

BLACKLINE MASTERS

HOUGHTON MIFFLIN HARCOURT

Response to Intervention

FOR THE COMMON CORE STATE STANDARDS FOR MATHEMATICS



TIER 1 LESSONS

GRADE 6

BLACKLINE MASTERS

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**PROVIDES Tier 1 Intervention for
Every Common Core Standard**

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HOUGHTON MIFFLIN HARCOURT

Statistics and Probability

► **Develop understanding of statistical variability.**

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► **Summarize and describe distributions.**

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LESSON
89

Recognize Statistical Questions

OBJECTIVE Recognize statistical questions.

A **statistical question** is a question about a set of **data** that can vary. To answer a statistical question, you need to collect or look at a set of data.

Identify the statistical questions about Jack’s homework time.

- A.** How many times did Jack spend longer than an hour on homework this week?
Statistical question. Jack is unlikely to do homework for the same amount of time each day, so the question asks about a set of data that can vary. You could answer it with data about Jack’s homework time for a week.
- B.** How long did Jack do homework today?
Not a statistical question. It asks about Jack’s homework time on one day. It does not refer to a set of data that varies.

Write a statistical question about your school’s cafeteria.

Think of what kind of data could vary in the situation. In this situation, it might be menu items, students, or activities.

These are both statistical questions:

- A.** How many students were in the cafeteria during fourth period each day for the past two weeks?
- B.** What was the greatest number of entrees served in one day in the cafeteria last month?

Identify the statistical question. Circle the letter of the question.

- 1.** **A.** How many people flew from New York to San Francisco yesterday?
B. How many people flew from New York to San Francisco each day this month?
- 2.** **A.** How many siblings does each of your classmates have?
B. How many siblings does your best friend have?

Write a statistical question you could ask in the situation.

- 3.** Hannah recorded the temperature in her yard every day for a week.

- 4.** Ian knows his scores for each time he has bowled this year.



Recognize Statistical Questions

Identify the statistical question. Explain your reasoning.

1. **A.** How many touchdowns did the quarterback throw during the last game of the season?

B. How many touchdowns did the quarterback throw each game of the season?

B; the number of touchdowns in each game can vary.

2. **A.** What was the score in the first frame of a bowling game?

B. What are the scores in 10 frames of a bowling game?

3. **A.** How many hours of television did you watch each day this week?

B. How many hours of television did you watch on Saturday?

Write a statistical question you could ask in the situation.

4. A teacher recorded the test scores of her students.

5. A car salesman knows how many of each model of a car was sold in a month.

Problem Solving

6. The city tracked the amount of waste that was recycled from 2000 to 2007. Write a statistical question about the situation.

7. The daily low temperature is recorded for a week. Write a statistical question about the situation.

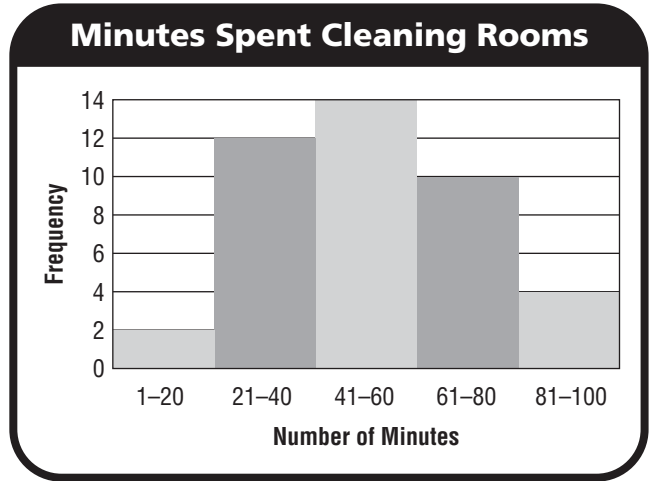
LESSON 90

Describe Distributions

OBJECTIVE Describe the distribution of a data set collected to answer a statistical question.

When interpreting data, it helps to make a graph and then analyze the distribution of data.

Mr. Chen asked all of his students how long it takes them to clean their rooms. He displayed the information in a histogram. Describe the data distribution.



Step 1
Look for clusters.

There are no groups of data that are separated from the rest, so there are no clusters of data.

Step 2
Look for gaps.

There are no intervals that contain no data, so there are no gaps in the data.

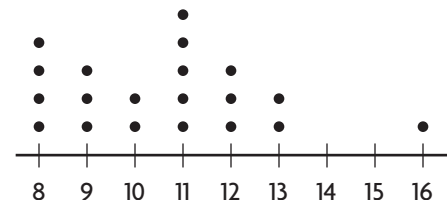
Step 3
Look for peaks.

There is one peak, at the interval 41–60.

Step 4
Look for symmetry.

Imagine folding the graph in half vertically, along the interval 41–60. The halves are not identical, but they are close. The graph has symmetry.

1. Sally has a restaurant. She recorded the cost of each person's dinner on Friday. Describe the distribution.



Cost (in dollars) of Dinners Ordered Friday

Describe Distributions

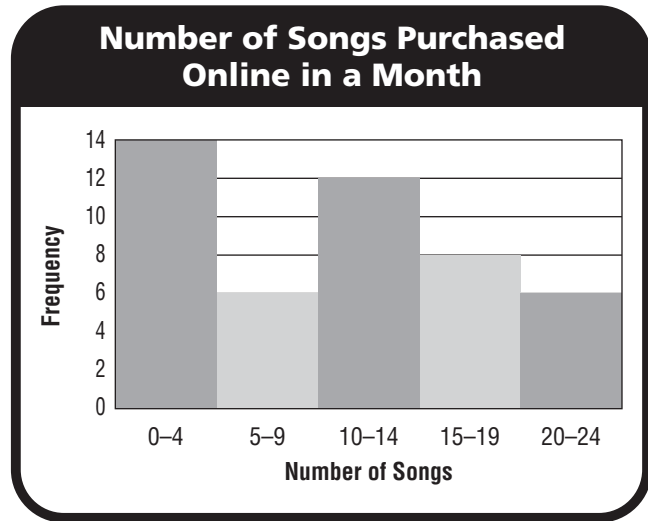
Chase asked people how many songs they have bought online in the past month. Use the histogram of the data he collected for 1–6.

1. What statistical question could Chase ask about the data?

What is the median number of
songs purchased?

2. Describe any peaks in the data.

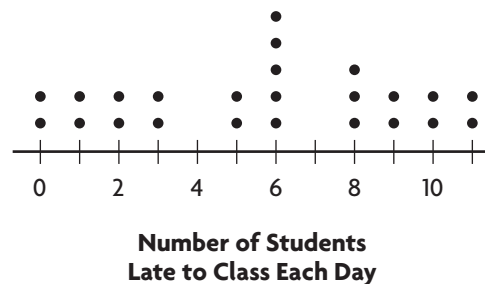
3. Describe any gaps in the data.



