\infty} -\frac{2x}{\cos\frac{1}{x}}$

18) $\lim_{x\to\infty} x\cos\frac{1}{x}$

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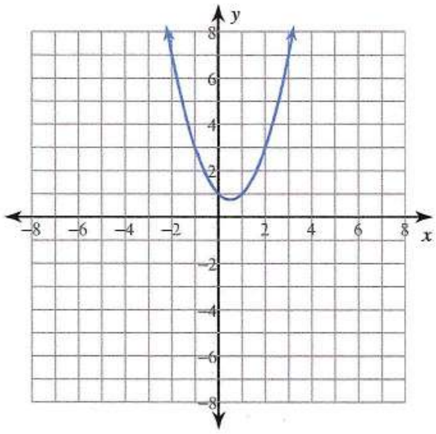
Name______________________

Average Rates of Change (1)

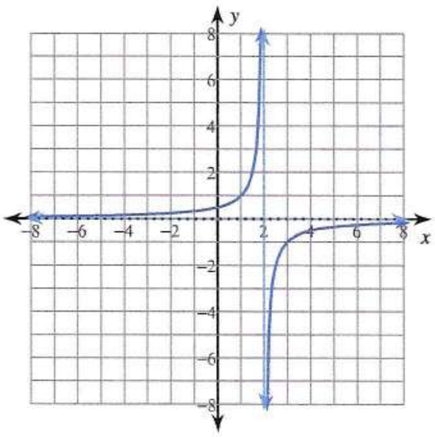
Date________________ Period____

For each problem, find the average rate of change of the function over the given interval.

1) $y = x^2 - x + 1;\ [0, 3]$

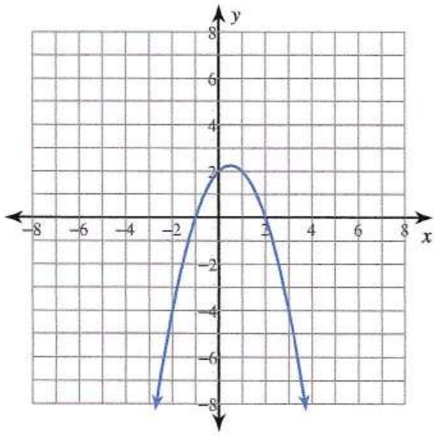


2) $y = -\dfrac{1}{x-2};\ [-3, -2]$

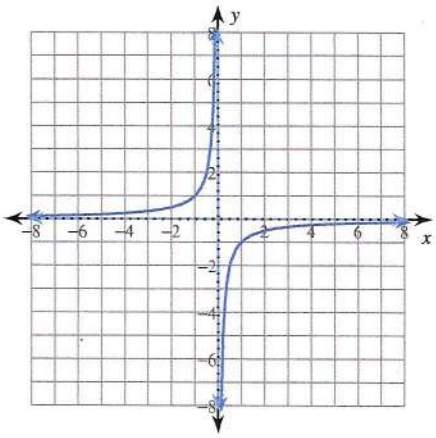


For each problem, find the equation of the secant line that intersects the given points on the function.

3) $y = -x^2 + x + 2;\ (-2, -4), (1, 2)$



4) $y = -\dfrac{1}{x};\ (1, -1), \left(3, -\dfrac{1}{3}\right)$



Av. Rate (2)

For each problem, find the average rate of change of the function over the given interval.

5) $y = x^2 + 2;\ [-2, -\frac{3}{2}]$

6) $y = 2x^2 - 2x + 1;\ [-1, -\frac{1}{2}]$

7) $y = -\frac{1}{x+2};\ [-1, -\frac{1}{2}]$

8) $y = 2x^2 + x + 2;\ [0, \frac{1}{2}]$

For each problem, find the equation of the secant line that intersects the given points on the function.

