Unit A - *Introduction to Single Variable Statistics*

Overview

Unit A begins with an overview of statistics and how they impact our lives. Students will examine univariate and bivariate data and make sense out of it using statistical methods and displays. Graphing calculators are used throughout the course.

21st Century Capacities: Analyzing

Stage 1 - Desired Results

<table>
<tr><td rowspan="4">ESTABLISHED GOALS/ STANDARDS

MP3 Construct viable arguments and critique the reasoning of others
MP5 Use appropriate tools strategically
MP7 Look for and make use of structure

Summarize, represent, and interpret data on a single count or measurement variable

CCSS.MATH.CONTENT.HSS.ID.A.1
Represent data with plots on the real number line (dot plots, histograms, and box plots).

CCSS.MATH.CONTENT.HSS.ID.A.2
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

CCSS.MATH.CONTENT.HSS.ID.A.3
Interpret differences in shape, center, and spread in the context of the data sets, accounting for</td><td colspan="2">Transfer:</td></tr>
<tr><td colspan="2">Students will be able to independently use their learning in new situations to...

1. Represent, summarize, and interpret patterns in data (Analyzing)
2. Use appropriate tools/methods to make mathematical concepts more concrete and accessible
3. Communicate effectively using appropriate vocabulary and format (verbally, symbolically, numerically, and graphically)</td></tr>
<tr><td colspan="2">Meaning:</td></tr>
<tr><td>UNDERSTANDINGS: Students will understand that:

1. Statistics can be used to describe data using graphs and numerical summaries.
2. Statisticians ask questions and attempt to answer them using scientific procedures.
3. Data is gathered through a variety of methods, some more valid than others.
4. Good samples reveal information about a population.
5. Statistics can be used to identify trends and make predictions.</td><td>ESSENTIAL QUESTIONS: Students will explore & address these recurring questions:

A. As consumers of information, how do we analyze the validity and reliability of statistics?
B. How does technology help to create meaning out of the data?
C. How can I best communicate to an audience what the statistics say?</td></tr>
</table>

possible effects of extreme data points (outliers). CCSS.MATH.CONTENT.HSS.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **Interpret linear models** CCSS.MATH.CONTENT.HSS.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. CCSS.MATH.CONTENT.HSS.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. CCSS.MATH.CONTENT.HSS.ID.C.9 Distinguish between correlation and causation.	**Acquisition:**	
	Students will know… 1. How to describe a distribution using center, shape, and spread 2. Statistics do not lie but people can bend the truth using statistics 3. The impact of outliers 4. The difference between a sample and a statistic 5. The empirical rule (68-95-99.7) 6. The area under the normal curve is 1 7. Understanding that correlational data do not imply cause and effect. 8. The direction, form, and strength of the overall pattern of a scatterplot 9. What the slope b and the intercept a mean in a equation $y = ax + b$ of a straight line. 10. The limitations of a prediction outside the range of available data 11. Vocabulary: categorical and quantitative variables, individuals, variables, observational and experimental studies, population, sample, census, stem plot, box plot, pie chart, dot plot, histogram, line plot, symmetry, skewness, 5-number summary, min, max, quartiles, median, mean, range, percentiles, z-scores, density curves, normal distribution, normal curve, empirical rule, scatterplots, correlation, regression, prediction, residual	*Students will be skilled at…* 1. Interpreting graphs and charts 2. creating a 5 number summary and displaying it in a box plot 3. Using graphing calculators to calculate single variable statistics 4. Using graphing calculators to create meaningful displays 5. Identify the best measure to use (mean or median) to describe central tendency 6. Identifying misleading statistics and displays 7. Using statistical models to make predictions and understanding the limitations of the model. 8. Recognizing the shape of Normal curves and estimate by eye both the mean and the standard deviation from such a curve. 9. Judging whether it is appropriate to use correlation to describe the relationship between two quantities 10. Using a regression line, given on a graph or as an equation, to predict y for a given x 11. Using a residual plot to examine how well a regression line fits the data 12. Assessing the strength of statistical evidence for a claim of causation, especially when experimentation is not possible

Unit B - *Research Design*

Overview

This unit explores the process of collecting and interpreting data. Chapter 5 investigates sampling as a method of understanding information about populations. It includes discussion of uncertainty in samples and how the margin of error narrows as sample size grows. Students review articles with sampling and review the validity of the statistical processes used to obtain data. Chapter 6 is about experimentation. Students learn about the basic principles of experiment design, including: explanatory versus response variables, the definition of statistical significance, adjusting for confounding variables, and double blind experiments. Students explore the ethical complexities of experimentation in a review of the movie Miss Evers' Boys, which is a historical account of the controversial Tuskegee Syphilis Study.