4. Does the graph have symmetry? Explain your reasoning.

Problem Solving

5. Mr. Carpenter teaches five classes each day. For several days in a row, he kept track of the number of students who were late to class and displayed the results in a dot plot. Describe the data.





LESSON 91

Problem Solving • Misleading Statistics

OBJECTIVE Draw conclusions about the distribution of a data set by using the strategy *work backward*.

Zaire wants to move to a town where the annual snowfall is no more than 5 inches. A real estate agent tells her that the mean annual snowfall in a certain town is 4.5 inches. Other statistics about the town are given in the table. Does this location match what Zaire wants? Why or why not?

Town Statistics for Annual Snowfall (in.)	
Minimum	0.5
Maximum	12
Median	8
Mean	4.5

Read the Problem		
<p>What do I need to find? I need to decide if the annual snowfall in the town is <u>typically less than 5 inches</u>.</p>	<p>What information do I need to use? I need the <u>statistics</u> in the table.</p>	<p>How will I use the information? I will work backward from the statistics to draw conclusions about the <u>annual snowfall</u>.</p>
Solve the Problem		
<p>The minimum annual snowfall is <u>0.5 in.</u> The maximum annual snowfall is <u>12 in.</u> The median annual snowfall is <u>8 in.</u> The mean annual snowfall is <u>4.5 in.</u></p>	<p>Think: The median is <u>8 in.</u>, which means that half of the data is equal to or greater than <u>8 in.</u></p>	
<p>So, the annual snowfall is usually <u>greater</u> than 5 inches because at least half of the annual snowfall values are <u>greater</u> than 5 inches. This location does not match what Zaire wants.</p>		

- Mack says he typically spends 4 hours per week practicing his piano. For the past 6 weeks, he has practiced for 1, 1, 1, 2, 10, and 9 hours. Do you agree with Mack? Explain.



Problem Solving • Misleading Statistics

Mr. Jackson wants to make dinner reservations at a restaurant that has most meals costing less than \$16. The Waterside Inn advertises that they have meals that average \$15. The table shows the menu items.

Menu Items	
Meal	Price
Potato Soup	\$6
Chicken	\$16
Steak	\$18
Pasta	\$16
Shrimp	\$18
Crab Cake	\$19

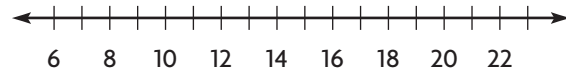
1. What is the minimum price and maximum price?

min = **\$6**

max = **\$19**

2. What is the mean of the prices?

3. Construct a box plot for the data.



4. What is the range of the prices?

5. What is the interquartile range of the prices?

6. What is the median of the prices?

7. Does the menu match Mr. Jackson's requirements? Explain your reasoning.

LESSON
92

Apply Measures of Center and Variability

OBJECTIVE Recognize what measures of center and variability indicate about a data set.

You can use measures of center and variability to compare sets of data.

Two math groups were given the same test.

Test Scores		
	Mean	Interquartile range
Group A	76.9	30
Group B	81.1	8

Compare the data.

Step 1 Compare the means.

Group B's scores are higher on average than Group A's scores because it has a greater mean.

Step 2 Compare the interquartile ranges.

Group B has a smaller interquartile range, which means their scores do not vary as much as Group A's scores.

Compare the data.

1.

Bowling Scores		
	Median	Range
Team X	66	11
Team Y	70	19

2.

Cantaloupes Weights in Pounds		
	Mean	Range
Farm 1	4	1.5
Farm 2	7	3



Apply Measures of Center and Variability

Solve.

1. The table shows temperature data for two cities. Use the information in the table to compare the data.

Daily High Temperatures (°F)		
	Mean	Interquartile Range
City 1	60	7
City 2	70	15

The mean of City 1's temperatures is less than the mean of City 2's temperatures.

The interquartile range of City 1's temperatures is less than the interquartile range of City 2's temperatures.

So, City 2 is typically warmer than City 1, but City 2's temperatures vary more than City 1's temperatures.

2. The table shows weights of fish that were caught in two different lakes. Find the median and range of each data set, and use these measures to compare the data.

Fish Weight (pounds)
Lake A: 7, 9, 10, 4, 6, 12
Lake B: 6, 7, 4, 5, 6, 4

Problem Solving

3. Mrs. Mack measured the heights of her students in two classes. Class 1 has a median height of 130 cm and an interquartile range of 5 cm. Class 2 has a median height of 134 cm and an interquartile range of 8 cm. Write a statement that compares the data.
4. Richard's science test scores are 76, 80, 78, 84, and 80. His math test scores are 100, 80, 73, 94, and 71. Compare the medians and interquartile ranges.

LESSON
93

Dot Plots and Frequency Tables

OBJECTIVE Display data in dot plots and frequency tables.

A dot plot displays data by placing dots above a number line. Each dot represents one data value.

Paloma sells produce at the farmers' market. The chart shows the number of pounds she sells each day. What was the most common number of pounds that Paloma sold?

Step 1 Draw a number line with an appropriate scale. The chart contains numbers from 11 to 20, so use a scale from 10 to 20.

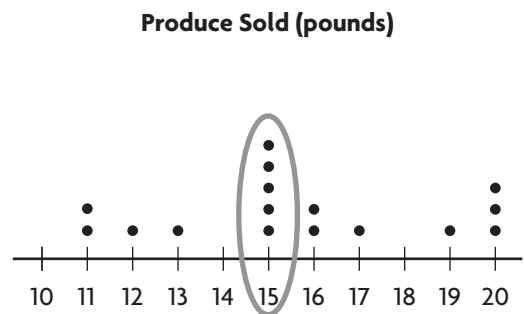
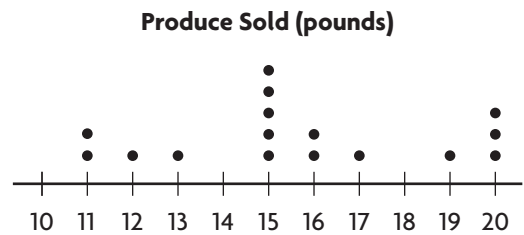
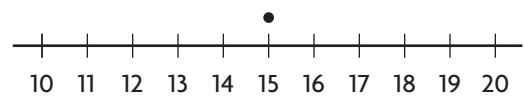
Step 2 For each data value in the chart, plot a dot above the number on the number line. The first data value in the chart is 15, so the dot is placed above 15 on the number line.

Complete the dot plot for the other values in the table. Since there are 16 data values, there should be 16 dots in all.

Step 3 The number of pounds Paloma sells most often is the value with the most dots. The stack with the most dots is at 15 pounds.

So, Paloma most often sells 15 pounds of produce.