9) $y = -x^2 - 2;\ (1, -3), \left(\frac{3}{2}, -\frac{17}{4}\right)$

10) $y = \frac{1}{x+3};\ \left(-1, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{2}{5}\right)$

11) $y = \frac{1}{x-1};\ \left(-2, -\frac{1}{3}\right), \left(-\frac{3}{2}, -\frac{2}{5}\right)$

12) $y = -\frac{1}{x};\ (1, -1), \left(\frac{3}{2}, -\frac{2}{3}\right)$

Critical thinking question:

13) The police have accused a driver of breaking the speed limit of 60 miles per hour. As proof, they provide two photographs. One photo shows the driver's car passing a toll booth at exactly 6 PM. The second photo shows the driver's car passing another toll both 31 miles down the highway at exactly 6:30 PM. Does the photo evidence prove that the driver broke the speed limit during this time?

Name____________________

Definition of the Derivative

Date________________ Period____

Use the definition of the derivative to find the derivative of each function with respect to x.

1) $y = -2x + 5$

2) $f(x) = -4x - 2$

3) $y = 4x^2 + 1$

4) $f(x) = -3x^2 + 4$

5) $y = -4x^2 - 5x - 2$

6) $y = 3x^2 + 3x + 3$

7) $y = \sqrt{-3x - 5}$

8) $f(x) = \sqrt{4x - 5}$

9) $y = \frac{1}{x + 2}$

10) $f(x) = -\frac{2}{2x - 1}$

Critical thinking question:

11) Use the definition of the derivative to show that $f'(0)$ does not exist where $f(x) = |x|$.

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Name________________________

Differentiation - Power, Constant, and Sum Rules (1)

Date________________ Period____

Differentiate each function with respect to x.

1) $y = 5$

2) $f(x) = 5x^{18}$

3) $y = 4x^5 + x$

4) $f(x) = 4x^4 - 5x - 3$

5) $y = 3x^{\frac{5}{4}}$

6) $y = \frac{5}{4}x^{\frac{2}{3}}$

7) $y = -4x^{-5}$

8) $y = \frac{3}{x^3}$

9) $y = x^{\frac{2}{3}}$

10) $f(x) = -2\sqrt[4]{x}$

Diff - Power .. (a)

11) $y = \frac{2}{3}x^4 + 5x - x^{-3}$

12) $y = -\frac{1}{2}x^4 + 3x^{\frac{5}{3}} + 2x$

Differentiate each function with respect to the given variable.

13) $y = -3r^5 - 5r^2$

14) $f(s) = -\frac{3}{s^2} - \frac{4}{s^4}$

15) $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{3}{5}}$

16) $h(s) = \sqrt{2} \cdot \sqrt[3]{s} + \sqrt{2} \cdot \sqrt[5]{s}$

Differentiate each function with respect to *x*. Problems may contain constants a, b, and c.

17) $y = 5c$

18) $y = 4ax^{3a} - bx^{3c}$

Name________________________________

Derivative at a Value

Date________________ Period____

For each problem, find the derivative of the function at the given value.

1) $y = x^2 + 4x$ at $x = -5$

2) $y = -x^3 + 4x^2 - 4$ at $x = 4$

3) $y = \frac{20}{x^2 + 5}$ at $x = 3$

4) $y = \frac{2}{x + 1}$ at $x = 5$

5) $y = (-x + 4)^{\frac{1}{2}}$ at $x = 0$

6) $y = (-3x + 9)^{\frac{1}{2}}$ at $x = -5$

7) $y = e^{-x + 2}$ at $x = 4$

8) $y = -\ln\left(x + 3\right)$ at $x = 5$

9) $y = 2\sin\left(2x\right)$ at $x = -\frac{\pi}{2}$

10) $y = -\tan\left(2x\right)$ at $x = -\pi$

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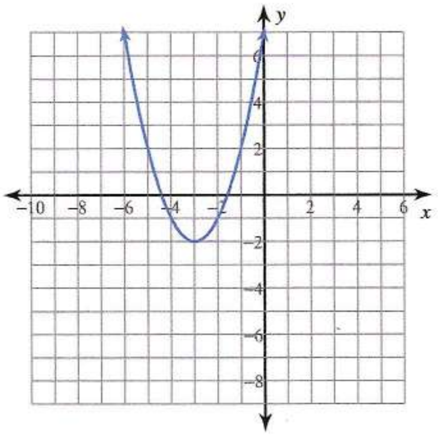
Slope at a Value