21st Century Capacities: Analyzing

Stage 1 - Desired Results

ESTABLISHED GOALS/ STANDARDS **MP2** Reason abstractly and quantitatively **MP3** Construct viable arguments and critique the reasoning of others **MP5** Use appropriate tools strategically **MP6** Attend to precision **MP7** Look for and make use of structure **Understand and evaluate random processes underlying statistical experiments** CCSS.MATH.CONTENT.HSS.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	***Transfer:*** *Students will be able to independently use their learning in new situations to...* 1. Draw conclusions about graphs, shapes, equations, or objects. (Analyzing) 2. Evaluate the accuracy and efficiency of a given solution. (Analyzing) 3. Justify reasoning using clear and appropriate mathematical language.	
	Meaning:	
	UNDERSTANDINGS: *Students will understand that:* 1. Mathematicians compare the effectiveness of various arguments, by analyzing and critiquing solution pathways. 2. Mathematicians continually evaluate their process and the reasonableness of the intermediate results. 3. Mathematicians select and use appropriate statistical methods and tools to analyze data, show trends, evaluate inference and/or describe or make predictions. 4. Mathematicians analyzed data to evaluate inferences, make predictions and/or communicate an decision.	ESSENTIAL QUESTIONS: *Students will explore & address these recurring questions:* A. How do we appropriately challenge the validity of sampling methods? B. How do we communicate the precision and confidence of sampling results? C. What makes experimentation statistically valid? D. How do ethics influence statistical practices?
	Acquisition:	
	Students will know… 1. Convenience sampling and voluntary response sampling are often biased	*Students will be skilled at…* 1. Identifying the population and

Make inferences and justify conclusions from sample surveys, experiments, and observational studies CCSS.MATH.CONTENT.HSS.IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. CCSS.MATH.CONTENT.HSS.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. CCSS.MATH.CONTENT.HSS.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. CCSS.MATH.CONTENT.HSS.IC.B.6 Evaluate reports based on data.	2. The characteristics of a simple random sample 3. The two types of error in estimation 4. How to manage bias and variability when sampling 5. The meaning of the phrase ‘95% confidence” 6. That the conclusion of a confidence statement applies to the population, not to the sample 7. That our conclusion about the population is never completely certain 8. A sample survey can choose to use a confidence level other than 95% 9. That to produce a smaller margin of error with the same confidence take a larger sample 10. Examples of sampling and of nonsampling errors 11. How to choose a stratified random sample 12. All good samples are probability samples (a sample chosen by chance) 13. The first goal of an experiment is to ensure that it will show us the effect of the explanatory variables on the response variables 14. The basic principles of experimental design (p265) 15. A good comparative study measures and adjusts for confounding variables 16. The advantages and disadvantages of a double-blind experiment (p276) 17. How to determine if findings from an experient are statistically significant 18. The role of the institutional review board, informed consent, and confidentiality is not the same as anonymity 19. Vocabulary: sample, populations, voluntary response, biased, convenience sampling, random digits, parameter, statistic, bias, variability, margin of error, sampling errors , random sampling error, nonsampling errors, processing errors, undercoverage, processing errors, response errors, nonresponse, strata, experiments, subjects, treatments, confounding, lurking variable, clinical trials, placebo effect, control group, randomize, statistically significant, comparative, matching, causes, non-adherers, dropouts, generalize, completely randomized designs, matched pairs design, block design, institutional review board, informed consent, confidential	parameter of interest 2. Recognizing bias due to voluntary response samples and other inferior sampling methods. 3. Choosing a simple random sample 4. Using a table of random digits to select a simple random sample (SRS) from a population 5. Finding the margin of error for 95% confidence roughly using $1/\sqrt{n}$ (n=sample size) 6. Asking good questions about a poll before paying attention to the poll results (p249) 7. Understanding the distinctions between sampling errors and nonsampling errors 8. Use random digits to select a stratified random sample from a population when the strata are identified.(level 2 only)

Unit C - *Chance*

Overview

In this unit, we discuss the basic ideas and methods of probability. Our goal is not just to help students answer questions like "What's the probability that you get no heads if you toss a fair coin 5 times?" We aim to show students the role that probability plays in statistical inference. Contrast the previous question with this one: "Suppose you toss a coin five times and get no heads. Is the coin fair?" That's a statistics question, but you need to understand probability to answer it.

Probability is about much more than coins, dice, and cards. It's about making decisions in the face of uncertainty. People use probability to assess the results of drug tests, to determine the strength of certain kinds of evidence in a court case, to set insurance premiums, to choose an investment strategy, and to weigh the risks and benefits of medical treatment options. Of course, probability also plays an integral role in games of chance, from state and national lotteries to casino favorites like slot machines, craps, roulette, and Texas Hold 'Em. In Unit C, we try to strike a balance between applications involving games of chance (which motivated the study of probability in the first place) and interesting uses of probability in everyday life.