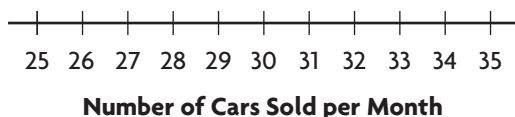
Produce Sold (pounds)			
15	19	15	16
20	16	17	20
11	12	15	20
15	13	11	15



Use the data in the chart at right.

1. Complete the dot plot.

Number of Cars Sold per Month					
26	32	35	29	30	26
25	29	28	31	29	26
35	26	26	28	26	30



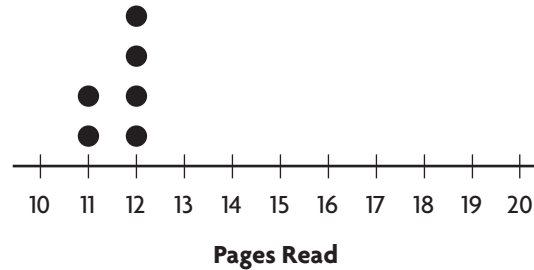
2. What is the most common number of cars sold per month?

Dot Plots and Frequency Tables

For 1–4, use the chart.

- The chart shows the number of pages of a novel that Julia reads each day. Complete the dot plot using the data in the table.

Pages Read				
12	14	12	18	20
15	15	19	12	15
14	11	13	18	15
15	17	12	11	15



- What number of pages does Julia read most often? Explain.

- Make a frequency table in the space below. Use the intervals 10–13, 14–17, and 18–21.

- Make a relative frequency table in the space below.

Problem Solving

- The frequency table shows the ages of the actors in a youth theater group. What percent of the actors are 10 to 12 years old?

Actors in a Youth Theater Group	
Age	Frequency
7–9	8
10–12	22
13–15	10

LESSON
94

Histograms

OBJECTIVE Display data in histograms.

A **histogram** looks like a bar graph without spaces between bars. When you have data to organize, it is helpful to group the data into intervals and let each bar show the frequency, or number of data, in that interval.

Complete the frequency table below, using the data to the right. Then make a histogram.

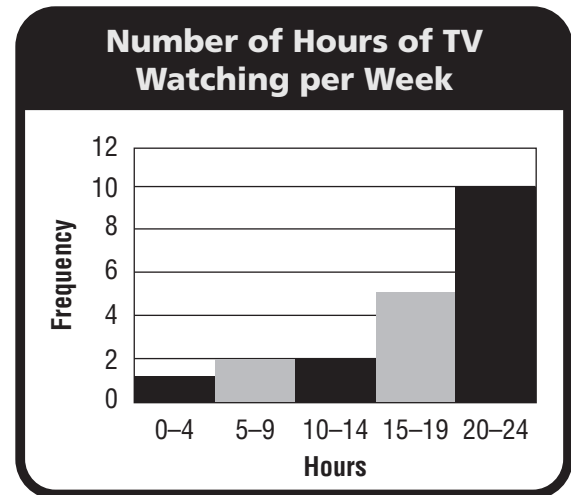
- Step 1** Sort the data into each interval.
 Only the 4 (1 item) is in the interval 1–4.
 8 and 5 (2 items) are in 5–9.
 10 and 14 (2 items) are in 10–14.
 17, 15, 19, 18, 19 (5 items) are in 15–19.
 24, 21, 21, 20, 23, 22, 24, 20, 22, 24
 (10 items) are in 20–24.

Number of Hours of TV Watching per Week				
4	14	24	17	10
21	21	15	20	23
5	22	19	18	8
24	19	20	22	24

Hours of TV/week	1–4	5–9	10–14	15–19	20–24
Frequency	1	2	2	5	10

- Step 2** Check that all 20 items in the table are in the frequency table by adding.
 $1 + 2 + 2 + 5 + 10 = 20$

- Step 3** Make the histogram of the data.
 Use a vertical scale from 0 to 12.
 Title and label the histogram.
 Draw a bar for each interval.
 Draw bars the same width.
 Draw the bar as high as the frequency.



For 1–2, use the table shown.

Minutes on Treadmill Each Day				
28	28	24	52	35
43	29	34	55	21
38	60	71	59	62
19	64	39	70	55

1. Complete the frequency table of the data.

Number of Minutes	0–19	20–39	40–59	60–79
Frequency				

2. Make a histogram of the data.



Histograms

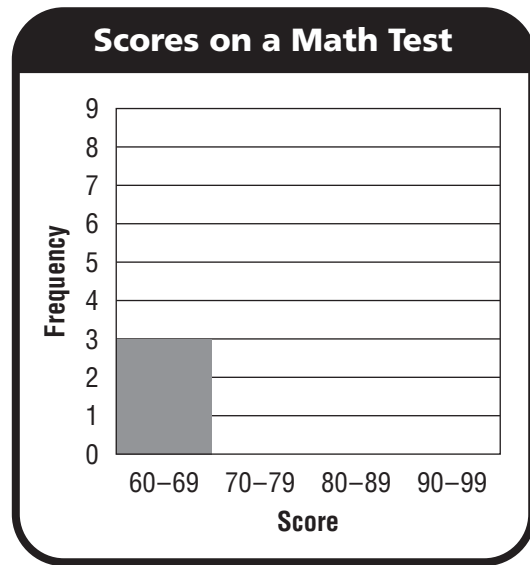
For 1–4 use the data at right.

- Complete the histogram for the data.
- What do the numbers on the y -axis represent?

Scores on a Math Test									
85	87	69	90	82	75	74	76	84	87
99	65	75	76	83	87	91	83	92	69

- How many students scored from 60 to 69?

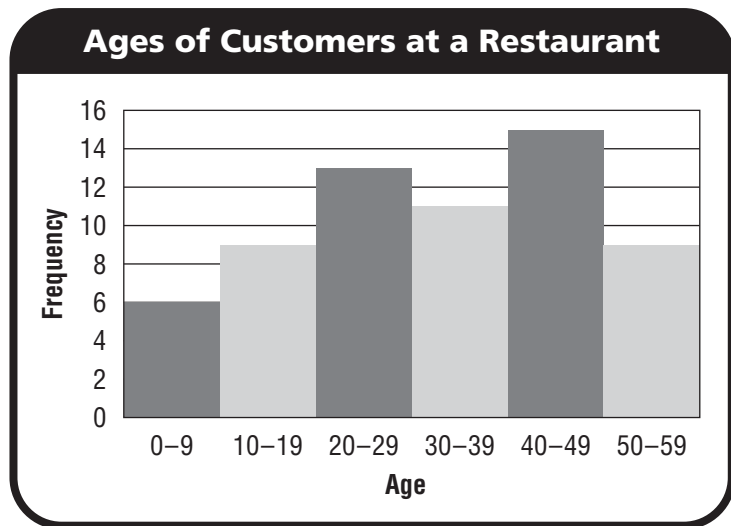
- Use your histogram to find the number of students who got a score of 80 or greater. Explain.