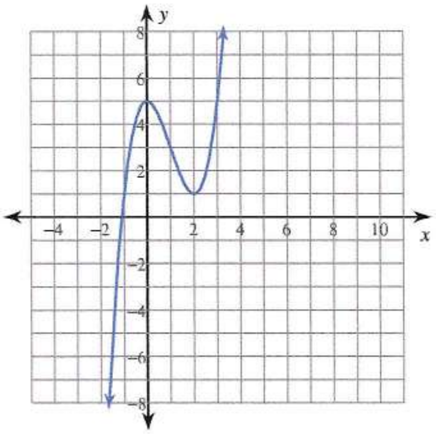
Date________________ Period____

For each problem, find the slope of the function at the given value.

1) $y = x^2 + 6x + 7$ at $x = -2$



2) $y = x^3 - 3x^2 + 5$ at $x = 3$



3) $y = x^3 - 6x^2 + 9x - 4$ at $x = 2$

4) $y = -x^3 - 6x^2 - 9x + 1$ at $x = -4$

5) $y = -\dfrac{1}{x^2 - 9}$ at $x = 2$

6) $y = -\dfrac{3}{x + 5}$ at $x = 1$

7) $y = -(x + 2)^{\frac{1}{3}}$ at $x = -1$

8) $y = -(-x + 2)^{\frac{1}{2}}$ at $x = -5$

9) $y = -\ln(-x + 2)$ at $x = -3$

10) $y = \sin(2x)$ at $x = -\pi$

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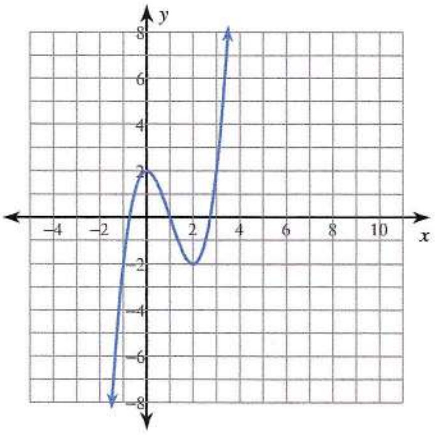
Tangent Lines

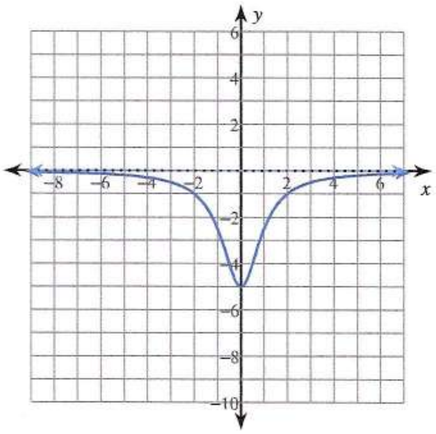
Date________________ Period____

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

1) $y = x^3 - 3x^2 + 2$ at $(3, 2)$



2) $y = -\frac{5}{x^2 + 1}$ at $\left(-1, -\frac{5}{2}\right)$



3) $y = x^3 - 2x^2 + 2$ at $(2, 2)$

4) $y = -\frac{3}{x^2 - 25}$ at $\left(-4, \frac{1}{3}\right)$

5) $y = -\frac{3}{x^2 - 4}$ at $(1, 1)$

6) $y = (5x + 5)^{\frac{1}{2}}$ at $(4, 5)$

7) $y = \ln(-x)$ at $(-2, \ln 2)$

8) $y = -2\tan(x)$ at $(-\pi, 0)$

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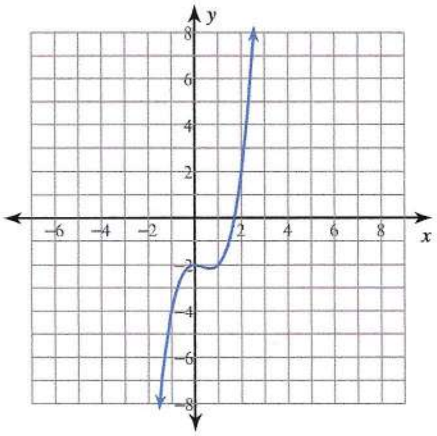
Normal Lines