21st Century Capacities: Analyzing

Stage 1 - Desired Results

<table>
<tr><td rowspan="4">ESTABLISHED GOALS/ STANDARDS

MP 1 Make sense sense of problems and persevere in solving them
MP2 Reason abstractly and quantitatively
MP5 Use appropriate tools strategically
MP7 Look for and make use of structure
MP8 Look for and express regularity in repeated reasoning

Understand independence and conditional probability and use them to interpret data

CCSS.MATH.CONTENT.HSS.CP.A.1
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of</td><td colspan="2">Transfer:</td></tr>
<tr><td colspan="2">Students will be able to independently use their learning in new situations to...
1. Represent, interpret, and draw conclusions in numbers, data and objects. (Analyzing)
2. Demonstrate fluency with math facts, computation and concepts.
3. Justify reasoning using clear and appropriate mathematical language.</td></tr>
<tr><td colspan="2">Meaning:</td></tr>
<tr><td>UNDERSTANDINGS: Students will understand that:
1. Mathematicians identify relevant tools, strategies, relationships, and/or information in order to draw conclusions or make predictions.
2. Mathematicians apply the mathematics they know to solve problems occurring</td><td>ESSENTIAL QUESTIONS: Students will explore & address these recurring questions:
A. What role does probability play in statistical inference?
B. How can we make decisions in the face of uncertainty?</td></tr>
</table>

other events ("or," "and," "not"). CCSS.MATH.CONTENT.HSS.CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. CCSS.MATH.CONTENT.HSS.CP.A.3 Understand the conditional probability of *A* given *B* as *P*(*A* and*B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. CCSS.MATH.CONTENT.HSS.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* CCSS.MATH.CONTENT.HSS.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **Use the rules of probability to compute probabilities of compound events.** CCSS.MATH.CONTENT.HSS.CP.B.6 Find the conditional probability of *A* given *B* as the fraction	in everyday life. 3. Mathematicians create or use models to examine, describe, solve and/or make predictions.	
	Acquisition:	
	Students will know… 1. The basic rules of probability (p331) 2. The general addition rule for two events 3. The notation for complement, intersection and union of events 4. The notation for conditional probability 5. Two mutually exclusive events can never be independent 6. The general multiplication rule 7. How to find a conditional probability 8. The multiplication rule for independent events 9. The law of large numbers 10. Properties of the Normal distribution 11. Multiplication counting principle 12. The four conditions for a binomial setting (Binary? Independent? Numbers? Success?) 13. Vocabulary: probability, random, independent, probability model, sample space, event, complement, intersections, union, mutually exclusive, disjoint, conditional probability, independent,random variable, probability distribution, sampling distribution, factorial,binary, binomial distribution	*Students will be skilled at…* 14. Using random digits from a table, calculator, or computer software to imitate chance behavior 15. Designing and using a simulation 16. Reading, finding probabilities from, and creating a two way table, Venn diagram, tree diagram 17. Reading and creating a probability distribution 18. Finding the expected value of a random variable 19. Finding expected values by simulation 20. Finding permutations and combinations 21. Using the Binomial theorem

of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. CCSS.MATH.CONTENT.HSS.CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. CCSS.MATH.CONTENT.HSS.CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. CCSS.MATH.CONTENT.HSS.CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems. **Calculate expected values and use them to solve problems** CCSS.MATH.CONTENT.HSS.MD.A.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. CCSS.MATH.CONTENT.HSS.MD.A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. CCSS.MATH.CONTENT.HSS.MD.A.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* CCSS.MATH.CONTENT.HSS.MD.A.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are		

assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **Use probability to evaluate outcomes of decisions** CCSS.MATH.CONTENT.HSS.MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. CCSS.MATH.CONTENT.HSS.MD.B.5.A Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* CCSS.MATH.CONTENT.HSS.MD.B.5.B Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* CCSS.MATH.CONTENT.HSS.MD.B.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).		

Unit D - *Inference*

Overview

Unit D deals with the reasoning of statistical inference. It presents methods for estimating and testing claims about a population proportion. Discussion about confidence intervals builds on foundations laid in previous learning about normal distributions and sampling. Unit D includes the course capstone PBA, Inference wherein students investigate a claim using the rules of hypothesis testing. Extensions are made to testing for an association between two categorical variables, and estimating and testing claims about a population mean.