Problem Solving

For 5–6, use the histogram.

- For which two age groups are there the same number of customers?
- How many customers are in the restaurant? How do you know?





LESSON
95

Problem Solving • Data Displays

OBJECTIVE Solve problems involving data by using the strategy *draw a diagram*.

The table shows the highest state populations in 2007, rounded to the nearest million. What percent of the states had at least 15 million residents?

2007 State Populations (in millions)					
18	10	6	9	6	9
6	37	13	12	6	11
24	8	6	6	19	6
10	6				

Read the Problem

What do I need to find?

I need to find the

percent of states

that had at least 15 million people.

What information do I need to use?

I will use the state

population data.

How will I use the information?

I will pick intervals for the data, find the frequency for each interval and use the frequencies to make a

histogram. I will use the information from the histogram to find a percent.

Solve the Problem

Make a frequency table.

Millions	5–9	10–14	15–19	20–24	25–29	30–34	35–40
Frequency	11	5	2	1	0	0	1

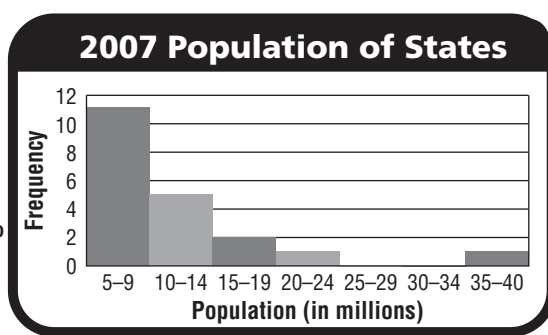
Use the frequency table to make a histogram.

States with at least 15 million: $2 + \underline{1} + \underline{1} = \underline{4}$

Total states: 20

Percent with at least 15 million: $\frac{\underline{4}}{\underline{20}} = \underline{0.20} = \underline{20}\%$

So, 20% of the states have populations over 15 million.



Use the data in the histogram above.

1. What percent of the states had between 5 million and 14 million residents?

States with 5–14 million: _____

Percent with 5–14 million: _____ %

2. What percent of the states had less than 10 million residents?

States with less than 10 million: _____

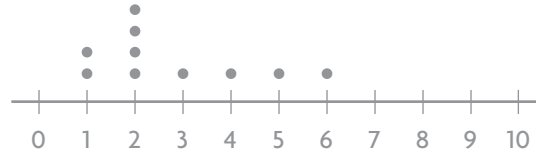
Percent with less than 10 million: _____ %



Problem Solving • Data Displays

Read each problem and solve.

1. Josie collected data on the number of siblings her classmates have. Make a data display and determine the percent of Josie’s classmates that have more than 2 siblings.



5, 1, 2, 1, 2, 4, 3, 2, 2, 6

40%

2. The following data show the number of field goals a kicker attempted each game. Make a data display and tell which number of field goals is the mode.

4, 6, 2, 1, 3, 1, 2, 1, 5, 2, 2, 3

3. The math exam scores for a class are shown below. Make a data display. What percent of the scores are 90 and greater?

91, 68, 83, 75, 81, 99, 97, 80, 85, 70, 89, 92, 77, 95,
100, 64, 88, 96, 76, 88

4. The heights of students in a class are shown below in inches. Make a data display. What percent of the students are taller than 62 inches?

63, 57, 60, 64, 59, 62, 65, 58, 63, 65, 58, 61, 63, 64

5. The ages of employees are shown below. Which age is the mode?

21, 18, 17, 19, 18, 23, 18, 16, 22, 18, 21, 18

LESSON
96

Box Plots

OBJECTIVE Display data in box plots.

The weights in ounces of 12 kittens are 20, 18, 22, 15, 17, 25, 25, 23, 13, 18, 16, and 22. A **box plot** for the data would show how the values are spread out.

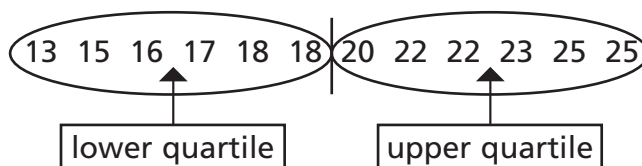
Make a box plot for the data.

Step 1 Write the numbers in order from least to greatest. Find the median and the least and greatest values.

13 15 16 17 18 **18 20** 22 22 23 25 25
 Since there is an even number of values, the median is the mean of the two middle values. The median is 19. The least value is 13, and the greatest value is 25.

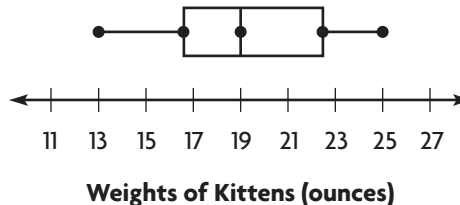
Step 2 Find the lower and upper quartiles.
 The **lower quartile** is the median of the lower half of the data.

The **upper quartile** is the median of the upper half of the data.



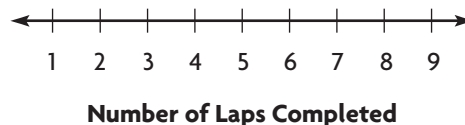
Draw a line where the median should be. Now the data set has been split into halves. (If there were an odd number of values in the data set, the median would be one of the data values, but you would not include it in the upper or lower half.) The lower quartile is 16.5, and the upper quartile is 22.5.

Step 3 Plot the five points on a number line, and construct the box and whiskers. Use an appropriate scale.



The numbers of laps completed on a track are 4, 5, 2, 7, 6, 8, 9, 8, and 6. Use the data for 1–4.

1. What is the median? _____
2. What is the lower quartile? _____
3. What is the upper quartile? _____
4. Make a box plot for the data.





Box Plots

Find the median, lower quartile, and upper quartile of the data.

1. the amounts of juice in 12 glasses, in fluid ounces:

11, 8, 4, 9, 12, 14, 9, 16, 15, 11, 10, 7

Order the data from least to greatest: **4, 7, 8, 9, 9, 10, 11, 11, 12, 14, 15, 16**

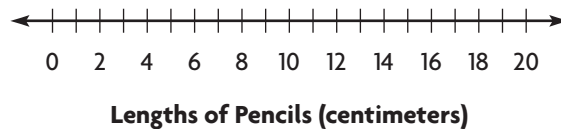
median: 10.5 lower quartile: 8.5 upper quartile: 13

2. the lengths of 10 pencils, in centimeters:

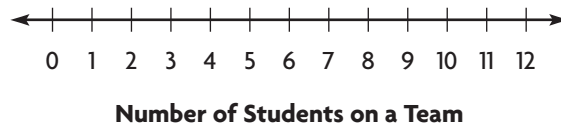
18, 15, 4, 9, 14, 17, 16, 6, 8, 10

median: _____ lower quartile: _____ upper quartile: _____

3. Make a box plot to display the data set in Exercise 2.



4. The numbers of students on several teams are 9, 4, 5, 10, 11, 9, 8, and 6. Make a box plot for the data.



Problem Solving

5. The amounts spent at a gift shop today are \$19, \$30, \$28, \$22, \$20, \$26, and \$26. What is the median? What is the lower quartile?
- _____

6. The weights of six puppies in ounces are 8, 5, 7, 5, 6, and 9. What is the upper quartile of the data?
- _____

LESSON
97

Describe Data Collection

OBJECTIVE Describe a data set by stating what quantity was measured and how it was measured.