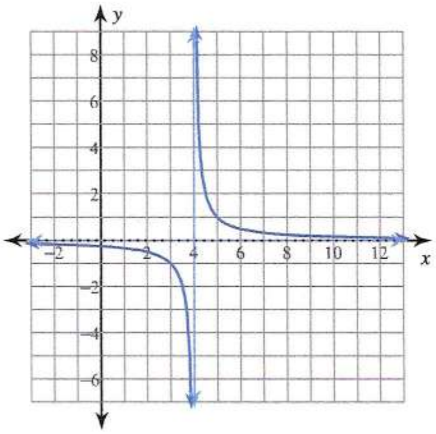
Date________________ Period____

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in slope-intecept form.

1) $y = x^3 - x^2 - 2$ at $(1, -2)$



2) $y = \frac{1}{x - 4}$ at $(5, 1)$



3) $y = -x^3 + 15x^2 - 72x + 110$ at $(4, -2)$

4) $y = \frac{2}{x - 3}$ at $(5, 1)$

5) $y = \frac{3}{x + 2}$ at $\left(4, \frac{1}{2}\right)$

6) $y = (2x - 8)^{\frac{1}{3}}$ at $(0, -2)$

7) $y = \ln(x + 4)$ at $(-3, 0)$

8) $y = -\sin(2x)$ at $\left(-\frac{\pi}{2}, 0\right)$

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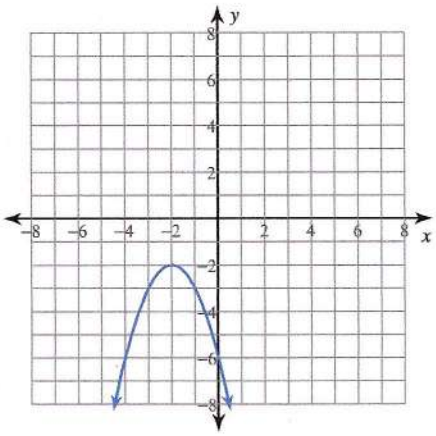
Horizontal Tangents

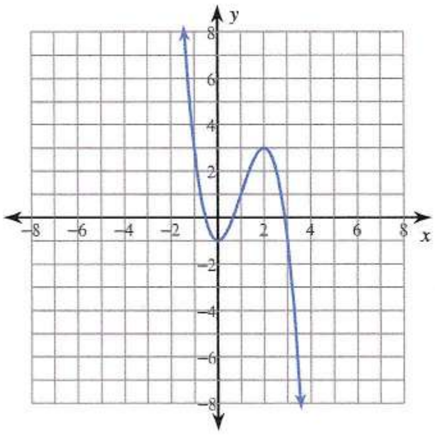
Date________________ Period____

For each problem, find the points where the tangent line to the function is horizontal.

1) $y = -x^2 - 4x - 6$



2) $y = -x^3 + 3x^2 - 1$



3) $y = -x^3 + x^2 - 2$

4) $y = \dfrac{1}{x^2 - 1}$

For each problem, find the points where the tangent line to the function is horizontal. Indicate if no horizontal tangent line exists.

5) $y = x^3 - 2x^2 + 2$

6) $y = -x^3 + \dfrac{9x^2}{2} - 12x - 3$

7) $y = -\dfrac{2}{x - 3}$

8) $y = -\dfrac{1}{x^2 + 1}$

9) $y = (-2x + 4)^{\frac{1}{2}}$

10) $y = -\csc(x);\ [-\pi, \pi]$