21st Century Capacities: Analyzing, Synthesizing

Stage 1 - Desired Results

<table>
<tr><td rowspan="4">ESTABLISHED GOALS/
STANDARDS

MP 1 Make sense sense of problems and persevere in solving them
MP3 Construct viable arguments and critique the reasoning of others
MP5 Use appropriate tools strategically
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CCSS.MATH.CONTENT.HSS.
MD.B.7
(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</td><td colspan="2">Transfer:</td></tr>
<tr><td colspan="2">Students will be able to independently use their learning in new situations to...

1. Model relationships among quantities. (Analyzing)
2. Make sense of a problem, initiate a plan, execute it, and evaluate the reasonableness of the solution. (Analyzing)
3. Apply familiar mathematical concepts to a new problem or apply a new concept to rework a familiar problem. (Synthesizing)
4. Justify reasoning using clear and appropriate mathematical language.</td></tr>
<tr><td colspan="2">Meaning:</td></tr>
<tr><td>UNDERSTANDINGS: Students will understand that:

1. Mathematicians identify relevant tools, strategies, relationships, and/or information in order to draw conclusions.
2. Mathematicians create or use models to examine, describe, solve and/or make predictions.
3. Mathematicians select and use appropriate statistical methods and tools to analyze data, show trends,evaluate inference and/or describe or make predictions.
4. Mathematicians analyze data to evaluate inferences, make predictions and/or communicate a decision.</td><td>ESSENTIAL QUESTIONS: Students will explore & address these recurring questions:

A. What beliefs are worth testing scientifically?
B. As consumers of information, how do we question the information we are presented as facts?
C. How is inference used to shape our world?</td></tr>
</table>

	Acquisition:	
	Students will know… 1. Statistical inference draws conclusions about a population based on data from a sample 2. Parameters describe the population 3. Statistics estimate the parameter based on a sample 4. If the sample is large enough: the sampling distribution of p is approximately normal, the mean of the sampling distribution is p, the formula for standard deviation of a sampling distribution 5. As we increase the number of samples, the distribution becomes normal, and 95% of samples approach the true parameter. 6. Significance tests assess the evidence provided by data about some claim concerning a population. 7. Significance tests try to prove that a claim is so unlikely to be true that it must not be true. 8. The smaller the P-value is, the stronger is the evidence against H0 provided the data. 9. If the P-value is smaller than a (alpha), we say that the data are statistically significant. 10. Significant does not mean important, it means not likely to happen by chance. 11. A significance test answers only 1 question: How strong is the evidence that the null hypothesis is not true? 12. The P-value of a significance test depends strongly on the size of the sample, as well as on the truth about the population. 13. There is no sharp border between "significant" and "insignificant," only increasingly strong evidence as the P-value decreases. 14. 95% confidence interval is guaranteed to capture the true population parameter in 95% of all samples 15. The sampling distribution of a statistic is the distribution of values taken by the statistic in all possible samples of the same size from the same population 16. Confidence interval for a parameter has two parts: an interval calculated from the data and a confidence level C which gives the probability that the interval will capture the true parameter value in repeated samples.	*Students will be skilled at…* 1. Identifying topics that can be tested with inference 2. Creating null and alternate hypotheses 3. Creating a random sample 4. Calculating a sample parameter 5. Calculating a standard deviation 6. Drawing P-values on a normal curve 7. Stating a 95% confidence interval as a confidence statement 8. Calculating and interpreting P-values at various degrees of confidence 9. Making a statement about the outcomes of hypothesis tests in the context of an experiment

	17. To create survey results with a desired margin of error, like ± 3 pts for gallup polls, solve for the margin of error algebraically. 18. The null hypothesis (H0 pronounced h-nought), is the claim which you will try to disprove 19. The alternate hypothesis (HA), is the claim you will support by disproving H0. 20. The probability, computed assuming that H0 is true, that the sample outcome would be as extreme or more extreme than the actually observed outcome is called the P-value of the test. 21. Good data is the foundation of good research 22. Questionable data creates weakness in any inference 23. Randomness of data sources must be addressed 24. Bias must be eliminated from samples and surveys 25. Confidence intervals estimate the unknown value of a parameter and also tell us how uncertain the estimate is, and * Even 99% confidence leaves 1% room for a false negative * High confidence comes at the cost of a wider confidence interval * If you want a higher confidence level and lower interval range, increase sample size. 26. Our methods require that the proportion (p) is significantly larger than the sample and that the sample is large enough so that the distribution of the sample proportion is close to normal 27. Always report a P-value with the sample size and statistics about the sample outcome. 28. Vocabulary: statistical inference, confidence interval, confidence level, sampling distribution, critical values, significance test, P-value, null hypothesis, alternative hypothesis, statistically significance at the 5% level	