To describe a set of data, describe these features:

Attribute: the characteristic being recorded or measured

Unit: the unit of measurement, such as inches or grams

Means: the tool used for the observations or measurements

Observations: the number of observations or measurements

Describe the data set shown in the chart.

Step 1 What attribute is measured?

The attribute is *length of time* spent walking a dog.

Step 2 What unit of measurement is used?

The time is shown in *minutes*.

Step 3 What means was likely used to obtain the measurements?

To measure time, you use a *clock, timer, or stopwatch*.

Step 4 How many observations were made?

Count the number of observations: 8

Daily Dog Walks			
Day	Time (min)	Day	Time (min)
1	35	5	60
2	40	6	25
3	25	7	90
4	55	8	20

Describe the data set by listing the attribute measured, the unit of measure, the likely means of measurement, and the number of observations.

1. Attribute: _____

Unit of measurement: _____

Means: _____

Number of observations: _____

Pet Weights (lb)				
5.2	8	9.5	48.4	0.9
4.7	10.5	32	18	12

2. Attribute: _____

Unit of measurement: _____

Means: _____

Number of observations: _____

Serving Volume (cups)			
Lettuce	2	Soup	1.5
Cheese	0.25	Ice Cream	0.75
Sauce	0.5		

Describe Data Collection

Describe the data set by listing the attribute measured, the unit of measure, the likely means of measurement, and the number of observations.

1. Daily temperature

Daily High Temperature (°F)				
78	83	72	65	70
76	75	71	80	75
73	74	81	79	69
81	78	76	80	82
70	77	74	71	73

Attribute: daily temperature; _____

unit of measure: degrees _____

Fahrenheit; means of measurement: _____

thermometer; number of _____

observations: 25 _____

2. Plant heights

Height of Plants (inches)				
10.3	9.7	6.4	8.1	11.2
5.7	11.7	7.5	9.6	6.9

3. Cereal in boxes

Amount of Cereal in Boxes (cups)							
8	7	8.5	5	5	5	6.5	6
8	8.5	7	7	9	8	8	9

4. Dog weights

Weight of Dogs (pounds)							
22	17	34	23	19	18	20	20

Problem Solving

5. The table below gives the amount of time Preston spends on homework. Name the likely means of measurement.

Amount of Time Spent on Homework (hours)							
5	3	1	2	4	1	3	2

6. The table below shows the speed of cars on a highway. Name the unit of measure.

Speeds of Cars (miles per hour)							
71	55	53	65	68	61	59	62
70	69	57	50	56	66	67	63

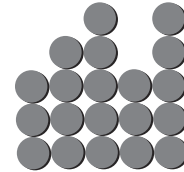
LESSON
98

Mean as Fair Share and Balance Point

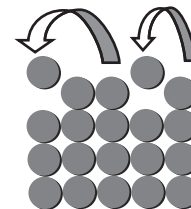
OBJECTIVE Understand the mean as a fair share and as a balance point.

Five students brought 3, 4, 5, 3, and 5 cups of flour to the cooking club. They divided it evenly so that each student got the same amount for cooking. Use counters to show how many cups each student got.

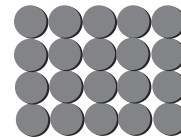
Step 1 Make 5 stacks of counters: one stack for each student.
Use one counter for each cup of flour.



Step 2 Take counters from taller stacks and put them on shorter stacks. Move counters until all the stacks are the same height.



Step 3 Count the counters in each stack. There are 4 counters in each stack.

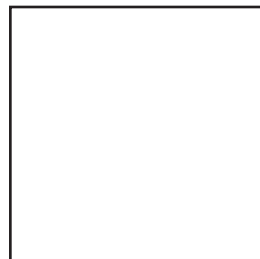


So, 4 is the mean of the data. When you divide the flour equally, each student gets 4 cups.

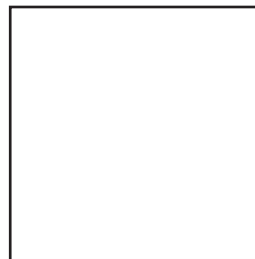
Use counters to find the mean of the data set.

1. 3, 5, 7, 5

Draw 4 stacks to show the data set.



Make the stacks the same height.

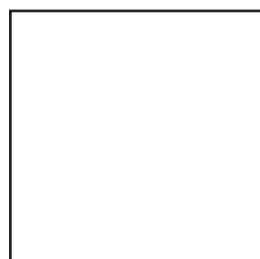


_____ counters in each stack.

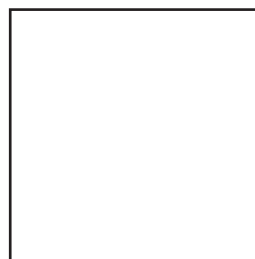
Mean: _____

2. 5, 7, 4, 3, 4, 1

Draw 6 stacks to show the data set.



Make the stacks the same height.



_____ counters in each stack.

Mean: _____



Mean as Fair Share and Balance Point

Use counters to find the mean of the data set.

- Six students count the number of buttons on their shirts. The students have 0, 4, 5, 2, 3, and 4 buttons.

Make 6 stacks of counters with heights 0, 4, 5, 2, 3, and 4.

Rearrange the counters so that all 6 stacks have the same height.

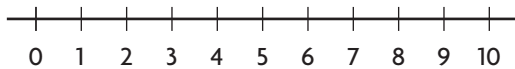
After rearranging, every stack has 3 counters.

So, the mean of the data set is 3.

- Four students completed 1, 2, 2, and 3 chin-ups. _____

Make a dot plot for the data set and use it to check whether the given value is a balance point for the data set.

- Sandy's friends ate 0, 2, 3, 4, 6, 6, and 7 pretzels. Sandy says the mean of the data is 4. Is Sandy correct?



The total distance from 4 for values less than 4 is _____.

The total distance from 4 for

values greater than 4 is _____.

The mean of 4 _____ a balance

point. So, Sandy _____ correct.

Problem Solving

- Three baskets contain 8, 8, and 11 soaps. Can the soaps be rearranged so that there is an equal whole number of soaps in each basket? Explain why or why not.

- Five pages contain 6, 6, 9, 10, and 11 stickers. Can the stickers be rearranged so that there is an equal whole number of stickers on each page? Explain why or why not.



LESSON
99

Measures of Center

OBJECTIVE Summarize a data set by using mean, median, and mode.

A **measure of center** is a single value that describes the middle of a data set.

The **mean** is the sum of all items in a set of data divided by the number of items in the set.

The **median** is the middle number or the mean of the middle two numbers when the items in the data set are listed in order.

The **mode** is the data value that is repeated more than other values. A data set can have more than one mode, or no mode.

Find the mean, median, and mode for the set of data.

80, 74, 82, 77, 86, 75

Find the mean.

Step 1 Find the sum of the data.

$$80 + 74 + 82 + 77 + 86 + 75 = 474$$

Step 2 Count the number of data items.

There are 6 items.

Step 3 Divide.

$$\frac{\text{sum}}{\text{number of items}} = \frac{474}{6} = 79$$

So, the mean is 79.

Find the median.

Step 1 Order the data.

74, 75, 77, 80, 82, 86

Step 2 Find the middle number.

There are two middle numbers:
77 and 80.

Step 3 Find their mean.

$$\frac{77 + 80}{2} = 78.5$$

So, the median is 78.5.

Find the mode.

Use the ordered list and look for numbers that repeat.

No numbers repeat. So, there is no mode.

Find the mean, median, and mode.

1. 31, 3, 14, 31, 11

mean: _____ median: _____

mode: _____

2. 95, 18, 51, 1, 22, 5

mean: _____ median: _____

mode: _____

3. 14, 22, 15, 7, 14, 0, 12

mean: _____ median: _____

mode: _____

4. 67, 103, 94, 65, 18, 114, 94, 63, 94, 27

mean: _____ median: _____

mode: _____



Measures of Center

Use the table for 1–4.

1. What is the mean of the data?

$$\frac{10 + 8 + 11 + 12 + 6}{5} = \frac{47}{5} = 9.4$$

9.4 points

2. What is the median of the data?
-

3. What is the mode(s) of the data?
-

Number of Points Blaine Scored in Five Basketball Games	
Game	Points Scored
1	10
2	8
3	11
4	12
5	6

4. Suppose Blaine played a sixth game and scored 10 points during the game. Find the new mean, median, and mode.
-

Problem Solving

5. An auto manufacturer wants their line of cars to have a median gas mileage of 25 miles per gallon or higher. The gas mileage for their five models are 23, 25, 26, 29, and 19. Do their cars meet their goal? Explain.
-
-
-

6. A sporting goods store is featuring several new bicycles, priced at \$300, \$250, \$325, \$780, and \$350. They advertise that the average price of their bicycles is under \$400. Is their ad correct? Explain.
-
-
-

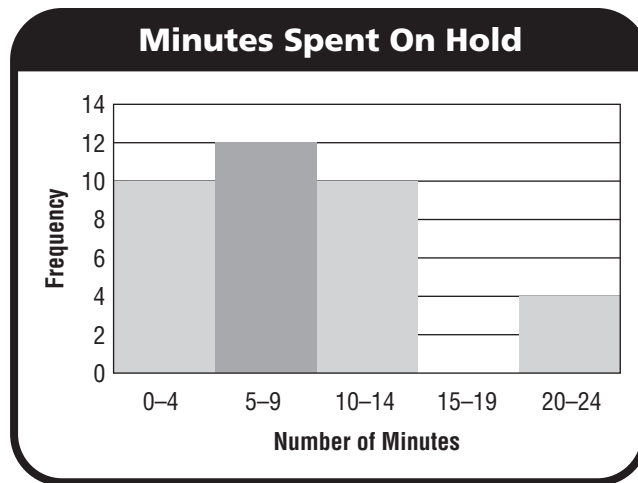
LESSON 100

Patterns in Data

OBJECTIVE Describe overall patterns in data, including clusters, peaks, gaps, and symmetry.

The histogram shows the number of minutes a caller had to be placed on hold before talking to a representative.

According to the graph, there were 10 people who were on hold for 0 to 4 minutes.



Does the graph contain any clusters or gaps? If so, where? Does the graph have symmetry?

Step 1 Look for a group of data points that lie within a small interval. These are clusters.

The bars for 0-4, 5-9, and 10-14 are in a group. This is a cluster of data.

Step 2 Look for an interval that contains no data. These are gaps.

There is no bar above the interval 15-19. This is a gap in the data. This means there were no people who were on hold for 15 to 19 minutes.

Step 3 Look for symmetry. If you draw a vertical line in the graph, the bars on the left and right sides will match if the graph has symmetry.

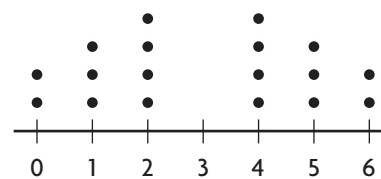
A line cannot be drawn anywhere on the graph and have the bars on either side match. There is no symmetry.

Use the dot plot to answer the questions.

1. Are there any clusters? If so, where?

2. Are there any gaps? If so, where?

3. Is there symmetry? If so, where can the line of symmetry be drawn?



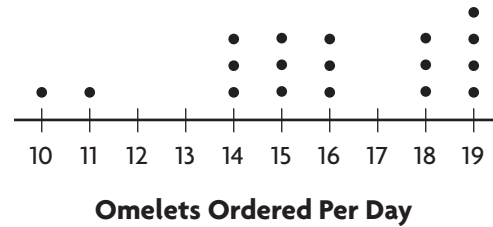
Number of Toppings on a Pizza

Patterns in Data

For 1–3, use the dot plot.

- The dot plot shows the number of omelets ordered at Paul’s Restaurant each day. Does the dot plot contain any gaps?

Yes; from 12 to 13, and at 17



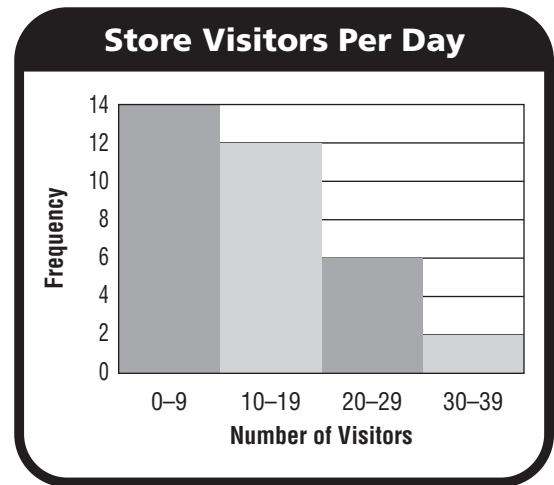
- Identify any clusters in the data.

- Summarize the information in the dot plot.

For 4–5, use the histogram.

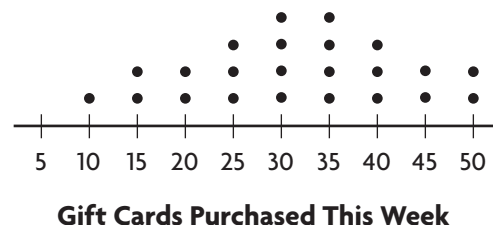
- The histogram shows the number of people that visited a local shop each day in January. How many peaks does the histogram have?

- Describe how the data values change across the intervals.



Problem Solving

- Look at the dot plot at the right. Does the graph have line symmetry? Explain.



LESSON 101

Mean Absolute Deviation

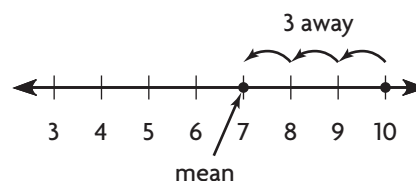
OBJECTIVE Understand mean absolute deviation as a measure of variability from the mean.

The **mean absolute deviation** tells how far away the data values are from the mean. A small mean absolute deviation means that most values are close to the mean. A large mean absolute deviation means that the data values are more spread out.

The prices of 8 lunches are \$10, \$8, \$3, \$5, \$9, \$6, \$7, and \$8.
The mean is \$7. Find the mean absolute deviation.

Step 1 Determine how far each data value is from the mean. You can use a number line.

Plot a value on the number line. Then count how many spaces you must move to reach the mean, 7.



Step 2 Make a list of all of the distances.

Data values: 10 8 3 5 9 6 7 8
Distance from mean: 3 1 4 2 2 1 0 1

Step 3 Find the mean of the distances by finding the sum and dividing by 8. The quotient is the mean absolute deviation.

$$\frac{3 + 1 + 4 + 2 + 2 + 1 + 0 + 1}{8} = \frac{14}{8} = 1.75$$

So, on average, each data value is 1.75 away from the mean.

Use counters or a number line to find the mean absolute deviation.

- 1.** ages of people on a team in years:
9, 12, 10, 8, 11
mean = 10 years

- 2.** Sam's test scores:
86, 71, 92, 84, 76, 95
mean = 84

distances from mean = _____
mean absolute deviation = _____

mean absolute deviation = _____

- 3.** prices of dinner menu items:
\$15, \$10, \$13, \$19, \$20, \$12, \$9, \$14
mean = \$14

- 4.** daily low temperatures, °F, in a city:
45, 39, 40, 52, 44
mean = 44°F

mean absolute deviation = _____

mean absolute deviation = _____



Mean Absolute Deviation

Use counters and a dot plot to find the mean absolute deviation of the data.

1. the number of hours Maggie spent practicing soccer for 4 different weeks:

9, 6, 6, 7

mean = 7 hours

$$\frac{2 + 1 + 1 + 0}{4} = \frac{4}{4} = 1$$

mean absolute deviation = 1 hour

2. the heights of 7 people in inches:

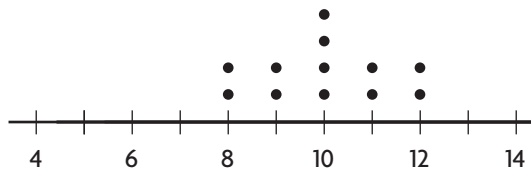
60, 64, 58, 60, 70, 71, 65

mean = 64 inches

mean absolute deviation = _____

Use the dot plot to find the mean absolute deviation of the data.

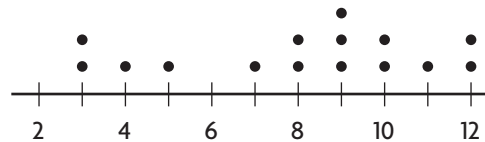
3. mean = 10



Ages of Students in Dance Class

mean absolute deviation = _____

4. mean = 8



Weekly Hours Spent Doing Homework

mean absolute deviation = _____

Problem Solving

5. In science class, Troy found the mass, in grams, of 6 samples to be 10, 12, 7, 8, 5, and 6. What is the mean absolute deviation?

6. Five recorded temperatures are 71°F, 64°F, 72°F, 81°F, and 67°F. What is the mean absolute deviation?

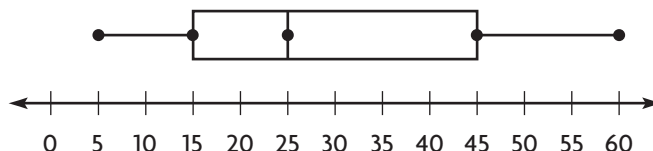
LESSON
102

Measures of Variability

OBJECTIVE Summarize a data set by using range, interquartile range, and mean absolute deviation.

A **measure of variability** is a single number that describes how far apart the numbers are in a data set. **Range, interquartile range,** and mean absolute deviation are all measures of variability.

The box plot shows the cost of various concert tickets. Find the range and interquartile range of the data in the box plot.



Step 1 To find the range, subtract the least value from the greatest value.

$$\begin{array}{rccccccc} 60 & - & 5 & = & 55 \\ \uparrow & & \uparrow & & \uparrow \\ \text{greatest} & & \text{least} & & \text{range} \end{array}$$

Step 2 To find the interquartile range, subtract the lower quartile from the upper quartile.

$$\begin{array}{rccccccc} 45 & - & 15 & = & 30 \\ \uparrow & & \uparrow & & \uparrow \\ \text{upper} & & \text{lower} & & \text{interquartile} \\ \text{quartile} & & \text{quartile} & & \text{range} \end{array}$$

Make a box plot for the data. Then find the range and interquartile range.

1. number of free throws made:
8, 13, 9, 4, 1, 6, 2, 2, 14, 6, 9, 11

range = _____

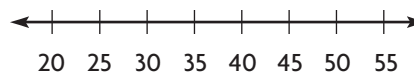
interquartile range = _____



2. minutes spent cooking dinner:
45, 38, 52, 29, 28, 31, 44, 40, 25

range = _____

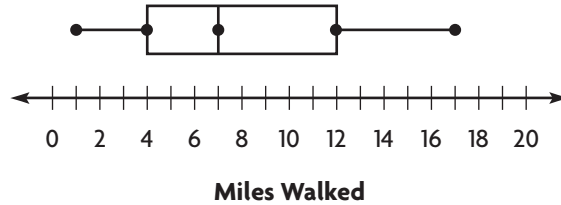
interquartile range = _____





Measures of Variability

1. Find the range and interquartile range of the data in the box plot.



For the range, find the difference between the greatest and least values.

$$\underline{17} - \underline{1} = \underline{16}$$

range: 16 miles

For the interquartile range, find the difference between the upper and lower quartiles.

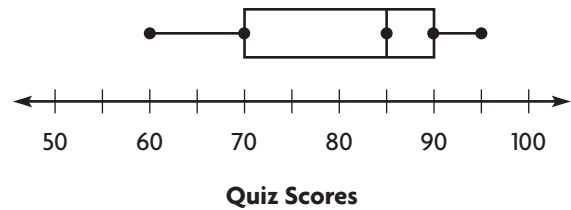
$$\underline{12} - \underline{4} = \underline{8}$$

interquartile range: 8 miles

Use the box plot for 2 and 3.

2. What is the range of the data?

3. What is the interquartile range of the data?



Find the mean absolute deviation for the set.

4. heights in centimeters of several flowers:

14, 7, 6, 5, 13

mean absolute deviation: _____

5. ages of several children:

5, 7, 4, 6, 3, 5, 3, 7

mean absolute deviation: _____

Problem Solving

6. The following data set gives the amount of time, in minutes, it took five people to cook a recipe. What is the mean absolute deviation for the data?

33, 38, 31, 36, 37

7. The prices of six food processors are \$63, \$59, \$72, \$68, \$61, and \$67. What is the mean absolute deviation for the data?

LESSON
103

Effects of Outliers

OBJECTIVE Determine the effects of outliers on measures of center and variability

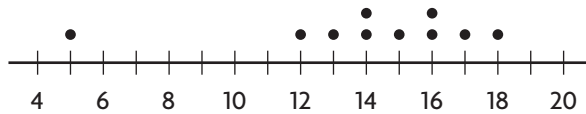
Sometimes a data set contains a number that is much less or much greater than the rest. This number is called an **outlier**. Taking note of outliers can help you understand a data set.

Use a dot plot to find the outlier for the quiz scores. Then tell how the outlier affects the mean and median.

Scores on 20-question Quiz				
15	16	17	13	18
12	5	14	14	16

Mean: 14 Median: 14.5

Step 1 Plot the data on the number line.



Step 2 Find the outlier.

Most of the points are between 12 and 18. 5 is much less than the rest, so it is an outlier.

Step 3 Find the median and mean without the outlier.

Median: Make an ordered list and find the middle value.

Mean: One value has been removed. Add the new list of values and divide by 9.

$$\frac{12 + 13 + 14 + 14 + 15 + 16 + 16 + 17 + 18}{9} = 15$$

12, 13, 14, 14, **15**, 16, 16, 17, 18

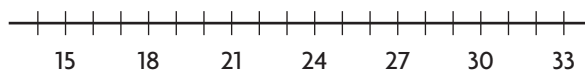
The new median is 15.

The new mean is 15.

Step 4 Describe the effect of the outlier. Without the outlier, the mean went up from 14 to 15. The median went up from 14.5 to 15.

Use the table for Problems 1–3.

1. Find the outlier by drawing a dot plot of the data.



Outlier: _____

Shirt Prices (\$)				
29	33	24	14	29
31	31	33		

Mean: \$28 Median: \$30

2. Find the mean and median without the outlier.

Median: \$ _____ Mean: \$ _____

3. Without the outlier, the mean _____.

The median _____.



Effects of Outliers

1. Identify the outlier in the data set of students in each class. Then describe the effect the outlier has on the mean and median.

Students in Each Class				
30	22	26	21	24
28	23	26	28	12

12; The outlier decreases the mean from about 25.3 to 24.

The outlier decreases the median from 26 to 25.

2. Identify the outlier in the data set of pledge amounts. Then describe the effect the outlier has on the mean and median.

Pledge Amounts			
\$100	\$10	\$15	\$20
\$17	\$24	\$32	\$36

3. In a set of points that Milton scored in basketball games, there is an outlier. Before one game, Milton injured his knee. Do you think the outlier is greater or less than the rest of the numbers of points? Explain.

Problem Solving

4. Duke's science quiz scores are 99, 91, 60, 94, and 95. Describe the effect of the outlier on the mean and median.

5. The number of people who attended an art conference for five days was 42, 27, 35, 39, and 96. Describe the effect of the outlier on the mean and median.



LESSON
104

Choose Appropriate Measures of Center and Variability

OBJECTIVE Choose appropriate measures of center and variability to describe data, and justify the choice.

Sometimes one measure of center or variability represents the data better than another measure of variability. For example, the median might be a better representation than the mean.

Cheeseburger prices at several different restaurants are \$5, \$3, \$2, \$6, \$4, and \$14. Should the mean, median, or mode be used to describe the data? Should the range or interquartile range be used?

Measure of Center	Measure of Variability
<p>Step 1 Find the mean, median, and mode.</p> <p>Mean: $\frac{5 + 3 + 2 + 6 + 4 + 14}{6} \approx \\5.67</p> <p>Median: 2 3 4 5 6 14 $\frac{4 + 5}{2} = \\$4.50$</p> <p>Mode = none</p>	<p>Step 1 Find the range and interquartile range.</p> <p>Range: $14 - 2 = \\$12$</p> <p>Interquartile range: $6 - 3 = \\$3$</p> <p>2 (3) 4 5 (6) 14</p>
<p>Step 2 Compare. There are six data values, and the mean is greater than four of them. The outlier of \$14 is causing this. So, the median is a better measure of center.</p>	<p>Step 2 Compare. All of the data values except one are between \$2 and \$6. The interquartile range is a better measure.</p>

- The times, in minutes, spent cleaning a room are 60, 50, 33, 28, and 44. Decide which measure(s) of center best describes the data set. Explain your reasoning.
- The amounts of snowfall, in inches, are 4, 3, 20, 6, 8, and 2. Decide which measure(s) of variability best describes the data set. Explain your reasoning.



Choose Appropriate Measures of Center and Variability

1. The distances, in miles, that 6 people travel to get to work are 14, 12, 2, 16, 16, and 18. Decide which measure(s) of center best describes the data set. Explain your reasoning.

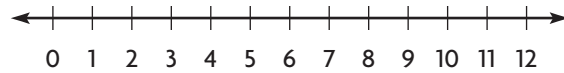
mean = 13 miles

median = 15 miles

mode = 16 miles

The _____ is less than 4 of the data points, and the _____ describes only 2 of the data points. So, the _____ best describes the data.

2. The numbers of pets that several children have are 2, 1, 2, 3, 4, 3, 10, 0, 1, and 0. Make a box plot of the data and find the range and interquartile range. Decide which measure better describes the data set and explain your reasoning.



range = _____

interquartile range = _____

Problem Solving

3. Brett's history quiz scores are 84, 78, 92, 90, 85, 91, and 0. Decide which measure(s) of center best describes the data set. Explain your reasoning.

mean = _____ median = _____

mode = _____

4. Eight students were absent the following number of days in a year: 4, 8, 0, 1, 7, 2, 6, and 3. Decide if the range or interquartile range better describes the data set, and explain your reasoning.

range = _____

interquartile range = _____



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